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# Teaching general concepts about sensors and transfer functions with a voltage divider

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**Abstract**

This work proposes the use of a simple voltage divider circuit composed by one potentiometer and one resistor to simulate the behavior of the electrical output signal of linear and nonlinear sensors. It is a low cost way to implement practical experiments in classroom and it also enables the analysis of interesting topics of electricity. This work induces naturally to a class guide where students can build and characterize a voltage divider to explore several concepts about sensors output signal. As the result of this teaching activity it is expected that students understand fundamentals of voltage divider, potentiometer operation, fundamental sensor characteristics, transfer function, and, besides, associate directly concepts of physics and mathematics with a practical approach.

**Keywords:** Sensors, potentiometers, transfer function.

**Resumen**

Este trabajo propone la utilización de un circuito divisor de voltaje sencillo compuesto por un potenciómetro y una resistencia para simular el comportamiento de la señal eléctrica de salida de los sensores lineales y no lineales. Este es un camino de bajo costo para implementar los experimentos prácticos en el aula y también permite el análisis de temas de interés de la electricidad. Este trabajo induce naturalmente a una guía de clase donde los estudiantes pueden construir y caracterizar un divisor de tensión para explorar varios conceptos acerca de la señal de salida de los sensores. Como resultado de esta actividad docente se espera que los estudiantes entiendan los fundamentos del divisor de tensión, el funcionamiento del potenciómetro, las características fundamentales del sensor, la función de transferencia, y, además, se asocian directamente los conceptos de Física y Matemáticas con un enfoque práctico.

**Palabras clave:** Sensores, potenciómetros, función de transferencia.

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## I. INTRODUCTION

Sensors compose an important tool for physics education with uses in subjects as mechanics, optics, electricity, instrumentation, etc. It is important to emphasize that sensor technology has had a significant development in the last years with a consequent cost reduction and performance improvement. It has collaborated to the creation of new and interesting scenarios in the area of physics education, creating motivational factors.

In spite of the advances, there are lots of schools in Latin America with lack of laboratories and financial support to implement practical experiments. Thacker [1] and Rak [2] emphasize that, in spite of the present technological development and globalization, not all the students have access to this development. Pearl [3] shows that it can be a more critical problem because it can endanger the student's motivation for physics studies and consequently the future graduation of new physicists.

Campos [4] shows cases where the school has technological apparatus for physics education, however the small apparatus quantity limits their use and the real experimentation by students is replaced by simple demonstrations in classroom. In other cases, Muit-Herzig [5] shows schools with many apparatus but with the lack of a concrete interaction between technology and pedagogical aspects. The simplistic and isolated use of technologies can only be a limited resource to the confirmation of previous theories and it can't be enough to ensure a concrete learning and the development of a critical sense. It means that, besides educational apparatus, teachers must have time, training, and motivation to learn about new technologies and to promote their pedagogical integration with the student's curricula.

The statements above have motivated the present work that proposes an apparatus based on a simple and inexpensive electrical circuit to introduce fundamental concepts about sensors and, besides, it concerns about voltage dividers and potentiometers. This study requires students with only a previous basic background in electricity. The low cost apparatus is important to enable its massive use in classroom and its simplicity is fundamental to enable an easy and quick insertion in pedagogical plans.

This work shows in section II fundamental concepts about sensors, including physical and mathematical concepts of the transfer function. Section III shows the study about voltage divider based on potentiometers which is a basis for sections IV and V which show the design of the proposed apparatus and its results, respectively. Finally, section V proposes the introduction of some specific mathematical concepts associated with the data analysis of nonlinear sensors output signal.

## II. SENSOR DEFINITION

The first step to introduce this subject in classroom is the definition of sensors, which are devices with internal characteristics directly affected by an external phenomenon (parameter), and, therefore, there is a direct relation between them. The external phenomenon can be temperature, humidity, pressure, etc, and the internal characteristic can be, for example, the resistance or capacitance.

There are active sensors that self generate an electrical output signal, as thermocouples, and sensors that require an external power source to provide an electrical output signal, as a LDR or a thermistor. They are called active and passive sensors, respectively. It means that according to the sensor type it may or may not require a battery to provide an electrical output signal. In some cases, the output signal can require a *signal conditioning*, which includes processing such as amplification, attenuation and filtering; and, it requires additional electronic components [6].

Sensors are also defined as a type of transducer [6], but other authors emphasize that a transducer is any device that converts one form of energy into another and, in many cases, without any association with sensors, such as a hydroelectric power that converts mechanical into electrical energy [7, 8].

It is important to emphasize that the definitions of active sensors, passive sensors and transducer are frequently misunderstood and it requires a special analysis in classroom. To avoid mistakes, this work suggests the use of the international vocabulary of metrology proposed by the International Bureau of Weights and Measures (BIPM) [9] and supported by other international institutions as the International Organization for Standardization (ISO) [10]. This vocabulary uses the term “measuring transducer” as “device, used in measurement, which provides an output quantity having a specified relation to the input quantity”. In other words, for a practical approach, the measuring transducer is a device that senses an external phenomenon and provides an electrical output signal [11, 12]. The vocabulary analysis in classroom is important to avoid future misunderstandings and to make students more concerned about that, in some cases, the same sensor concept can be referred with different words.

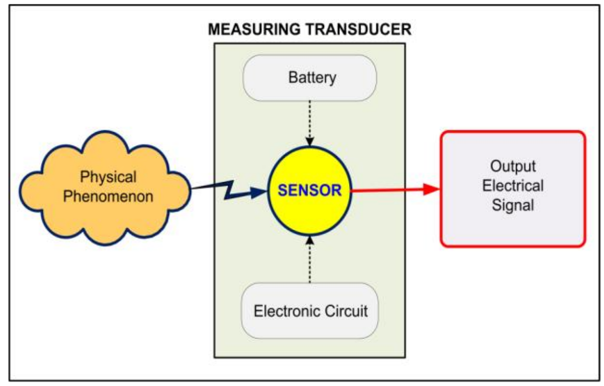


**FIGURE 1.** A measuring transducer structure. It may or may not require a battery and an additional electronic circuit.

Fig. 1 shows a measuring transducer system structure that may or may not require a battery and an additional electronic circuit, according to its sensor type.

## III. THE SENSOR TRANSFER FUNCTION

The transfer function is a mathematical function which represents the relation between a physical measured parameter, also called stimulus or phenomenon, and the *system response* which is an electrical output signal, whose relation can be expressed as $S = f$ (p), where S is the electrical output signal and $p$ is the stimulus. The output voltage variation can be either linear or nonlinear and it represents an inverse function expressed as $f^{-1}(p)$ or F($S$). It means that, if you know the electrical output signal magnitude you can know the value of the physical measured parameter [13]. Note that, the output signal depends on some specific sensor characteristics variation and, therefore, both responses can be directly associated and usually have the same graphical curve. Fig. 2 shows that, basically, the transfer function is defined as either linear or nonlinear function.

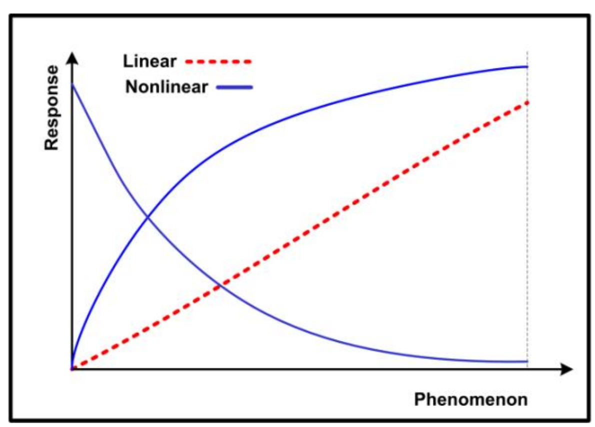


**FIGURE 2.** Transfer functions types. The phenomenon variation causes a variation of the measuring transducer output signal.

### A. The Linear Transfer Function

The simplest transfer function is the linear transfer function which graphics is a straight line expressed as:

$$S = b + m.p, \quad (1)$$

where:

*b* is the output signal when the stimulus is zero,
*m* is the slope or line gradient,
*p* is the stimulus intensity.

Note that, b and m are constants, and S (output) varies according to *p* (input).

Fig. 3 shows that the linear transfer function can be easily demonstrated in classroom with the sensor of temperature LM35 that provides a linear output signal of 10mV/$^{o}$C. It only requires an external power source from 4 to 30Volts and a voltmeter to measure the output signal. In this case, the transfer function is expressed as:

$$S = 0 + 10mV\ p, \quad (2)$$

where *p* is temperature in ºC.

The voltmeter must be set to the milivolts scale to provide a direct temperature scale measurement. For example, a voltage of 253mV represents a temperature of 25.3ºC.

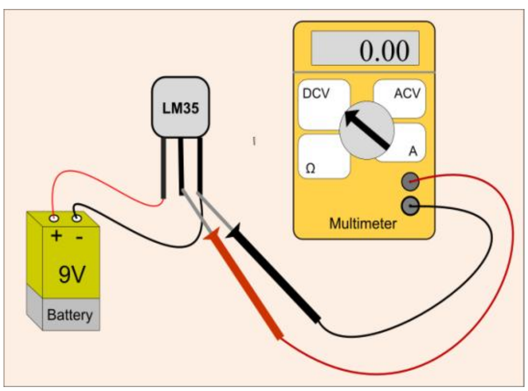


**FIGURE 3.** Measuring directly the temperature with a LM35 sensor, using only a multimeter set in the milivolts scale.

### B. Experiment: The Slope Computation and the Sensitivity Concept

This section proposes an experiment to verify the LM35 line slope. With the structure shown in Fig. 3, the student must measure the environmental temperature and subsequently the temperature close to hot source, as a bulb. It represents two temperatures, called x1 and x2, respectively, and two output voltages, called y1 and y2, reactively. Based on these values, it is possible to compute the slope (m) using the point-slope formula:

$$y2 - y1 = m\ (x2 - x1). \quad (3)$$

In this case, the student will verify that the slope is 10, according to theoretical sensor specifications. Besides the slope computation, this exercise is interesting to introduce the concept of *sensitivity* which specifies how much the output signal varies per input unit. It is a type of conversion efficiency that is 10mV/ºC for the LM35.

It is also interesting to show that the LM35 datasheet [14] refers to the sensitivity as "sensor gain" or "average slope"; however, works as [15, 16] analyze the LM35 sensor and use the term sensitivity to refer this parameter. This observation is very much relevant to reemphasize the importance of the vocabulary analysis in classroom.

### C. Analysis of Other Fundamental Sensor Characteristics Based on the Transfer Function

The term sensitivity is frequently misunderstood with *resolution* that represents the smallest increment of the input stimulus that can be sensed.

For example, an image *sensor* with a spatial resolution of 1 meter will be able to detect objects smaller than this size. A thermal sensor with resolution of 0.5°C cannot detect a thermal variation from 20.0ºC to 20.1ºC because it requires step variation of 0.5°C. For a better signal manipulation, the ideal would be a high sensitivity and a low resolution.

Other two import terms are Precision and Accuracy that are frequently misunderstood. *Accuracy* represents how close the measurement is to the real value, and, in other words, accurate has a sense of "correct". *Precision* means "repeatability" and it represents the result variation when the same measurement is repeated under the same conditions [13].

Fig. 4 shows a transfer function and its association with some fundamental concepts about sensors. It also shows the concepts of *threshold*, *full output scale* and the *span* which represent respectively the minimum phenomenon level that can be sensed, the maximum output signal, and the phenomenon range that can be sensed.

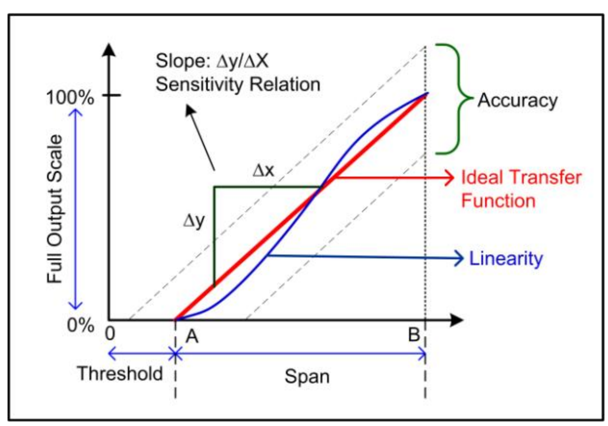


**FIGURE 4.** Analysis of fundamental sensor characteristics associated with the transfer function.

Fig. 4 shows that, in some cases, the real sensor transfer function line is not perfect. It is a factor called linearity and represents the difference level from theoretical and real lines. The ideal sensor behavior is a linearity deviation as small as possible.

### D. The Nonlinear Transfer Function

There are sensors with internal characteristics variation not proportional to the external phenomenon variation [13]. Therefore, they have a nonlinear behavior and the sensitivity is not constant. Sensors as LDR (Light Dependent Resistor*)*, NTC *(*Negative Temperature Coefficient), and PTC (Positive Temperature Coefficient) are simple examples of nonlinear sensors.

The output signal of nonlinear measuring transducer can be represented by several equations such as exponential or logarithmic. Mathematical functions with an exponential growth can be given by equation:

$$y = a^x. \tag{4}$$

If the variable *a* represents a rational number smaller than zero then you may not get a real number. For example, it occurs for $(-2)^{0,5}$. If A is zero or one, then the result is a straight line. Therefore, the variable *a* is limited to $a>=0$ and $a \neq 1$. When *x* is negative the equation is equivalent to $(1/a)^x$ and its graphic behavior is a reflection of the y-axis. The graphics can also be translated using an auxiliary constant (*b*) using the expression:

$$y = b\, a^{x}. \tag{5}$$

In Eq. 5, *b* is a constant that represents the initial value of *y* when *x* is zero. Note that *a* is the function base and it represents the general case for the traditional exponential function given by:

$$y = e^{x}, \tag{6}$$

Where: $e \cong 2.7182$.

The exponential function has an inverse function computed as the logarithmic function. Supposing the Eq. $x = a^{y}$, the logarithmic function is given by:

$$y = log_{a} x, \tag{7}$$

Where *b* is the base and must be greater than zeros and different from one.

According with [13], the Eqs. 6 and 7 can be rewritten, respectively, as Eqs. 8 and 9:

$$y = a\, e^{km}, \tag{8}$$

$$y = a + b \log(m). \tag{9}$$

Where variables *a* and *b* are parameters and *k* is the power factor

Few selected mathematical equations can compose a background to understand different typical behavior of linear and nonlinear sensors. In the next sections, this background is associated with the real generation of electrical signals similar to sensors transfer functions. It is done with voltage divider circuits.

## IV. VOLTAGE DIVIDER AND POTENTIOMETERS

Fig. 5 shows a voltage divider circuit that provides an output voltage (Vo) proportional to the input voltage (Vin), expressed as:

$$Vo= I * R2, \tag{10}$$

Where I = Vin/(R1+R2)

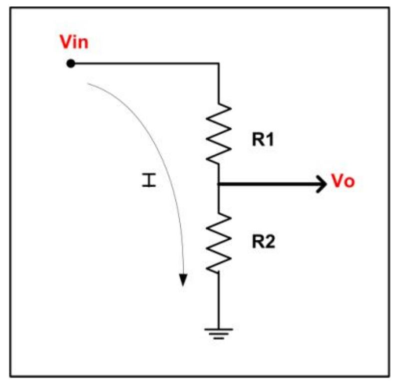


**FIGURE 5.** The voltage divider schematic. The output voltage (Vo) is a proportion of the input voltage (Vin).

Fig. 6 shows that a potentiometer could replace the resistor R2 to provide an adjustable output signal. It is a very simple circuit based only on a resistor and a potentiometer, but it is a very powerful tool to introduce important sensor concepts in classroom. In this case, the voltage relation is expressed as:

$$Vo= I * Ra, \tag{11}$$

Where: I = Vin/(R1+Pr)
Pr=Ra+Rb

Fig. 7 shows the potentiometer that is a three-terminal resistor whose resistance between the centered terminal and the end terminals depends on the centered terminal position. It means that the centered terminal rotation causes a variation of the resistance between it and the other two terminals, and, this variation can be linear or logarithmic. The middle terminal is named *wiper terminal* and the end terminals area usually named Counter Clockwise (CCW) and Clockwise (CW) for a front view, respectively. Fig. 6 shows that in this work the terminals are named A, B and C to ensure an easier understanding.

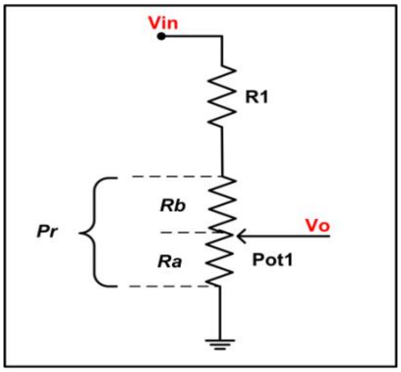


**FIGURE 6.** The voltage divider schematic based on a potentiometer to provide an adjustable output voltage.

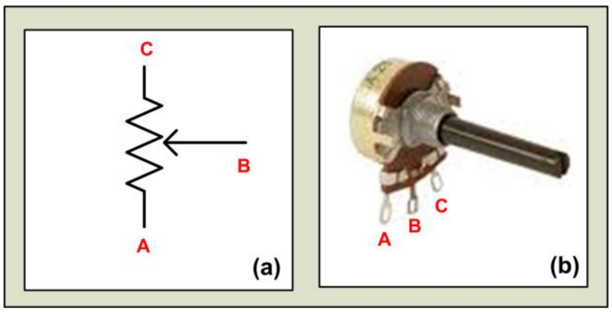


**FIGURE 7.** Electronic symbol of a potentiometer (a). Photo of a real potentiometer (b).

The characterization of a potentiometer includes electrical, mechanical and environmental specifications; however, for an educational approach, this work limits the specification basically on the potentiometer resistance (Pr) and the rotational angle (Ar). The *potentiometer resistance* (Pr) has other names as total resistance, value of the potentiometer or maximum resistance, and, it defines the resistance between the end terminals. In other words, it is the resistance measured directly between the terminal A and C. The *rotational angle* (Ar) represents the angle between the terminals A and B (wiper terminal) when terminal B is completely turned to the right and connected directly to terminal C. Remember that theoretical and real values of potentiometer characteristics can be different due to errors that vary from each component. Therefore, the verification of the Pr and Ar values of each potentiometer is fundamental.

## IV. THE PROPOSED APPARATUS DESIGN

Fig. 8 shows a proposed apparatus to generate an output signal similar to the output signal generated by a measuring transducer.

**FIGURE 8**. The proposed apparatus to generate different output signals. The potentiometer is centered in the protractor and it has three wires to connect a parallel resistor, according to the Figs. 9 and 12 circuits.

It is designed with a protractor with 360º, a paper arrow, and a linear potentiometer. This apparatus is designed with a linear potentiometer but it can generate linear and nonlinear curves, similar to real transfer function to assembly the apparatus:

1. The linear potentiometer is positioned in the protractor center and turned totally to the left, with Ra at 0º;
2. The paper arrow is inserted in the potentiometer shaft;
3. The knob is also inserted in the potentiometer shaft and glued to the paper arrow.

The knob movement also moves the potentiometer shaft and the paper arrow that indicates the Ra value that can be directly verified in the protractor.

In this work, the direct angle measured between terminals A and B is called *Wiper Angle* (Wa) and its proportion in relation to Ar is called *Wiper Angle Percentage* (Wp) and computed as:

$$Wp=Ar/Wa. \quad (12)$$

For example, supposing a potentiometer with Pr of 1.5KΩ and Ar of 270°. For Wp at 40%, the theoretical angle between terminals A and B would be 108° and the resistance between both terminals would be 600Ω.

Note that, when the same voltage divider experiments are done with different potentiometer, the output values can change; however, the behavior must have the same trend. It means that, using this concept, students are motivated to concentrate on the conceptual idea more than on numerical results.

A detailed potentiometer analysis can show an incorrect linearity due to problems such as the intrinsic potentiometers manufacture characteristics or parasitic inductance or capacitance in the winding features. Besides, the measurement instrument can induce an error in the measurement due to the "loading effect" that can occurs because, in some cases, the voltmeter resistance represents a parallel resistance to the potentiometer on measurement and it can generate a measurement error. This error can be neglected by instruments with very great impedance in relation to the potentiometer. Usually, commercial voltmeters have high input impedance, but this problem is interesting to be considered for didactical approaches.

## V. EXPERIMENTS AND RESULTS

This section shows the experiments and results for the linear and nonlinear signals generated with the apparatus shown in Fig. 8.

### A. Linear Output Signal Generation

The first proposed experiment with linear potentiometers is the analysis of its real linearity. In spite of datasheets specification, students must verify the real values of Ar and Pr. The real Pr value is verified with the measurement of the resistance between terminals A and C. The Ar angle is measured directly with the apparatus shown in Fig. 8, turning the potentiometer knob completely to the right.

To verify the potentiometer linearity, the experiment requires the measurement of the resistance between terminals A and B for several Wp values. These values are a percentage of the Ar angle fixed at 0%, 12.5%, 25%, 37.5%, 50%, 62.5, 75%, and 100%. Besides, this work also proposes the voltage variation analysis on the potentiometer terminals. It is done measuring both voltages between terminals A and B, and between terminals B and C. Fig. 9 shows that in this work these voltages are called Va and Vc, respectively.

The potentiometer used in this work had a Rotational Angle (Ar) of 305°, and, therefore, the Wp values are respectively equivalent to the angles 0º, 38º, 76º, 113º, 152º, 191º, 229º, 267º, and 305º. In this work, these angles are called *symmetrical angles* (Sa).

The students in classroom must turn the potentiometer knob and, for each of the eight symmetrical angles they must:

1. Measure Ra
2. Measure Va and Vc
3. Compute the theoretical values of Ra, called TRa and defined as TRa=Pr*Wp,

Fig. 10 shows the graphics of Ra and TRa as a function of Wp. The theoretical graphics based on the TRa values is a straight line (blue) but the real measurement based on Rm (red) shows a potentiometer with curve not exactly linear. It represents a deviation of the theoretical value, whose maximum difference is referred as independent linearity error.

The linearity problems of a potentiometer can be caused when the wiper terminal acts as a load due to its resistance or electrical current intensity. Another cause can be an imperfect mechanical positioning of the rotational axes. Fig. 10 shows that the potentiometer used in this work has a linearity error, but it is small and, therefore, it will be neglected in the sequence of this work.

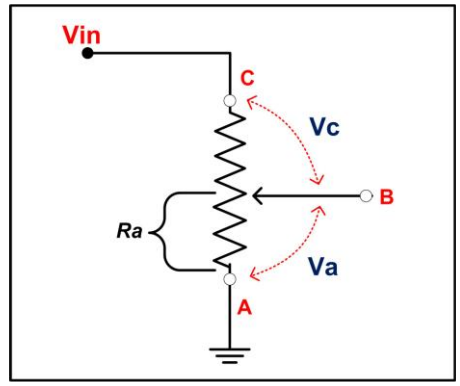


**FIGURE 9.** The circuit for Ra, VA and Vc measurements.

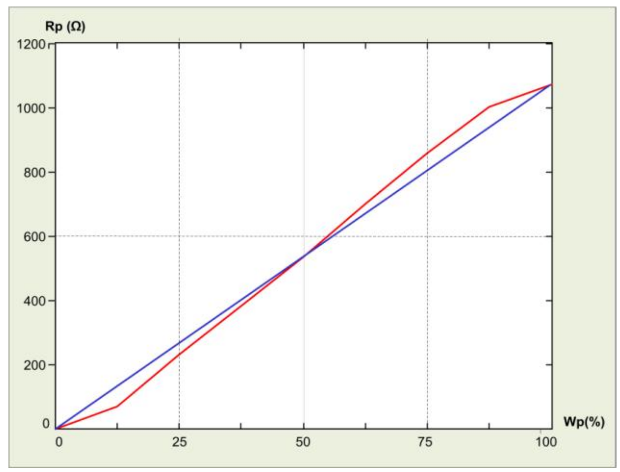


**FIGURE 10**. Analysis of the potentiometer linearity.

Fig. 11 shows the Va and Vc variation as a function of Wp, for an input voltage between terminals A and C, called Vin, fixed at 5 Volts.

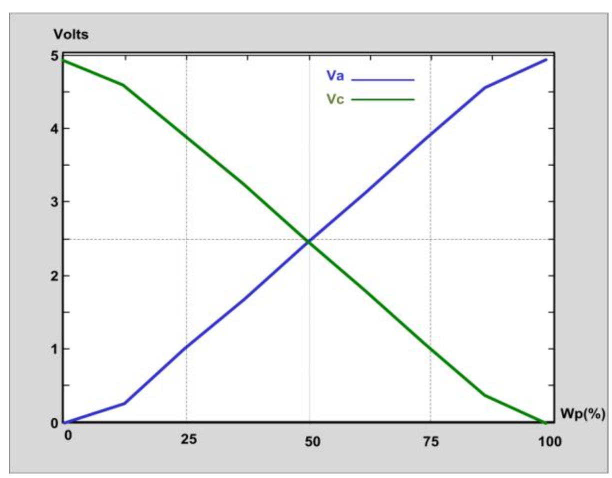


**FIGURE 11.** Variation of Va and Vp as a function of WP, for Vin=5V.

### B. Nonlinear Linear Output Signal Generation

A linear potentiometer and one resistor are enough to design of a circuit to generate a nonlinear output signal similar to the transfer function. Fig. 12 shows a circuit with an auxiliary resistor, called Rx, connected in parallel to the potentiometers terminals B and C. The voltages Va and Vp have a nonlinear variation as a function of Wp and they vary according to the Rx value.

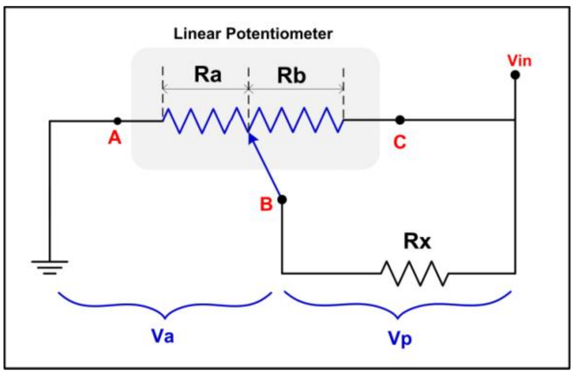


**FIGURE 12**. Circuit that provides two output nonlinear signal (Va and Vp) according to the Rx value.

This type of output voltage could be generated by a nonlinear potentiometer; however, the proposed circuit has advantages as the generation of different curves based on the auxiliary resistor, the didactical approach to conciliate the study of sensor and electrical circuits based on potentiometers and, besides, the linear potentiometer is cheaper and easier to buy in the marked.

The resistance between terminals A and B is called Ra, and the resistance between terminals B and C is called Rb. The resistance measured on Rx parallel to Rb is called Rp. The voltages between these terminals are called Va and Vp, respectively. The values of Rp and the total resistance (RT) between A and C are respectively computed as:

$$Rp = (Rb*Rx)/(Rb+Rx), \tag{13}$$

$$RT = Ra + Rp. \tag{14}$$

Fig. 13 shows the Rp variation for a potentiometer with Pr of 1KΩ and Rx values at 100Ω, 250Ω, 1KΩ, and 10KΩ, respectively. Note that the Rx values influence directly the graphic trend which varies from an extreme curve to a straight line.

Fig. 14 shows the variation of RT as a function of Wp, for different Rx values and Pr of 1KΩ. It shows a great variation on the graphics behavior according to the Rx value.

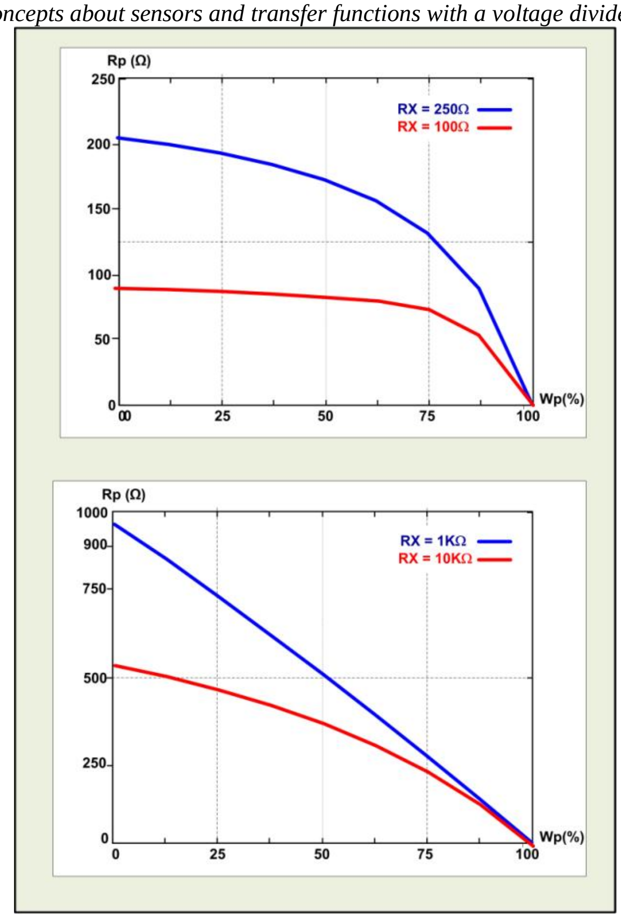


**FIGURE 13.** Rp variation for a potentiometer with a fix Rp of 1KΩ and Rx values at 100Ω, 250Ω, 1KΩ, and 10KΩ.

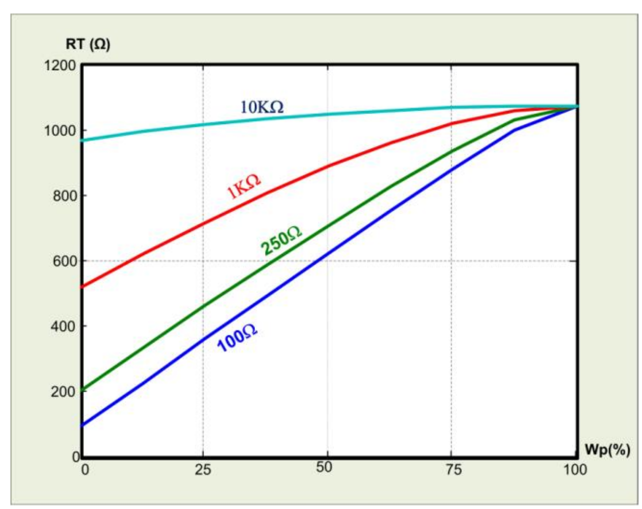


**FIGURE 14.** Variation of Rt as a function of Wa, for different values of Rx

### C. Output voltage variation

This work verified the behavior of Va and Vp output voltage as a function of Wp for different values of Rx. In this case, the potentiometer Pr was 1KΩ and the Rx values were fixed at 100Ω, 250Ω, 1KΩ, and 10KΩ. The output voltages Vp and Va can be, respectively, computed as:

$$Vp = (Vcc/RT) *Rp, \quad (15)$$

$$Va = (Vcc/RT)*Ra. \quad (16)$$

Fig. 15 shows the measured Va variation, whose behavior depends on the Rx value. The graphics is a curve for small RX values, but it trends to a straight line when Rx increases.

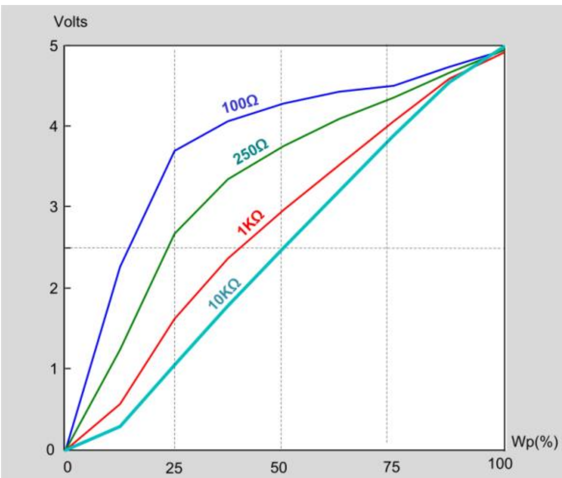


**FIGURE 15.** Variation of Va as a function of Wa for different values of Rx.

Fig. 16 shows the variation of Vp a function of Wp for different values of Rx. The difference between Rx and Rp also influences the graphic trend, but on the contrary of Va, the Vp value decreases when Wp increases.

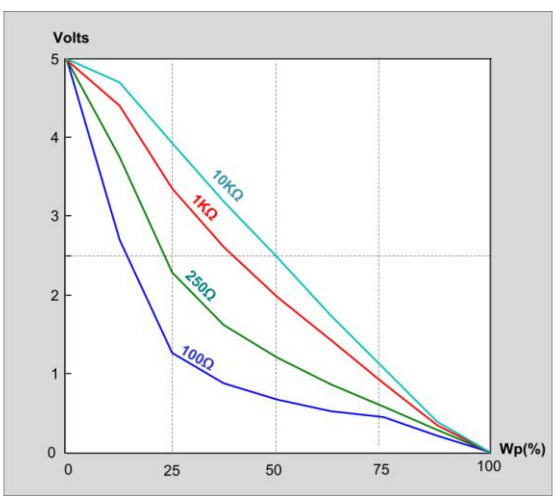


**FIGURE 16.** Variation of Vp as a function of Wp for different values of Rx.

Figs. 15 and 16 are important in this work because they represent different output signal that can be associated with output signal of several nonlinear sensors. Note that, the graphics extremes present a discontinuity due to the potentiometer linearity characteristic shown in Fig. 10. This work could be developed with another potentiometer without linearity problem to achieve a more continuous graphic trend, but the original potentiometer was used intentionally because this type of linearity problem is common and its concernment becomes convenient in this type of article.

## VI. MATHEMATICAL ANALYSIS OF NONLINEAR OUTPUT SIGNAL

The relation between the phenomenon and the output signal of a nonlinear sensor, in many cases, can't be expressed directly as a simple and direct mathematical equation. In this case, one way to solve the problem is the *curve fitting* usage, which enables the construction of a curve, or mathematical function, that fits to a series of previous data points known. In other words, if we know some sensor transfer function points, we can determine a polynomial function, which graphic can be very much similar to the sensor transfer function. Remember that a polynomial is a mathematical expression made with constants, variables and exponents, which largest exponent represents the polynomial degree.

For example, Table I shows nine selected points extracted from Fig. 15 for the variation of Va as a function of Wa when Rx is 250Ω. Remember that it is related to the Fig. 12 circuit.

**TABLE I.** Measured values of Va as a function of Wa, for the Fig. 12 circuit with a potentiometer resistance (Pr) of 1KΩ and Rx of 250Ω.

| *Wa (Degrees)* | *Va (Volts)* |
|---|---|
| 0 | 0 |
| 38 | 1.23 |
| 76 | 2.67 |
| 113 | 3.34 |
| 152 | 3.75 |
| 191 | 4.08 |
| 229 | 4.35 |
| 267 | 4.66 |
| 305 | 4.93 |

To determine the polynomial function, which curve fits on these nine points, we fix arbitrarily a polynomial degree and use a solution method as the *least squares* to determine the polynomial function.

The polynomial degree usually influences the polynomial efficiency to compute the transfer function relation and, if the final result is not satisfactory, the computation method can be repeated with another polynomial degree. In this work, the polynomial degree was arbitrary defined as three, and, therefore, the polynomial function ($\hat{y}$) is expressed as:

$$\hat{y} = a_3 * x^3 + a_2 * x^2 + a_1 * x + a_0 . \qquad (17)$$

The first step is the computation of the variables $a_i$($i=0, 1, 2,\ldots, m$) that are computed with a linear systems, which can be represented in the matrix form as:

$$Ax = b. \qquad (18)$$

where ***A*** is the coefficients square matrix of order $m$+1; ***x*** is the column vector of variables, and ***b*** is the column vector of solutions.

In this case, each element of the matrices *X* and the vector *b* is a sum and the system [18] is rewritten as:

$$\begin{pmatrix} n & \sum x_i & \sum x_i^2 & \cdots & \sum x_i^m \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \cdots & \sum x_i^{m+1} \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \cdots & \sum x_i^{m+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum x_i^m & \sum x_i^{m+1} & \sum x_i^{m+2} & \cdots & \sum x_i^{m+m} \end{pmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix} = \begin{pmatrix} \sum y_i \\ \sum x_i * y_i \\ \sum x_i^2 * y_i \\ \vdots \\ \sum x_i^m * y_i \end{pmatrix}. \qquad (19)$$

Table II shows the sum used to compute the elements of the matrix *A*, and Table III shows the sum to compute the elements of the vector b. Note that, in this example, *m* is three due to the polynomial degree chosen.

**TABLE II.** Computation of the *A* matrix elements.

| **i** | $x_i$ | $x_i^2$ | $x_i^3$ | $x_i^4$ | $x_i^5$ | $x_i^6$ |
|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 38 | 1444 | 54872 | 2085136 | 79235168 | 3010936384 |
| 3 | 76 | 5776 | 438976 | 33362176 | 2535525376 | 192699928576 |
| 4 | 113 | 12769 | 1442897 | 163047361 | 18424351793 | 2081951752609 |
| 5 | 152 | 23104 | 3511808 | 533794816 | 81136812032 | 12332795428864 |
| 6 | 191 | 36481 | 6967871 | 1330863361 | 254194901951 | 48551226272641 |
| 7 | 229 | 52441 | 12008989 | 2750058481 | 629763392149 | 144215816802121 |
| 8 | 267 | 71289 | 19034163 | 5082121521 | 1356926446107 | 362299361110569 |
| 9 | 305 | 93025 | 28372625 | 8653650625 | 2639363440625 | 805005849390625 |
| *Sum* | *1371* | *296329* | *71832201* | *18548983477* | *4982424105201* | *1374682711622390* |

**TABLE III.** Computation of the *y* vector elements.

| i | $x_i$ | $y_i$ | $x_i * y_i$ | $x_i^2 * y_i$ | $x_i^3 * y_i$ |
|---|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 | 0 |
| 2 | 38 | 1,23 | 46,74 | 1776,12 | 67492,56 |
| 3 | 76 | 2,67 | 202,92 | 15421,92 | 1172065,92 |
| 4 | 113 | 3,34 | 377,42 | 42648,46 | 4819275,98 |
| 5 | 152 | 3,75 | 570 | 86640 | 13169280 |
| 6 | 191 | 4,08 | 779,28 | 148842,48 | 28428913,68 |
| 7 | 229 | 4,35 | 996,15 | 228118,35 | 52239102,15 |
| 8 | 267 | 4,66 | 1244,22 | 332206,74 | 88699199,58 |
| 9 | 305 | 4,93 | 1503,65 | 458613,25 | 139877041,25 |
| *Sum* | *1371* | *29,01* | *5720,38* | *1314267,32* | *328472371,12* |

Based on Tables II and III, the linear systems (19) can be written as:

$$\begin{pmatrix} 9 & 1371 & 296329 & 71832201 \\ 1371 & 296329 & 71832201 & 18548983477 \\ 296329 & 71832201 & 18548983477 & 4982424105201 \\ 71832201 & 18548983477 & 4982424105201 & 1374682711622390 \end{pmatrix} * \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 29,01 \\ 5720,38 \\ 1314267,32 \\ 328472371,12 \end{pmatrix}.$$

This article doesn't detail the liner systems solutions because it is not the article focus. However, there are several theoretical books, articles and web sites that explain the different mathematic methods to solve linear systems, such as the substitution method, the elimination method and the Cramer's rule. Each method has advantages and disadvantages and its selection depends on each case [17] and this work considers that physics teachers will not have problems to remember this mathematical subject. The above system solution is:

$$\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} -0,0747000 \\ 0,0462087 \\ -0,0001758 \\ 0,0000003 \end{pmatrix}.$$

Based on the linear system solution, the polynomial function (17) can be written as:

$$\hat{y} = 0{,}0000003x^3 - 0{,}0001758x^2 + 0{,}0462087x - 0{,}07470.$$

Fig. 17 shows a graphic with the mine previous known points from in Table I, and the curve designed with the polynomial $\hat{y}$.

Fig. 17 shows that the points of the Eq. ($\hat{y}$) curve are very close to the nine previous points coordinates. Therefore, the Eq. $\hat{y}$ has a high degree of reliability and the article has proved that output signal for any potentiometer angle of the Fig. 12 circuit, can be computed directly with the polynomial function $\hat{y}$.

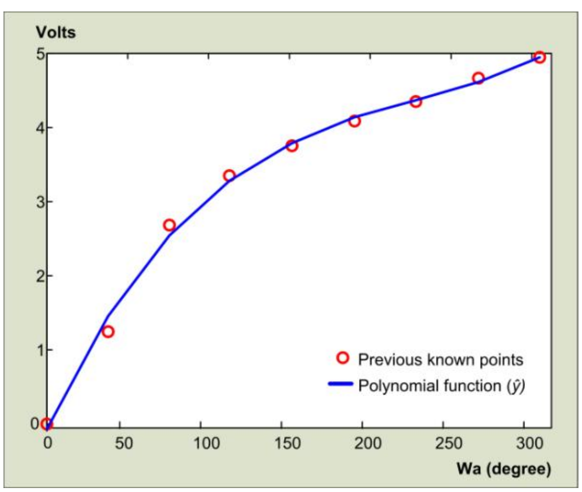


**FIGURE 17.** The nine previous known points (blue) and the curve of the polynomial function ($\hat{y}$).

## VII. CONCLUSIONS

This article has presented a review about some fundamental sensor concepts and shown a very simple voltage divider apparatus to generate linear and nonlinear output signals similar to real sensors. It proves that an inexpensive apparatus can be an efficient tool to teach concepts associated with sensors as transfer function, sensitivity, resolution, linearity, etc. It also proves that this subject is an interesting way to integrate physics and mathematics, including aspects of electricity, sensors, functions and interpolation.

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