The effect of gravity hypothesis

Adrián G. Cornejo
Electronics and Communications Engineering from Universidad Iberoamericana, Calle Santa Rosa 719, C. P. 76138, Col. Santa Mónica, Querétaro, Querétaro, Mexico.

E-mail: adriang.cornejo@gmail.com

(Received 22 August 2011; accepted 28 November 2011)

Abstract

In this paper, we consider a hypothetical scenario where gravity results as an inertial effect of combine both, curvature of space-time and its hypothetical acceleration as a non-inertial frame of reference that follows the accelerated expansion of the universe. Thus, effect of gravity can be described by applying mainly classical mechanics concepts as an inertial effect due to the acceleration of space-time, where "field of gravity" concept is not involved. Then, we derive a general expression for accelerated curvature of space-time which, by applying some equivalent expressions, can be transformed to an approximated equivalent expression of the so-called "compact form of Einstein's field equation".

Keywords: Newtonian gravitation, Einstein's field equation, Equivalence principle, Non-inertial frame of reference.

Resumen

En este trabajo, consideramos un escenario hipotético donde la gravedad resulta como un efecto inercial de combinar la curvatura del espacio-tiempo y su hipotética aceleración como un sistema de referencia no-inercial que sigue la expansión acelerada del universo. Así, el efecto de gravedad puede ser descrito aplicando principalmente conceptos de la mecánica clásica como un efecto inercial debido a la aceleración del espacio-tiempo, donde el concepto de "campo de gravedad" no está involucrado. Entonces, derivamos una expresión general para la curvatura acelerada del espacio-tiempo, la cual aplicando algunas expresiones equivalentes puede ser transformada en una expresión equivalente aproximada de la llamada "forma compacta de la Ecuación de campo de Einstein".

Palabras clave: Gravitación Newtoniana, Ecuación de campo de Einstein, Principio de equivalencia, Marco de referencia no-inercial.

PACS: 04.20.-q, 04.25.Nx, 04.50.Kd **ISSN 1870-9095**

I. INTRODUCTION

As known, gravitation is considered one of the four fundamental interactions of nature, together with the electromagnetism, weak interaction and strong interaction [1]. As background, gravity was described by Newton in Principia (1867) as the force exerted by a central force acting upon point masses around it. In propositions 70 to 71 [2] he proves the inverse-square law of gravitation whereby a point mass *m* situated outside a sphere of mass *M* is attracted towards the centre of the sphere with a force *F* inversely proportional to the square of its distance *r* from the centre. Later, Laplace attempted to model gravity as some kind of radiation field or fluid [3]. Thus, since the 19th century, gravity has been usually explained in terms of a "field model", rather than undergone forces by the bodies as result of the experienced impulse due to the acceleration of their masses, in accordance to the Newton's second law.

In addition, Einstein's General Theory of Relativity (GTR) [4] also considers gravity due to a gravitational field which causes attractive forces between the bodies, where that field is determined as the solution of Einstein's field equations. These equations are dependent on the distribution of matter and energy in a region of space (described by the stress-energy tensor), unlike Newtonian gravity, which is dependent only on the distribution of matter (described by the mass). Curvature of space-time is one of the main consequences of GTR, which states that gravity is the effect or consequence of the curved geometry of space-time. GTR states that the bodies within a gravitational field follow a curved spatial trajectory (named geodesic, being the minimum straight line between two bodies in the space-time). In addition, GTR deducts the equivalence principle (introduced by Einstein in 1907) [5], which assume the complete physical equivalence of a gravitational field and a corresponding accelerated frame of reference. From this principle, Einstein concluded that free-fall is actually inertial motion. In this way, gravitational "force" as experienced locally while standing on a massive body (such as those of the Earth or the Sun) is actually the same as the non-inertial (also called pseudo-force) experienced by an observer in a non-inertial (accelerated) frame of reference.

Robert H. Dike in 1959 first proposed to make a distinction between the wake and the strong equivalence principle (SEP) [6] which suggests that gravitation has a nature purely geometrics (it means, metric determinates the effects of the gravity) and it does not contain any field associated with it. In such a concept, the fields themselves represent the curvature of space-time.

Thus, rather than two particles attracting each other, the massive bodies distort space-time (as a distorted surface) via their mass, and this distortion is what is perceived subjectively as a "force". In fact there is no force in such a model, rather matter is simply responding to the rate of curvature of space-time itself. Nevertheless, nature of gravity has not been enough clarified and some theories and hypothesis have been developed to explain its nature.

On the other hand, Edwin Hubble in 1929 from his observations of distant galaxies where red shift increases with distance deducted the expansion of the universe [7]. The observed velocity of distant galaxies, taken together with the Einstein's cosmological principle [8] (homogeneity and isotropy structure of the universe), was the first observational support for the Big Bang theory which had been proposed by Georges Lemaître in 1927, from which is considered that time and the universe began from the Big Bang (when time equals zero) [9].

In addition, according to the observations of the Supernova Legacy Survey (SNLS) [10], it is considered that the universe currently is in accelerated expansion.

In this paper, we consider a hypothetical scenario where gravity results as an inertial effect of combine both, curvature of space-time (for instance, distorted by the presence of a massive body on the space-time, as it was proposed by Einstein) [4] and its hypothetical acceleration as a non-inertial frame of reference that follows the accelerated expansion of the universe. Then, we derive a general expression for the curvature of space-time in acceleration. Applying some equivalent expressions (that relate speed of light and orbital velocity) it can be transformed to an approximated equivalent expression of the so-called "compact form of Einstein's field equation" (EFE), but it derived from an inertial effect where "field of gravity" is not involved.

II. CLASSICAL THEORY OF GRAVITY REVISITED

In this section we briefly revisit some of the fundamental descriptions of the properties of inverse-square force law of Newton and its equivalences with the dynamical expressions of a given body rotating around a massive body as center of mass, which will be later applied in this paper.

Although most orbits are elliptical in nature, a special case is the circular orbit, which can be considered as an ellipse of zero eccentricity. This consideration simplifies the calculations to the case of circular orbit. Formula for velocity of a given body in a circular orbit around a center of mass as central point [11] is given by

$$v_o^2 = \frac{GM}{r}, \tag{1}$$

where v_o is the orbital velocity of the given body, G is the Newtonian constant of gravitation, M is the center of mass of a massive body (as that of the Sun for the Solar System case) and r is the distance from such a center of mass where M exists. Thus, we can apply its equivalence with the accelerated circular motion, hence

$$GM = v_o^2 r = ar^2 \therefore a = \frac{GM}{r^2}, \tag{2}$$

where a is the acceleration of the given body (as a planet rotating around a center of mass as that of the Sun). Thus, according to the Newton's second law, undergone force is defined as the mass m of given body by its acceleration, hence

$$F = \frac{d}{dt}(mv_o) = ma = \frac{GMm}{r^2}. \tag{3}$$

III. THE NON-INERTIAL FRAME IN RADIAL ACCELERATION

As commented, Einstein's equivalence principle considers that a gravitational field is equivalent to an accelerated frame of reference. In the hypothesis here presented, we consider that such accelerated frame of reference corresponds to the hypothetical acceleration of space-time that follows the accelerated expansion of the universe.

Non-inertial frame of reference is traditionally derived by a coordinate transformation. Thus, in order to derive the undergone effect on a non-inertial frame of reference, let us consider a given body in circular motion with constant velocity v and radius r circumgyrating around a central point O on the x and y-axes. From the classical mechanics [12], its position vector is given by

$$r' = v_t t, \tag{4}$$

where v_t is the tangential velocity of the given body and t is the time.

If such a body in circular motion is also uniformly accelerated towards the vertical direction (it is, along the z-axis), then its position vector is given by

$$r_0 = v_0 t + \tfrac{1}{2} a t^2, \tag{5}$$

where v_0 is the initial velocity of the given body and a is its acceleration along that z-axis. Having that relative velocity is the velocity of a body (or a frame of reference) with respect to other; it is related only in systems of two bodies (or two frames of reference). Thus, relation between both, position in a fixed coordinate system and positions in the accelerated system, for a fixed observer is given by

$$r = r' + r_0 = v_t t + \left(v_0 t + \tfrac{1}{2} a t^2\right), \tag{6}$$

where its components in a three-dimensional frame of reference are given by

$$\begin{cases} r = v_t t \therefore t = \dfrac{r}{v_t}, \\ z = v_0 t + \tfrac{1}{2} a t^2. \end{cases} \tag{7}$$

Finding out time from the first expression in (7) and replacing it in the second expression of it, we have the equation of its trajectory as it is seen by a fixed observer on the given body, hence

$$z = v_0 \left(\frac{r}{v_t}\right) + \tfrac{1}{2} a \left(\frac{r}{v_t}\right)^2, \tag{8}$$

which is a parabola. If that acceleration starts from the rest, then initial velocity equals zero and expression (8) is reduced, giving

$$z = \frac{a r^2}{2 v_t^2}. \tag{9}$$

We can generalize expression (9) for a spherical scenario extending vertical acceleration from along only one z-axis to several radial "z-axes" starting each one of them from a common central point. Then, in a homogeneous acceleration, a sphere (by simplicity) in accelerated dilation is formed. Thus, equation of the radial motion will be equivalent to the radius R of the formed sphere, hence

$$R = \frac{a r^2}{2 v_t^2}. \tag{10}$$

IV. SCALAR CURVATURE FOR THE CURVED SPACE-TIME IN ACCELERATION

In this hypothesis, we can consider that R term is a parameter which with acceleration increases its magnitude along a radial w-axis, being orthogonal to the x, y-axes at each point of the sphere.

In order to derive the scalar curvature of a spherical surface in accelerated dilation when is distorted by the presence of a massive body placed on such a surface, related by the orbital velocity of a given body along the curvature of space-time in acceleration, we can write expression (10) in terms of the resultant area. Area or surface of such a formed figure can be derived by multiplying expression (10) by $4\pi R$, hence

$$4\pi R^2 = 4\pi \frac{a^2 r^4}{\left(2 v_t^2\right)^2}. \tag{11}$$

According to the differential geometry [13], Gaussian curvature of a surface is the real number $K(P_0)$ which measures the intrinsic curvature in each regular point P_0 of such a surface. That curvature can be calculated from the determinants of the first and second fundamental forms of the surface, given by

$$K(P_0) = \frac{b_{11} b_{22} - {b_{12}}^2}{g_{11} g_{22} - {g_{12}}^2}. \tag{12}$$

This Gaussian curvature in general varies from a point to other of the surface and it is related with the main curvatures of each point (k_1 and k_2) through the expression $K = k_1 k_2$, where to the spherical surface of radius r, which has the same curvature in all of its points, from expression (12) the Gaussian curvature (2-spfere) is the same for all of its points, defined as

$$K(S^2) = \frac{1}{r^2} > 0. \tag{13}$$

Thus, finding out the radius of curvature for a given section of the distorted spherical surface from expressing (11), yields

$$\frac{1}{r^2} = \frac{4\pi \cdot a^2 r^2}{\left(2 v_t^2\right)^2 4\pi R^2}. \tag{14}$$

Then, considering a gravitational system given by a center of mass M (as that of the Sun), described by expression (2), and replacing it in expression (13), yields

$$\frac{1}{r^2} = \frac{4\pi \cdot G M a}{\left(2 v_t^2\right)^2 4\pi R^2}. \tag{15}$$

Furthermore, having the cosmological principle [8] from GTR (which considers a homogeneous and isotopic structure of the universe), where space-time (as the universe) hypothetically could has a spherical surface (by simplicity) currently in accelerated dilation, where its area in S^3 is defined by $A = 4\pi R^2$, which can be distorted by a massive body M on such a spherical surface while R accelerated increases.

When a massive body M is on such an accelerated surface, it must resist while the massive body on such a surface is carried out along the radial w-axis. Then, it produces a force F per unit of area A on such a surface [14]; where for the case of sphere is giving by

$$T'_{\mu,\nu} = \frac{F}{A_{\mu,\nu}} = \frac{Ma}{4\pi R^2}, \qquad (16)$$

where $T'_{\mu,\nu}$ is the "stress tensor" and indexes μ, ν run 1, 2, 3.

Expression (16) represents distortion of the spherical surface due to the pressure exerted by a massive body *M*, applied on it as a force per unit of area. Stress tensor could be considered as an approximated equivalent for the stress-energy tensor ($T_{\mu,\nu}$) considered in GTR [15]. Nevertheless, stress tensor is a scalar magnitude and no energy part (as a field) is involved in expression (16), but the acceleration of the frame of reference as source for the dynamics.

Thus, substituting stress tensor from expression (16) in expression (15), yields

$$\frac{1}{r^2} = \frac{4\pi G}{\left(2v_t^2\right)^2} T'_{\mu,\nu}. \qquad (17)$$

In two dimensions (for a given area), scalar curvature is exactly twice the Gaussian curvature [16]. For an embedded surface in Euclidean space, this means that for expression (17), yields

$$S = \frac{2}{\rho_1 \rho_2} = \frac{8\pi G}{\left(2v_t^2\right)^2} T'_{\mu,\nu}, \qquad (18)$$

where ρ_1, ρ_2 are the principal radii of the surface. For example, scalar curvature in S^3 of a sphere with radius *r* is equal to $2/r^2$.

The 2-dimensional Riemann tensor has only one independent component and it can be expressed in terms of scalar curvature and metric area form. In any coordinate system, one thus has

$$2R_{1212} = S \det(g_{ij}) = S[g_{11} g_{22} - (g_{12})^2]. \qquad (19)$$

If we now consider that each point on the radial *w*-axis that forms the surface has its own *z*-axis (orthogonal to the *x*, *y*-axes), then we can extend expression (18) to a four-dimensional frame of reference in function of (*w*, *x*, *y*, *z*), and expressing it in Riemann tensor terms (applying tensor Einstein's notation), yields

$$S^4 = \frac{8\pi G}{\left(2v_t^2\right)^2} T'_{\mu,\nu}, \qquad (20)$$

where indexes μ, ν run 1, 2, 3, 4.

V. THE EQUIVALENCE WITH THE EINSTEIN FIELD EQUATION

We have that expression (20) extended to a four-dimensional frame of reference has not any term of speed of light, but it is related with the tangential velocity of the given body in orbit. Nevertheless, we can attempt to derive an approximation to the relativistic expression [4] by considering a given body orbiting around a center of mass [17], where radius from the center of mass is tending to the Schwarzschild radius [18], hence

$$r_o \to r_S ; \frac{GM}{v_t^2} \to \frac{2GM}{c^2}, \qquad (21)$$

where r_o is the distance from the center of mass, r_S is the Schwarzschild radius and *c* is the speed of light in vacuum, and *M* is the mass of a massive body. Considering the mass of Sun, for instance, the correspondent Schwarzschild radius is approximately of 2.95km. Applying the equivalent expression which relates orbital velocity with speed of light [19], also simplifying common terms in expression (21) and reordering, yields

$$c^2 = 2v_o^2 \left(\frac{r_o}{r_S} \right) = 2v_t^2. \qquad (22)$$

Then, replacing expression (22) in expression (20), hence

$$S^4 = \frac{8\pi G}{\left[2v_o^2 \left(\frac{r_o}{r_S} \right)\right]^2} T'_{\mu,\nu}, \qquad (23)$$

where indexes μ, ν run 1, 2, 3, 4.

Expression (23) describes scalar curvature of a spherical surface in accelerated dilation when is distorted by the presence of a massive body placed on such a surface, related by the orbital velocity of a given body along the curvature of space-time in acceleration. Thus, such a given body should undergo an inducted motion along the distorted surface experiencing as inertial force acting upon the given body (which could apparent an attractive force between the bodies).

Replacing expressions (22) in expression (20), hence

$$G_{\mu,\nu} \approx S^4 = \frac{8\pi G}{c^4} T'_{\mu,\nu}, \qquad (24)$$

where $G_{\mu,\nu}$ is the Einstein's tensor and indexes μ, ν run 1, 2, 3, 4.

Expression (24) is an equivalent expression to the so-called "compact form of Einstein's field equation" [4, 8], but it derived from a non-relativistic way which does not consider energy from a field of gravity.

VI. CONCLUSIONS

This paper aims to offer a hypothetical alternative physical explanation for a celebrated effect, the gravity (no merely a reformulation of the previous knowledge). Nevertheless, this hypothesis is based on an inertial effect which is explained by the classical mechanics and also considers some deductions from the main theories about gravity, as curvature of space-time and the equivalence principle regarding to the comparison of gravity and a non-inertial frame of reference, which are considered in GTR.

Then, by supposing a hypothetical scenario where space-time behaves as a surface in accelerated dilation (as a non-inertial frame of reference) following the accelerated expansion of the universe, gravitation is related with the curvature of space-time (for instance, distorted by a massive body) and its hypothetical acceleration, where both together would contribute to produce an inertial effect experienced as a motion due to the inertial force (which could be undergone like the effect of gravity) by a given body looking for the equilibrium on the curved surface in acceleration.

In addition, an approximated equivalent expression with the "compact form of Einstein's field equation" is derived mainly from classical mechanics concepts by consider space-time as a non-inertial frame of reference, where derived "stress tensor" is also an approximated equivalent with the stress-energy tensor of GTR, but those without the concept of mutual attractive forces of gravity exerted as a "field of gravity" to produce effect of gravity.

Thus, this hypothesis shows that gravity could have a nature geometrics and dynamics nature, where acceleration of curved space-time could be enough sources to provide of inertial motion to the celestial bodies (instead of through a field of gravity where mutual "forces" are exerted between the bodies).

Extension of the curvature of space-time from the center of mass of a massive body could be very longer around a given massive body (given by the product GM), which geometrically could explain the longer reach considered for gravity.

It is applicable in education, where the main Newtonian and relativistic concepts and the main principles for gravity are revisited. It is showed the possibility to apply the current knowledge in order to propose alternative explanations of the natural phenomena. As a possible interpretation, we represent space-time as a dynamics surface in accelerated dilation, in order to apply classical concepts also to attempt explain some effects of nature that are traditionally explained by the relativistic theories.

ACKNOWLEDGEMENTS

The author would like to thank Professor Sergio S. Cornejo for the review and comments for this paper.

REFERENCES

[1] Davies, P., *The Forces of Nature*, 2nd Ed. (Cambridge Univ. Press, UK, 1986).

[2] Newton, I., *Philosophiæ Naturalis Principia Mathematica*, (Cambridge, UK, 1867), Props. 70-71.

[3] Laplace, P. S., *Exposition du Système du Monde*, 1st Ed. (Cercle-Social, Paris, 1796).

[4] Einstein, A., *The Foundation of the General Theory of Relativity*, Annalen der Physik **49** (1916), pp. 284–337.

[5] Einstein, A., *Über das Relativitätsprinzip und die aus demselben gezogene Folgerungen*, Jahrbuch der Radioaktivitaet und Elektronik **4** (1907), pp. 412–462.

[6] Dicke, R. H., *New Research on Old Gravitation*, Science **129**, 621-624 (1959).

[7] Hubble, E., *A Relation between Distance and Radial Velocity Among Extra-galactic Nebulae*, PNAS **15**, 168–173 (1929).

[8] Einstein, A., *The Meaning of Relativity*, (translated to the English, Methuen, UK, 1922).

[9] Hawking, S., *The Beginning of Time* (University of Cambridge, Cambridge, UK, 2008).

[10] Astier, P., Guy, J., *et al*., *Astronomy and Astrophysics* **447**: 31-48 (2006), pp. 2-3 [arXiv:astro-ph/0510447v1].

[11] Tan, A., *Theory of Orbital Motion* (World Scientific Publishing Co. Pte. Ltd., Singapore, 2008), pp. 26-44.

[12] Tocaci, E., *Relativistic Mechanics,Time,and Inertia*, (C. W. Kilmister Ed., Cambridge University Press, Cambridge, UK, 1984), pp. 183-251.

[13] Kühnel, W., *Differential Geometry: Curves - Surfaces – Manifolds*, 2nd Ed. (American Mathematical Society, Rhode Island, USA, 2006), pp. 73-74.

[14] Chen, W. and Han, D., *Plasticity for structural engineers*, (J. Ross Publishing, Florida, USA, 2007), pp. 46–71.

[15] Misner, C. W., Thorne, K. S. and Wheeler, J. A., Gravitation, (W. H. Freeman and Company, New York, USA, 1973), pp. 141–142.

[16] Kobayashi, S. and Nomizu, K., *Foundations of Differential Geometry*, Vol. 1, (Interscience, New York, USA, 1963).

[17] Chang, H. and Hsiang, W., *The epic journey from Kepler's laws to Newton's law of universal gravitation revisited,* Am. J. Phys. **6**, 7–17 (2008), [arXiv:0801.0308v1].

[18] Schwarzschild, K., *Over the gravitational field of a point mass according to the Einstein's theory* (Minutes of meetings of the German Academy of Sciences to Berlin, class for mathematics, physics and technology, Berlin, 1916), p. 424.

[19] Cornejo, A. G., *The Equivalent Expressions Between Escape Velocity and Orbital Velocity*, Lat. Am. J. Phys. Educ. **4**, 575–579 (2010).