The classical Doppler effect as a rendezvous problem; 3D case

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(Received 15 February 2013, accepted 2 October 2013)

Abstract

We derive the classical Doppler shift formula as a rendezvous problem, unifying all possible relative movements between the source and the observer.

Keywords: Doppler effect, Kinematics, rendezvous problem.

Resumen

En este trabajo se deriva la fórmula para el corrimiento Doppler como si tratásemos un problema de encuentro, unificando todos los posibles movimientos relativos entre la fuente y el observador.

Palabras clave: Efecto Doppler, Cinemática, Problemas de encuentro.

PACS: 01.40.gb, 42.25.Bs **ISSN 1870-9095**

I. INTRODUCTION

The frequency change of a wave as measured by an observer *O (=f')* with respect to the frequency emitted by a source *S (=f)* when there are a relative motion between *O* and *S*, is known as the Doppler effect. This effect occurs for acoustic waves, when are valid the Galileo transformation equations, as well as for electromagnetic waves, when govern the Lorentz transformation equations. In this work, we treat only the mechanical waves.

Although this topic is treated in practically all elementary texts (always for material media at rest), the deduction of the relation *f' vs f* is presented in a highly non-unified form. Generally speaking, the texts: i) consider separately the condition source at rest - observer in motion from the reciprocal case: source in motion - receptor at rest. After presenting these two particular cases, is considered the case of the relative motion of source and observer with respect to the quiet media. But this last condition is not deduced but appears phrases like *"it can be deduced..."* [1] or *"... the previous results can be combined..."* [2], and ii) in this last case, usually presented in the form [1]

$$f'/f = (v \pm v_{obs})/(v \mp v_{source})$$

is lacking in clarity, because the student may believe that there are only two cases, one in which the upper signs are used and one in which the lower signs are used. There are, of course, four cases, as said clearly in Ref. [3]. In some texts, the used sign convention suggests that the positive direction is (necessarily!) from the observer to the source, whereas in others, the positive direction is from left to right [4], as if it were mandatory.

It is interesting to note that, oddly, two renowned specific texts about waves are silent (or almost silent) about the Doppler effect. In facts, the book by Crawford [5] not treat this topic, whereas the treatment by French [6] is brief and not general. The more recent text by Hirose and Lonngren [7] make only a elementary discussion, not treating the general case of relative motion of source and receptor.

Such dispersion of particular cases is against the philosophy taught in Kinematics, where the recipe is something like: take a coordinate origin, a direction *arbitrarily* defined as positive and a time origin, and refers the position and the time with respect to the above defined origins. And respects such conventions in a given problem and change it, if you like, in other ones!

In the last years, diverse authors submitted to the Journals better and more unified deductions, as the cases, e.g. of Kapoutlitsas [8], Neipp *et. al.* [9] and Ma *et. al.* [10], to cite a few. Our brief contribution is to show that the Doppler effect can be deduced similarly to the acclaimed case of a rendezvous between Achilles and the tortoise, unless in our case we need two tortoises. Here, Achilles play the observer role whereas the two tortoises play the roles of two succesive wave crests (or pulses): *the Doppler effect can be treated as a rendezvous problem,* typical of Kinematic chapters.

We will show that it is necessary only one formula, condensing all cases, and is

$$\frac{f'}{f}=\frac{v-\vec{v}_O.\vec{n}}{v-\vec{v}_S.\vec{n}};\qquad(1)$$

all the particular cases are deduced from Eq. (1), as will be explained at the end of the text.

II. THEORY

We consider the general case where the positions of both the wave source and the observer are described in *3D*; the basic relation between the received frequency and the emitted one is found considering the classical Doppler effect as a rendezvous problem between the observer and two successive crest waves.

Let a material medium at rest and a source *S* whose motion equation, with respect to the medium, is given by $\vec{r}_S(t)=\vec{r}_{S0}+\vec{v}_St$, emitting crest waves (or pulses) with frequency *f*. Also, let an observer (or receiver) *O*, whose motion is described by $\vec{r}_O(t)=\vec{r}_{O0}+\vec{v}_Ot$, and $\vec{n}$ an unit vector directed from the point at which the waves are emitted to the point at which they were received [3], such the wave velocity with respect to the isotropic rest medium is *v*. Projecting the above equations on axis $\vec{n}$, we have

$$\vec{r}_S(t).\vec{n}=\vec{r}_{S0}.\vec{n}+(\vec{v}_S.\vec{n})t\,,\qquad(2)$$

and

$$\vec{r}_O(t).\vec{n}=\vec{r}_{O0}.\vec{n}+(\vec{v}_O.\vec{n})t\,.\qquad(3)$$

At *t=0*, when *S* is at $\vec{r}_S(0)=\vec{r}_{S0}$, emits the first pulse p_1, traveling with velocity *v* $(v>v_S,v_O)$. After a period *T(=1/f)*, when *S* travelled $\vec{v}_ST$ and therefore is at $\vec{r}_S(T)=\vec{r}_{S0}+\vec{v}_ST$, emits the second pulse p_2. Their respective motion equations are, after projecting on $\vec{n}$

$$\vec{p}_1(t).\vec{n}=\vec{r}_{S0}.\vec{n}+vt\,,\qquad(4)$$

and

$$\vec{p}_2(t).\vec{n}=\vec{r}_{S0}.\vec{n}+(\vec{v}_S.\vec{n})T+v(t-T).\qquad(5)$$

The first encounter between *O* and *S* occurs in $t=t_1$, when $\vec{r}_O(t_1)=\vec{p}_1(t_1)$; from Eqs. (3) and (4), t_1 is given by

$$t_1=\frac{(\vec{r}_{O0}-\vec{r}_{S0}).\vec{n}}{v-\vec{v}_O.\vec{n}}.\qquad(6)$$

Analogously, for $t=t_2$, when $\vec{r}_O(t_2)=\vec{p}_2(t_2)$, from Eqs. (3) and (5)

$$t_2=\frac{(\vec{r}_{O0}-\vec{r}_{S0}).\vec{n}+T(v-\vec{v}_S.\vec{n})}{v-\vec{v}_O.\vec{n}}.\qquad(7)$$

The period measured by the observer is $T'=t_2-t_1$; from Eqs. (6) and (7)

$$T'=\frac{T(v-\vec{v}_S.\vec{n})}{v-\vec{v}_O.\vec{n}}\,,\qquad(8)$$

and, therefore, the *general basic relation* is the *unique* formula

$$\frac{f'}{f}=\frac{v-\vec{v}_O.\vec{n}}{v-\vec{v}_S.\vec{n}}.\qquad(9)$$

If the source and the observer move along the same direction, say on x-axis, where $\vec{n}=\vec{i}$, then we have the simpler relation

$$\frac{f'}{f}=\frac{v-v_O}{v-v_S}\,,\qquad(10)$$

found in standard textbooks (see, for example, Refs. [1] and [7]). In some texts, the above equation is written in the form

$$\frac{f'}{f}=\frac{v\pm v_O}{v\mp v_S}\,,$$

indicating that there are four cases. However, this is unnecessary if we take into account that, fixing the positive sign for *v*, the other celerities must be measured with respect to the same convention. From Eqs. (9) and (10), posterior disquisitions can be made as can be found, for example, in the papers by Kapoulitsas [8] and Neipp *et al.* [9] or in undergraduate textbooks.

ACKNOWLEDGEMENTS

The supports from the Facultad de Ciencias Exactas (UNCPBA) and the Consejo Nacional de Investigaciones Científicas y Técnicas, Argentina, are gratefully acknowledged.

REFERENCES

[1] Resnick, R. y Halliday, D., *Física*, Parte I (C.E.C.S.A., México, 1970).
[2] Tippler, P. A., *Física*, Parte I (Reverté, Barcelona, 1995).
[3] Erlichson, H., *The classical Doppler effect and intuition*, Am. J. Phys. **45**, 1227-1228 (1977).
[4] Alonso, M. y Finn, E. J., *Física*, Volumen II (Fondo Educativo Interamericano, México, 1985).

[5] Crawford, F. S., *Ondas* (Reverté, Barcelona, 1971).
[6] French, A. P., *Vibraciones y Ondas* (Reverté, Barcelona, 1974).
[7] Hirose, A. and Lonngren, K. E., *Introduction to Wave Phenomena* (Krieger Publishing Company, Malabar, 1985).
[8] Kapoulitsas, G. M., *On the non-relativistic Doppler effect*, Eu. J. Phys. **2**, 174-177 (1981).
[9] Neipp, Hernández, C. A., Rodes, J. J., Márquez, A., Beléndez, T. and Beléndez, A., *An analysis of the classical Doppler effect*, Eu. J. Phys. **24**, 497-505 (2003).
[10] Ma, L., Yang, J. and Nie, J., *Doppler Effect of Mechanical Waves and Light*, Lat. Am. J. Phys. Educ. **3**, 550-552 (2009).