A simple electric circuit for teaching one-dimensional characterization of piezoelectric plates

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(Received 19 October 2011, accepted 23 December 2011)

Abstract
This work presents a method based on elementary concepts of electric network for characterization of piezoceramics. A simplified version of Mason's equivalent circuit is obtained applying basic theorems of electricity. The methodology employed to simplify the circuit is mandatorily presented in the issues addressed in introductory courses of electric circuits. The experimental development can be performed in laboratories for the teaching activities without further specific equipment. Using this methodology stimulates students because it is performed on devices very employed in technological application. Furthermore, it facilitates the understanding of physical principles present in the study of piezoelectric elements.

Keywords: Thévenin; piezoelectric; transducer.

Resumen
Este trabajo presenta un método basado en los conceptos elementales de la red eléctrica para la caracterización de piezocerámicas. Una versión simplificada del circuito equivalente de Mason se obtiene aplicando los teoremas básicos de la electricidad. La metodología empleada para simplificar el circuito es presentada como obligatoria en los temas abordados en los cursos introductorios de los circuitos eléctricos. El desarrollo experimental se puede realizar en los laboratorios para las actividades de enseñanza, sin equipamiento específico adicional. Utilizando esta metodología se estimula a los estudiantes ya que se realiza en dispositivos muy empleados en aplicaciones tecnológicas. Además, facilita la comprensión de los principios físicos presentes en el estudio de los elementos piezoeléctricos.

Palabras clave: Thévenin, piezoeléctricas, transductor.

PACS: 77.65.-j, 01.40.-d **ISSN 1870-9095**

I. INTRODUCTION

Piezoelectric ceramics are largely used in technological applications [1, 2]. They are the heart of ultrasonic systems. In some cases, the ultrasonic system is basically composed of a single piezoceramic element. The characterization of these piezoceramics allows us to obtain important subsidies for designing the ultrasonic systems.

Often, the modeling with equivalent electric circuits is used for studying the behavior of piezoelectric discs. The procedures based on electric network theory are easy and largely disseminated. When the equivalent electric circuits are used, the characterization of the piezoelectric device is restricted to one-dimensional modeling. Even so, this simplification is enough for piezoceramic characterization, since the vibrations occur mainly in one direction. Several one-dimensional modeling of piezoelectric transducers based on electric networks are found in the literature [3, 4, 5, 6, 7, 8, 9].

The theories of electricity and acoustics are based on similar differential equations. This fact becomes evident when the modeling for studying acoustic systems uses equivalent electrical circuits because arise parameters that can be dealt in a similar way. Furthermore, the equivalence between theories allows that this issue be approached with teaching purposes. In physics courses, the unified study of these topics is very stimulating and helps the comprehension of similarity of the electric and acoustic laws formulation. The electric formulation embraces the description of the electric circuits based on voltage and electric current. The acoustic formulation is the same when dealt with force and velocity, respectively [10].

The understanding of the physical properties of the piezoelectric ceramics using equivalent electric circuits is not easy for students of introductory semesters. For instance, Mason's model [6, 7] uses virtual concepts such as negative capacitance and electromechanical transformer to describe the behavior of one-dimensional vibration of a piezoceramic plate.

This article aims to illustrate the characterization of piezoelectric ceramics through laws of the network electric studied in elementary classes of electricity. The equivalent electric circuit of a piezoelectric ceramic, described by Mason's model, can be simplified using Thévenin's Theorem [11, 12, 13]. The simplification eliminates the virtual concepts mentioned in the early paragraph and enables their use for educational purposes. Besides, this study improves a didactical proposal for a work previously published by the authors [14].

To reach this objective, we have considered piezoelectric ceramic plates working in transmission mode, *i.e*. converting electrical into mechanical energy, on harmonic excitation and vibrating in the thickness mode. On these conditions, a simple experimental procedure is performed. The experiments and the modelling of electric circuit lead to a full one-dimensional characterization. The full characterization of a piezoceramic consists of determination of the velocity of wave propagation, the acoustic impedance and the dielectric, elastic and piezoelectric coefficients along 3-axis (perpendicular axis relative to the flat surface of the ceramic).

II. THEORY OF EQUIVALENT ELECTRIC CIRCUITS FOR PIEZOELECTRIC PLATES

There are some models based on equivalent electric circuit used to model the behavior of piezoelectric ceramics plates [8]. Mason's model is the most known. Often, the piezoelectric ceramics employed in technological applications are circular shaped (disk or ring) and the radial dimensions are larger than the thickness. The flat surfaces of these devices are coated with metallic electrodes. In the most of the applications, the vibration mode excited is that which the mechanical strain is perpendicular to the flat surface of the ceramics (3-axis). It is called thickness mode. The resonant frequency of the thickness mode of a thin piezoelectric plate is far higher than that of this transversal mode. Fig. 1 shows the typical oscillations of a thin piezoelectric disc.

Mason´s equivalent electric circuit for piezoelectric devices is shown in Fig. 2. A complete description of this circuit is presented in reference [6].

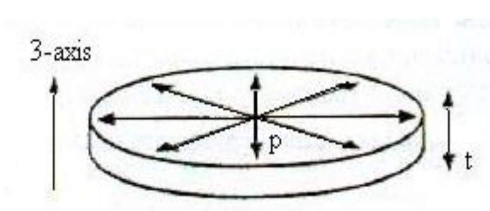

FIGURE 1. Piezoelectric disc and vibration modes representation. The arrows marked by "p" and "t" indicate the planar (or transversal) and the thickness vibration mode, respectively.

The mechanical impedances of this circuit are given by Eqs. (1) and (2)

$$Z_1 = jZ_0 \tan\left(\frac{\omega l_c}{2v_c}\right), \tag{1}$$

$$Z_2 = -jZ_0 \csc\left(\frac{\omega l_c}{v_c}\right), \tag{2}$$

where

v_c is the wave propagation velocity in the piezoelectric medium in m/s;

l_c is the half of the thickness of the piezoelectric ceramic in m;

ω=2πf is the angular frequency in rad/s and f, frequency, given in Hz;

$Z_0 = Z_c A_c$;

Z_c is the acoustic impedance of the ceramic given in kg/m^{2}s;

A_c is the area of the flat surface of the ceramic in m^2.

Besides, the electromechanical transformer in Fig. 2 has turn ratio given by Eq. (3)

$$n = h_{33} C_0 \, , \tag{3}$$

where

C_0 is the intrinsic capacitance of the piezoelectric ceramic in F, when the strain is null, n is the conversion factor of the electromechanical transformer given by Eq. (3).

h_{33} is the piezoelectric coefficient, given in N/C.

The transducers considered in this work are only composed of piezoelectric ceramics. Thévenin's equivalent circuit is derived in a straightforward manner. The center of the piezoelectric ceramic is considered clamped, so the port F_1F_0 is left as open-circuit and the mechanical stress on port F_2F_0 is determined. The impedance for Thévenin's equivalent is "seen" from the port F_2F_0 with the voltage source short-circuited. Using the ideal transformer relations, the reflected equivalence of the mechanical part on the electric one is obtained.

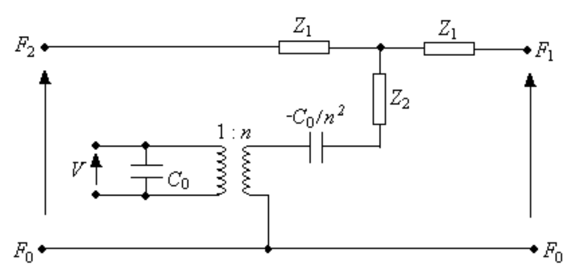

FIGURE 2. Mason´s equivalent electric circuit for a piezoelectric ceramic.

Equations 4 and 5 show the expressions for voltage source and electric impedance correspondent to mechanical part, respectively.

$$V_{Thel} = V_1, \quad (4)$$

where

V_1 is the voltage between the primary terminals of the electromechanical transformer. This voltage is the same to that applied in the ceramic electrodes.

$$Z_{Thel} = \frac{-jZ_0 \cot\left(\frac{\omega}{v_c} l_c\right)}{n^2} - \frac{1}{j\omega C_0}. \quad (5)$$

Fig. 3 shows the equivalent electric circuit obtained. We can see that this circuit is a modified Thévenin's equivalent. Any complex network can be reduced to Thévenin equivalent circuit. The circuit of Fig 3 has a capacitor in parallel to the ac source. The capacitor C_0 could be included in Thévenin impedance, however, the separation in two branches improves and becomes evident the didactic purpose of the circuit. This circuit has two branches: one electrical, formed by the capacitor C_0 and other mechanical, assigned by Z_{Thel}. The energy driven by a voltage source is divided between these branches. The electrical branch stores the electric energy provided by voltage source. The energy that is converted in the piezoelectric process goes to mechanical impedance given by Eq. (5).

The circuit of Fig. 3 represents only one half of the ceramic from the clamped center to the one end. Another equivalent electric circuit should be used to represent the second half of the ceramic. However, in this case, both circuits are the same because the whole system is symmetrical. Thus, it is not necessary to use two circuits to characterize the piezoelectric ceramic.

The voltage source that provides harmonic signals is connected to the electrodes of the ceramic at frequency near to thickness mode resonance. Being the transducers set up only by ceramics, these can vibrate freely because the mechanical impedance of the load (air) is negligible. Thus, the outputs of the circuit of Fig. 3, denoted by A and B, are short-circuited to represent this operation condition.

If the signal driven by source is at anti-resonance of the ceramic, the effects of the electrical and mechanical branches reactancies cancel themselves. This means that at anti-resonance the values of the reactancies of C_0 and Z_{Thel} have the same modula, but opposite signals. Eq. (6) is obtained from this equality

$$0 = Z_0 \cot\left(\frac{\omega_{ac}}{v_c} l_c\right), \quad (6)$$

where

ω_{ac} is the angular anti-resonance of the piezoelectric ceramic in thickness mode.

Putting the argument of cotangent function of Eq. (6) as $\pi/2$, the wave propagation along 3-axis in the ceramic is determined. Jointly with density and cross section, that can be easily measured, the mechanical impedance, Z_0, is determined too.

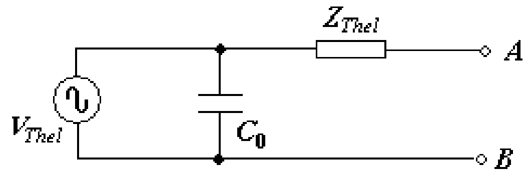

FIGURE 3. Modified Thévenin's equivalent circuit for piezoelectric ceramic.

At resonance, the reactance of the mechanical branch is null. This means that Z_{Thel}, from Eq. (5), is zero and, consequently, we obtained Eq. (7).

$$\frac{n^2}{\omega_{rc} C_0} = Z_0 \cot\left(\frac{\omega_{rc}}{v_c} l_c\right), \quad (7)$$

where ω_{rc} is the angular resonance of piezoelectric ceramic in the thickness mode.

The value of n is calculated replacing the experimental values of resonance, f_r, and C_0 in the Eq. 7. So, by using Eq. 3, h_{33} is calculated too.

Equations (8) and (9) allow us to determine the dielectric (ε_{33}^S, in F/m) and elastic (c_{33}^D, in N/m^2) coefficients of the ceramics.

$$\varepsilon_{33}^S = C_0 \frac{2l_c}{A_c}, \quad (8)$$

$$c_{33}^D = \rho_c v_c^2, \quad (9)$$

where ρ_c is the density of the ceramic.

III. EXPERIMENTAL PROCEDURE

The simple electric model shown in Fig. 3 is now applied to fully characterize various piezoelectric samples. Four piezoelectric ceramics were used in the experiments. These ceramics are three discs (named PC1, PC2 and PC3) and one ring (PC4). The masses and the dimensions of these ceramics are listed in Table I.

The thickness vibration mode of these ceramics has resonance at range from 0.3 to 1.2MHz. Usually, the ceramic discs are employed in ultrasonic transducers for applications in physiotherapy, fetal detection and cleaning tanks. The piezo ring is used in sandwiched transducer for high power applications. The proper ceramic material for these transducers is either PZT4 or similar itself. In this work, the experimental results are compared with physical characteristics of PZT4. Reference [6] gives a full table with these physical characteristics.

From values presented in Table I are determined the area of the electrodes, the volume of the ceramics and the density.

The resonance and the anti-resonance relative to the thickness vibration mode are measured by using the resonance method [15]. Fig. 4 shows the experimental setup for this measurement. In Fig. 4, G is the signal generator (Tektronix – AFG 3021B), R is the shunt resistor R = 51Ω, T is the piezoelectric ceramic and CH1 and CH2 are the channels of the osciloscope (Tektronix – TDS 1002). Since the voltage in CH1 is constant, the electric current that flows in the circuit is proportional to the voltage in CH2. Therefore, the maximum (minimum) voltage in CH2 corresponds to the maximum (minimum) current and the minimum (maximum) impedance of the ceramic. The minimum and the maximum values for the electric impedance of the ceramic are found at resonance and anti-resonance, respectively.

TABLE I. Mass, thickness and diameter of the piezoelectric ceramics.

Ceramic	PC1	PC2	PC3	PC4
Mass (g)	0.50	44.72	7.44	47.37
Thickness (mm)	2.0	3.0	2.0	6.3
Inner Diameter (mm)	X	X	X	12.6
Outer Diameter (mm)	6.4	50.0	25.2	38.0

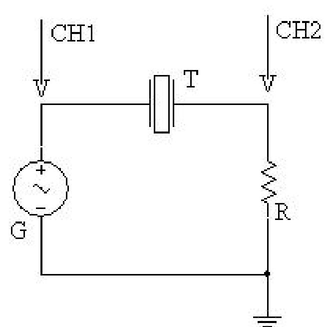

FIGURE 4. Experimental setup used for measuring of resonances of the piezoelectric ceramics.

From experimental values of resonance and anti-resonance, the electromechanical coupling factor is determined using Eq. 10.

$$k_t^2 = \frac{\pi}{2}\frac{f_{rc}}{f_{ac}}\tan\left[\frac{\pi}{2}\left(1-\frac{f_{rc}}{f_{ac}}\right)\right]. \quad (10)$$

The electromechanical coupling factor is an indicator of the efectiveness with which the piezoelectric ceramic converts energy. The subscript $_t$ indicates the thickness vibration mode.

The measurements of capacitance of the ceramics are done at frequency of 1 kHz by using a capacimeter. In this case, the obtained value corresponds to C_0^T, that is, the capacitance of the ceramic on null stress in the surfaces. The value required by modeling is the capacitance of the ceramic with null strain in the surface, C_0, which is measured at frequencies very large and superior to the fundamental vibration modes. From the point of view of the experimental practice, this can be problematic due the high frequencies. An alternative to overcome this problem is to use the value of C_0^T and determine C_0 with Eq. 11.

$$C_0 = C_0^T(1-k_t^2)\,. \quad (11)$$

From Eq. 11, the dielectric permittivity is calculated through of Eq. 8.

All steps of this procedure, the ceramics were placed in a proper support to avoid the effects of the contact with mechanical loads.

Inserting the set of experimental values in Eqs. (6) to (9), the wave propagation velocity, acoustic impedance and piezoelectric, dielectric and elastic coefficients of the piezoelectric ceramics are determined.

IV. RESULTS AND DISCUSSIONS

Table II shows the values of resonance, anti-resonance, electromechanical coupling factor, and the capacitances C_0^T and C_0.

The values of the velocity of wave propagation, the acoustic impedance and the dielectric, elastic and piezoelectric coefficients from PZT4 [6] and the experimental procedure are shown in Table III.

TABLE II. Experimental values of intrinsic capacitance, resonance and anti-resonance.

	PC1	PC2	PC3	PC4
f_{rc} (MHz)	1.044	0.682	1.038	0.318
f_{ac} (MHz)	1.145	0.762	1.118	0.343
k_t	0.45	0.48	0.41	0.41
C_0^T (nF)	0.27	8.18	3.50	3.15
C_0 (nF)	0.22	6.27	2.92	2.62

TABLE III. Values of wave propagation velocity, acoustic impedance and dielectric, elastic and piezoelectric coefficients from PZT4 and experimental procedure.

	PC1	PC2	PC3	PC4	PZT4
v_c(m/s)	4580	4572	4472	4322	4600
Z_0(kg/m^2 s) $\times 10^6$	35.6	34.9	33.2	32.2	34.5
ε_{33}^S (nF/m)	13.4	9.6	11.7	16.4	12.0
c_{33}^D(N/m^2) $\times 10^{10}$	16.3	16.0	14.8	13.9	15.9
h_{33}(N/C) $\times 10^8$	22.0	27.9	20.4	16.9	26.8

The comparison of experimental and expected values for PZT4 shows that the discrepancy is small. The major differences of the results can be attributed to the manufacturer process and chemical composition variation of the ceramic material.

Some characteristics of the proposed methodology can be listed as significant and enabler in the apprenticeship process:

- In the proposed modeling, abstract concepts such as negative capacitance [16] and electromechanical transformer that arise in Mason's model are not explicitly shown. From the point of view of the teaching for students from introductory semesters of physics courses, with small or none knowledge in piezoelectricity, the purpose will help them to simplify the understanding of the phenomenon;
- The new circuit is a current divisor, but bring an own physical sense. The branch of the capacitor (C_0) stores the energy that is not converted in mechanical strain. This energy is stored in the ceramic such as a conventional capacitor does. The other branch (Z_{Thel}) stores the elastic energy resultant from the piezoelectric conversion, responsible for the vibration of the ceramic's surface.
- The experimental data necessary to the determination of the ceramic parameters are obtained through a very simple experimental procedure. The experimental setup used in this work employs conventional oscilloscope and function generator. These equipments are in according to the sophistication pattern of a typical teaching laboratory. Though, the measurements performed with these equipments are enough to illustrate and point out the resonance and anti-resonance in the process, for the finest precision, it is recommended to use an Impedance Analyzer;
- The procedure allows us to study the electric behavior of the piezoelectric ceramics and illustrates the application of Thévenin's Theorem in teaching for students with elementary knowledge about electric network theory;
- Resonance and anti-resonance are forwardly obtained by experimental procedure. The analysis of the equivalent circuit links the resonance and anti-resonance with the impedances of the mechanical and electrical branches;
- The new circuit has an important property: The representation of the mechanical parts in Mason's model is dealt as an electric device. This fact resumes the characterization of the transducer to the analysis of an electric circuit and the conversion between mechanical and electric parameters is defined by *n*. The physical sense of *n* is revealed with the virtual electromechanical transformer presented in Fig. 2. This electromechanical transformer has ratio of turns of 1:*n*. The relation of an ideal electromechanical transformer are $V_p/V_s = 1/n$ and $I_p/I_s = n$, where the subscripts "*p*" and "*s*" mean primary (electric circuit) and secondary (mechanical circuit), respectively. For instance, the electric current flux through the secondary represents the vibration velocity in the ceramic's surface. This velocity can be calculated by the product of the electric current in the primary and *n*. Therefore, as the circuit presented in this paper is purely electric, just as electric current flux in Z_{Thel} is converted into vibration velocity;
- The results encourage use hybrid devices for teaching electric networks. The use of piezoelectric devices employed in several technological applications results in motivation for the students.

V. CONCLUSIONS

This method can be applied in classes and laboratory and allows to the instructor other developments for showing the methods of analysis of electric networks. Both the theoretical and the experimental parts are simple and allow to the student of introductory semesters and with basic knowledge in physics to be able to understand the process for characterization of piezoelectric plates. Furthermore, the proposed method introduces a new tool to teach, simultaneously, concepts about electric networks and piezoelectricity.

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