The rotating reference frame and the precession of the equinoxes

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(Received 7 August 2013, accepted 27 December 2013)

Abstract
In this paper, angular frequency of a rotating reference frame is derived, where a given body orbiting and rotating around a fixed axis describes a periodical rosette path completing a periodical "circular motion". Thus, by the analogy between a rotating frame of reference and the whole Solar System, angular frequency and period of a possible planetary circular motion are calculated. For the Earth's case, the circular period is calculated from motion of the orbit by the rotation effect, together with the advancing caused by the apsidal precession, which results the same amount than the period of precession of the equinoxes. This coincidence could provide an alternative explanation for this observed effect.

Keywords: Rotating reference frame, Lagranian mechanics, Angular frequency, Precession of the equinoxes.

Resumen
En este trabajo, se deriva la frecuencia angular de un marco de referencia rotante, donde un cuerpo dado orbitando y rotando alrededor de un eje fijo describe una trayectoria en forma de una roseta periódica completando un "movimiento circular" periódico. Así, por la analogía entre un marco de referencia rotacional y el Sistema Solar como un todo, se calculan la frecuencia angular y el periodo de un posible movimiento circular planetario. Para el caso de la Tierra, el periodo circular es calculado a partir del movimiento de la órbita por el efecto de la rotación, junto con el avance causado por la precesión absidal, resultando el mismo valor que el periodo de la precesión de los equinoccios. Esta coincidencia podría proveer una explicación alternativa para este efecto observado.

Palabras clave: Marco de referencia rotante, Mecánica Lagraniana, Frecuencia angular, Precesión de los equinoccios.

PACS: 03.65.Sq, 96.12.De, 96.30.Bc **ISSN 1870-9095**

I. INTRODUCTION

As known, one of the Earth's movements is the precession of the equinoxes (also called precession of the equator). This movement is currently considered an effect by the Earth's axial movement, whereby the axis slowly moves tracing out an imaginary cone with respect to the "fixed" stars, rotating completely around 360° in a period of about 25,771.5 years (about 0.0139689° per year) called Platonic year. During such a period, the visible positions of stars as measured in the equatorial coordinate system will slowly change. Over this cycle the Earth's North axial pole moves from where it is now in a circle about the ecliptic pole, with an angular radius in average of 23.45°. A consequence of the precession of the equinoxes is a changing pole star. For instance, currently Polaris star has the position of the North celestial pole. According to the observations [1], in the year 2100 AC, the Earth's North Pole will appoint at only 0.5º of Polaris. Later, at around the year 14000 AC, Earth's North Pole will appoint to Vega star (in the constellation Lyra).

On the other hand, we have that a given body orbiting and rotating around a fixed axis describes a periodical rosette path completing a "circular motion" in a given period. Thus, by the analogy between a rotating reference frame and the whole Solar System, angular frequency and period of a possible planetary circular motion are calculated. For instance, in the Earth's case, the circular period is calculated from the motion of the orbit by the rotation effect, together with the advancing caused by the apsidal precession, which results the same amount than the period of precession of the equinoxes. This coincidence could provide an alternative explanation for this observed effect.

II. ORBITING AROUND A FIXED AXIS IN A STATIC FRAME OF REFERENCE

In order to describe the difference between path of a body orbiting around a fixed axis in a static frame of reference, and path of a body orbiting and rotating around a fixed axis in a rotating reference frame, let us first briefly to describe each path. The orbit of a body around a fixed axis in a static frame of reference (it means, where only the bodies are in motion, but not the frame of reference), orbital path can be described by the Kepler's Laws [2], which states that the orbit of every planet is an ellipse with the Sun at one of the two foci. Third Kepler's law states that the square of the orbital period of a planet is proportional to the cube of the semi-major axis of its orbit, giving

$$T^2 \propto a^3 \therefore \frac{a^3}{T^2} \propto K, \tag{1}$$

where a is the semi-mayor axis of the ellipse, T is the period to complete an orbit and K is a constant. Elliptical path in polar coordinates is given by

$$r = \frac{a\left(1-e^2\right)}{1-e\cdot\cos\omega}, \tag{2}$$

where e is the eccentricity of the elliptical path and ω is the angular frequency (also called angular velocity) of the periodic orbit, like a simple harmonic oscillator, given by

$$\omega = \frac{2\pi}{T}. \tag{3}$$

Although most orbits are elliptical in nature, a special case is the circular orbit, which can be considered as an ellipse of zero eccentricity, where $a \rightarrow r$, being r the radius of the circular orbit. This consideration simplifies further calculations. Then, for a circular orbit, expression (1) is reduced, giving

$$\frac{r^3}{T^2} \propto K. \tag{4}$$

In the circular orbit, velocity is defined as

$$v = \frac{2\pi r}{T}, \tag{5}$$

where v is the orbital velocity. In order to find out constant K, both terms of expression (5) can be multiplied by vr, and reordering, yields

$$v^2 r = \frac{(2\pi)^2 r^2}{T^2} r \therefore \frac{r^3}{T^2} = \frac{v^2 r}{(2\pi)^2} = \frac{\mathrm{a} r^2}{(2\pi)^2} = K, \tag{6}$$

where a is the centripetal acceleration of the given body in the circular system and K keeps constant. Then, acceleration must be also constant. Thus, for a circular orbit, relation between accelerated circular motion and the Newtonian constant of gravitation multiplied by the mass is defined by

$$v^2 r = \mathrm{a} r^2 = GM \therefore \mathrm{a} = \frac{GM}{r^2}, \tag{7}$$

where G is the Newtonian constant of gravitation, M is the mass of a large body (as that of the Sun) and r is the distance from the center of the mass M. Substituting expression (7) in (6), yields

$$\frac{r^3}{T^2} = \frac{\mathrm{a} r^2}{(2\pi)^2} = \frac{GM}{(2\pi)^2} = K, \tag{8}$$

which is the Newton's solution for the planetary motion [3].

On the other hand, as observed, other rotational effects are present in the dynamics of a body in elliptical orbit around a fixed axis, such as the apsidal precession that increases during the perihelion. This effect slowly changes the position of a body orbiting as a displacement of precession towards the direction of rotation. For the planetary apsidal precession, angle of precession $d\varphi$ per cycle due to the advance of perihelion, from the General Relativity [4] is defined as

$$d\varphi = \frac{24\pi^3 a^2}{c^2 T^2\left(1-e^2\right)} = \frac{6\pi GM}{a\left(1-e^2\right)c^2}, \tag{9}$$

where c is the speed of light. Thus, angular frequency of apsidal precession is given by

$$\omega_\varphi = \frac{\mathrm{d}\varphi}{t_\mathrm{P}} = \frac{6\pi GM}{a\left(1-e^2\right)c^2 t_\mathrm{P}}, \tag{10}$$

where t_P is the time of each period in seconds.

Expression for the ellipse in polar coordinates with the pole at a focus, also including the rate of apsidal precession, is given by

$$r = \frac{a\left(1-e^2\right)}{1-e\cdot\cos(\theta+d\phi)} = \frac{a\left(1-e^2\right)}{1-e\cdot\cos(\omega t+\omega_\phi t)}, \tag{11}$$

where angle $\theta = \omega t$ and $d\phi = d\omega_\phi t$. Replacing data of the Sun-Earth system in expression (11), with the maximums distance r (during the aphelion) and the related minimum orbital velocity v, being t_P the time in seconds of a sidereal year (365.256 days), angular frequency of apsidal precession equals 1.86123×10^{-7} radians per cycle (3.8391" of arc per century). Nevertheless, according to the observations [5], apsidal precession of the Earth is measured in about of 5" ± 1.2 of arc per century.

III. ORBITING AND ROTATING AROUND A FIXED AXIS IN A ROTATING REFERENCE FRAME

Let us now consider the general case of a rotating reference frame and fixed frame being related by translation and rotation [6]. Position of a point P according to the fixed frame of reference is named r', while some position of a same point according to the rotating reference frame is named r, and

$$r' = R + r, \tag{12}$$

where R denotes the position of the origin of the rotating frame according to the fixed frame. Since the velocity of the point P involves the rate of change of position, we can define two time-derivative operators, for $(d/dt)_f$ or $(d/dt)_r$, respectively. The velocities of point P as observed in the fixed and rotating frames are defined as

$$v_f = \left(\frac{dr'}{dt}\right)_f, v_r = \left(\frac{dr}{dt}\right)_r, \tag{13}$$

respectively. The relation between the fixed-frame and rotating-frame velocities is expressed as

$$v_f = \left(\frac{dR}{dt}\right)_f + \left(\frac{dr}{dt}\right)_f = V + v_r + \Omega\times r, \tag{14}$$

where $V = (dR/dt)_f$ denotes the translation velocity of the rotating-frame origin (as observed in the fixed frame of reference) and Ω is the angular frequency of the rotating reference frame. Using expression (13), expressions for the acceleration of point P as observed in the fixed and rotating frames of reference are given by

$$\mathrm{a}_f = \left(\frac{dv_f}{dt}\right)_f, \mathrm{a}_r = \left(\frac{dv_r}{dt}\right)_r, \tag{15}$$

respectively. Hence, using expression (14), we find

$$\mathrm{a}_f = \left(\frac{dV}{dt}\right)_f + \left(\frac{dv_r}{dt}\right)_f + \left(\frac{d\Omega}{dt}\right)_f \times r + \Omega\times\left(\frac{dr}{dt}\right)_r. \tag{16}$$

Solving expression (16) and reordering, yields

$$\mathrm{a}_f = A + \mathrm{a}_r + 2\Omega\times v_r + \dot{\Omega}\times r + \Omega\times(\Omega\times r), \tag{17}$$

where $A = (dV/dt)_f$ denotes the translational acceleration of the rotating-frame origin (as observed in the fixed frame of reference). We can now write an expression for the acceleration of point P as observed in the rotating frame as

$$\mathrm{a}_r = \mathrm{a}_f - A - \Omega\times(\Omega\times r) - 2\Omega\times v_r - \dot{\Omega}\times r, \tag{18}$$

which represents the sum of the net inertial acceleration (a_f - A), where the centrifugal acceleration is given by $-\Omega \times (\Omega \times r)$, and the term $-2\Omega \times v$ is the Coriolis acceleration a_c which only depends on velocity.

IV. LAGRANGIAN FORMULATION OF NON-INERTIAL MOTION

The Lagrangian [7] for a particle of mass m moving in a non-inertial rotating frame (with its origin coinciding with the fixed-frame origin) in the presence of the potential $U(r)$ is expressed as

$$L(r,\dot{r}) = \frac{m}{2}\left|\dot{r} + \Omega\times r\right|^2 - U(r), \tag{19}$$

where Ω is the angular velocity vector. It can be expressed as

$$\left|\dot{r} + \Omega\times r\right|^2 = \left|\dot{r}\right|^2 + 2\Omega\cdot(r\times\dot{r}) + \left[\Omega^2 r^2 - (\Omega\cdot r)^2\right]. \tag{20}$$

Applying the Lagrangian (19), we can derive the general Euler-Lagrange equation for the r term. Thus, expression for the momentum is defined as

$$p = \frac{\partial L}{\partial \dot{r}} = m(\dot{r} + \Omega\times r), \tag{21}$$

and

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) = m(\ddot{r} + \dot{\Omega}\times r + \Omega\times\dot{r}). \tag{22}$$

Partial derivative is given by

$$\frac{\partial L}{\partial r} = -\nabla U(r) - m\left[\Omega\times\dot{r} + \Omega\times(\Omega\times r)\right], \tag{23}$$

so that the Euler-Lagrange equation is defined as

$$m\ddot{r} = -\nabla U(r) - m\left[\dot{\Omega}\times r + 2\Omega\times\dot{r} + \Omega\times(\Omega\times r)\right]. \tag{24}$$

Here, the potential energy term generates the fixed-frame acceleration, $-RotU = m\mathrm{a}_f$, and thus the Euler-Lagrange equation (24) yields expression (18).

Coriolis acceleration in a rotating reference frame implies a displacement of a given body by the drag effect [8]. Equivalence between Coriolis acceleration in a rotating reference frame and the velocity in a circular frame is given by

$$\mathrm{a}_c = \frac{v^2}{r} = 2\Omega v \therefore v = 2\Omega r. \tag{25}$$

where a_c is the Coriolis acceleration.

Having the equivalence where $v = \mathrm{a}t$, expression (25) with respect to the time gives

$$\mathrm{a}_c = 2\Omega \mathrm{a}t. \tag{26}$$

Integrating two times with respect to the time, yields

$$x_c = \frac{1}{3}\Omega a t^3, \quad (27)$$

where x_c is the displacement towards the opposite direction of rotation.

In order to get an idea of the magnitude of the displacement x_c, let us consider a free-falling object which dropped down a 100-meter shaft at the equator. The time to reach the ground is given by

$$h = \frac{1}{2}gt^2 \therefore t = \sqrt{\frac{2h}{g}}, \quad (28)$$

where h is the height and g is the gravitational acceleration in the Earth (9.8 m/s^2). Then, replacing expression (28) in (27), final displacement is defined as

$$x_c = \frac{1}{3}\Omega g\left(\frac{2h}{g}\right)^{3/2}. \quad (29)$$

Thus, replacing values in expression (29) total displacement is about 2.2 cm. This is small in comparison to the 100 meter drop, but it is certainly measurable.

In addition, we can derive the angular frequency of the rotating reference frame related by the velocity from the expression (27) for a radius r from a fixed axis, where $x_c = r$, and reducing terms, yields

$$r = \frac{1}{3}\Omega a t^3 = \frac{1}{3}\Omega r t, \quad (30)$$

reordering expression (30) and replacing with velocity given in (25), yields

$$v = \frac{r}{t} = 2\Omega r = \frac{1}{3}\Omega r. \quad (31)$$

Solving for the square of angular frequency, hence

$$\Omega^2 = \frac{1}{6r}\Omega^2 r \therefore \Omega = \sqrt{\frac{a}{6r}} = \sqrt{\frac{v^2}{6r^2}} = \frac{1}{\sqrt{6}}\frac{v}{r}, \quad (32)$$

which represents the angular frequency of a rotating reference frame related by the velocity.

V. THE ANGULAR FREQUENCY IN TERMS OF SPEED OF LIGHT

It is possible to derive equivalent expression for (32) related with the speed of light by considering the relation between the escape velocity exerted by a central body and the orbital velocity of a second body orbiting such a central body [9]. Thus, considering a body in elliptical orbit around of a central body, distance of maximum approach from the center of mass is r_1 and the one of maximum distance is r_2, where $r_1 < r_2$, so that the velocity for each position are v_1 and v_2, respectively, where $v_1 > v_2$. The constant of angular momentum and of the energy allows one to relate these four magnitudes, giving

$$mr_1v_1 = mr_2v_2, \quad (33)$$

where m is the mass of the body orbiting, r is the radius from the fixed axis and v is the velocity.

We can express the velocity v_2 in terms of the escape velocity v_e of the central body [9], giving

$$v_2 = \frac{v_e^2}{v_1} - v_1. \quad (34)$$

Substituting equivalence (34) in expression (33), we get

$$r_1v_1 = r_2\left(\frac{v_e^2}{v_1} - v_1\right), \quad (35)$$

and finding out for square of escape velocity multiplied by the radius r_2, yields

$$v_e^2 r_2 = v_1^2(r_1 + r_2). \quad (36)$$

Considering the special case of the circular orbit (which can be considered as an ellipse of zero eccentricity), where the radius of the circumference is $r_o = r_1 = r_2$, then equivalence (36) takes the form

$$v_e^2 r_e = v_o^2(r_o + r_o) = 2v_o^2 r_o = v_o^2 D_o, \quad (37)$$

where r_o is the radius of the circular orbit, v_o is the orbital velocity and D_o is the diameter of the circular orbit. Then, rate between escape velocity and orbital velocity, yields

$$v_e^2 = 2v_o^2\left(\frac{r_o}{r_e}\right) \therefore \frac{v_o^2}{v_e^2} = \frac{v_o^2}{2v_o^2\left(\frac{r_o}{r_e}\right)} = \frac{r_e}{2r_o}. \quad (38)$$

Then, including rate between orbital velocity and escape velocity, multiplying expressions (32) by (38), yields

$$\Omega_v^2 = \Omega^2\frac{v_o^2}{v_e^2} = \frac{a}{6r}\frac{v_o^2}{v_e^2} = \frac{ar^2}{6r^3}\frac{v_o^2}{2v_o^2\left(\frac{r_o}{r_e}\right)}. \quad (39)$$

where Ω_v is the angular frequency with the proportion between orbital velocity and escape velocity. When the escape velocity is tending to the speed of light [9], the equivalence (38) can be written as

$$v_e^2 = 2v_o^2\left(\frac{r_o}{r_e}\right) \rightarrow c^2 = 2v_o^2\left(\frac{r_o}{r_S}\right), \tag{40}$$

where r_s is the Schwarzschild radius [10]. Replacing expression (40) in (39), hence

$$\Omega_v^2 = \frac{\left(\mathrm{a}r^2\right)v_o^2}{6c^2r^3} = \frac{GMv_o^2}{6c^2r^3} \therefore \Omega_v = \sqrt{\frac{GMv_o^2}{6c^2r^3}}. \tag{41}$$

Furthermore, having the contribution in the circular motion of the apsidal precession given by the slight motion of the planet in the direction of rotation, total angular frequency Ω_S for a period of circular motion (for a given body orbiting and rotating) is approximately the addition of both angular frequencies, (10) and (41), hence

$$\Omega_S = \Omega_v + \omega_\varphi = \sqrt{\frac{GMv_o^2}{6c^2r^3}} + \frac{6\pi GM}{a\left(1-e^2\right)c^2t_P}. \tag{42}$$

Then, for one revolution (2π radians), total period is given by

$$T_S = \frac{2\pi}{\Omega_S} = \frac{2\pi}{\sqrt{\dfrac{GMv_o^2}{6c^2r^3} + \dfrac{6\pi GM}{a\left(1-e^2\right)c^2t_P}}}. \tag{43}$$

VI. THE ROSETTE PATH OF A BODY ORBITING AND ROTATING AROUND A FIXED AXIS

The elliptical path of a body periodically orbiting around a fixed axis in a static circular frame of reference as describe by the Kepler's Laws shows a simple harmonic oscillator with angular frequency ω. Elliptical path in parametric equations, is given as

$$\begin{aligned} x &= a\cdot\cos(\omega t) \\ y &= b\cdot\sin(\omega t), \end{aligned} \tag{44}$$

where a is the semi-major axis, b the semi-minor axis and t the parameter.

In addition, elliptical path in a rotating reference frame shall be also rotating with angular frequency Ω of the whole system. Thus, a body orbiting and rotating should traces out a simple harmonic motion within the rotating reference frame, which we can be separated in two components with the same frequency but perpendicular directions out phased by 90° between them.

Position of a point in a non-inertial frame in parametric equations is given by

$$\begin{aligned} x' &= x\cdot\cos(\Omega t) - y\cdot\sin(\Omega t) \\ y' &= x\cdot\sin(\Omega t) + y\cdot\cos(\Omega t). \end{aligned} \tag{45}$$

Thus, replacing x and y terms of expression (45) in (44), the parametric components of a simple harmonic motion for a non-inertial frame are given by

$$\begin{aligned} x' &= a\cdot\cos(\omega t)\cos(\Omega t) - b\cdot\sin(\omega t)\sin(\Omega t) \\ y' &= a\cdot\cos(\omega t)\sin(\Omega t) + b\cdot\sin(\omega t)\cos(\Omega t). \end{aligned} \tag{46}$$

The given body should follow one of both motions, elliptical or circular orbit; it depends of the relation between the a and b semi-axes.

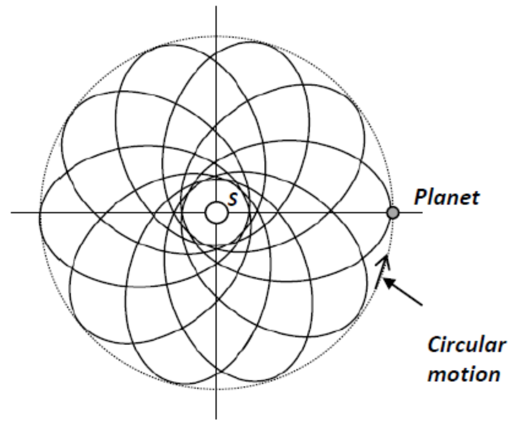

FIGURE 1. Rosette path of a planet orbiting and rotating in a rotating reference frame.

We can write expression (46) in function of eccentricity to the ellipse with a focus at the origin by replacing a and b terms by r term from expression (11). Reducing expression (46) by trigonometric identities, path of the body along of the circular motion including the rate of apsidal precession traces out an elliptical rosette shape (Fig. 1), given by

$$\begin{aligned} x' &= \frac{a\left(1-e^2\right)}{1-e\cdot\cos(\omega t + \mathrm{d}\omega_\phi t)}\cos(\omega t + \Omega t) \\ y' &= -\frac{a\left(1-e^2\right)}{1-e\cdot\cos(\omega t + \mathrm{d}\omega_\phi t)}\sin(\omega t + \Omega t), \end{aligned} \tag{47}$$

and in polar coordinates, yields

$$r_{\mathrm{Max}} = \frac{a\left(1-e^2\right)}{1-e\cdot\cos(\omega t + \mathrm{d}\omega_\phi t)}\cos(\omega t + \Omega t), \tag{48}$$

where r_{Max} is the maximum distance from the body to the centre of rotation (during the aphelion).

VII. THE ROTATING REFERENCE FRAME AND THE PRECESSION OF THE EQUINOXES

According to the expression (48), a body in a rotating reference frame must be orbiting and rotating with angular frequency Ω around a fixed axis describing a rosette path and changing its position with respect to the other bodies; but apparently this effect is not perceived from the Earth.

Nevertheless, having the expression (41), we can determine the angular frequency value for the Sun-Earth system. Thus, replacing the Earth's data [5] in expression (41), with the maximums distance *r* (during the aphelion) and the related minimum orbital velocity *v*, and t_P the time in seconds of a sidereal year, angular frequency of the rotating reference frame equals 7.71965×10^{-12} radians per second. Furthermore, replacing data of the Sun-Earth system in expression (42) that includes apsidal precession, total angular frequency equals 7.72554×10^{-12} radians per second (50.2881" of arc per year, or 0.0139689° per year).

For one revolution (2π radians), total period T_S in seconds for the Earth's case equals 8.133×10^{11} seconds (25,771.5 years), which coincide with the observed period of the precession of the equinoxes [1].

We can explain this result by proposing an analogy between a rotating reference frame and the whole Solar System (as a rotating system). According to this analogy, the same effect of precession of the equinoxes must be observed from the Earth orbiting in a rotating reference frame, where path of a body orbiting and rotating defines an elliptical orbit that also rotates with the rotating reference frame around a fixed axis at the Sun, as shown in Fig. 2, giving 25,771.5 turns around of the Sun in one circular period.

Then, for the Earth's case, about the year 2100 AC (point A in Fig. 2), the Earth's North Pole should be appointing near of Polaris (point C); and later, when planet travels out by half period of the Platonic year to the opposite side (point B), Earth's North Pole should be appointing near to Vega star (point D) about the year 14000 AC, thus changing the Earth its position within the Solar System and also maintaining in average the same angular radius (α in Fig. 2) with respect to the ecliptic, then changing its linear reference with respect to the "fixed" stars, resulting in the precession of the equinoxes effect as observed.

Considering respective maximum distance from the Sun to each planet and their respective minimum orbital velocity at such a position, we can find out from expression (43) a general expression to determine the respective period of the possible heliocentric circular motion for each planet, hence

$$T_{Sn} = \frac{2\pi \cdot T_{Pn}}{\Omega_n + \omega_n}, \tag{49}$$

where Ω_n is the angular frequency at the distance *r* in radians per second of any respective planetary ring, ω_n is the angular frequency in radians per second of respective apsidal precession of any planet and T_{Pn} is the period in seconds of any respective planet orbit. Thus, substituting respective planetary data [11] in expression (49), period of the possible heliocentric circular motion (as their apparently precession of the equinoxes) are derived for each planet, building the Table I, which results could be verified by the observation of the planetary motion with respect to the "fixed" stars.

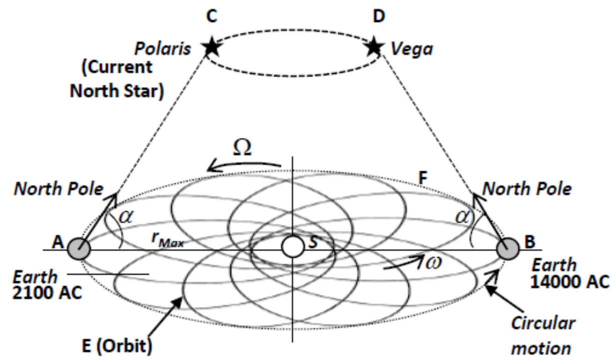

FIGURE 2. Rosette path of the Earth orbiting and rotating in a rotating reference frame with respect to the fixed stars.

VIII. CONCLUSIONS

This paper aims to offer a hypothetical alternative physical explanation for a celebrated effect, the precession of the equinoxes (no merely a reformulation of the previous knowledge). Nevertheless, this proposing is based on the knowledge about the dynamics behavior in a rotating reference frame, where Coriolis acceleration plays an important role in the precession of the equinoxes.

TABLE I. Data of planets with the predicted heliocentric circular period.

Planet	*Maximum Sun-planet distance (10^6 km)*	*Minimum orbital velocity (km/sec)*	*Calculated precession of perihelion (arc sec per century)*	*Sidereal orbit period (days)*	*Sidereal orbit period Ratio (Planet / Earth)*	*Predicted heliocentric circular period (years)*
Mercury	69.82	38.86	43.0133	87.969	0.241	6,022.09
Venus	108.94	34.79	8.6262	224.701	0.615	13,109.40
Earth	152.10	29.29	3.8391	365.256	1	25,771.50
Mars	249.23	21.97	1.3513	686.98	1.881	71,770.84
Jupiter	816.62	12.44	0.0623	4,332.589	11.862	749,613.52
Saturn	1,514.50	0.09	0.0137	10,759.22	29.457	2,581,214.51
Uranus	3,003.62	6.29	0.0024	30,685.40	84.011	10,334,913.87
Neptune	4,545.67	5.37	0.0008	60,189	164.79	22,398,514.91

Then, planets periodically could be changing their position with respect to the whole Solar System, completing a circular period (in terms of thousands of years, being almost imperceptible) that from the Earth could be perceived as the periodical change its linear reference with respect to the considered "fixed" stars, as is observed in the precession of the equinoxes.

Dynamics of the rotating reference frame can be also extended to the exoplanets of other stellar systems. Under this scenario, considered angular frequency shows the possibility that space in a uniform gravitational system given by a fixed axis is rotating, which allows a better understanding of galactic formations, spiral arms and planetary systems formation.

Regarding to the education, classical dynamics, planetary motion, Newtonian theory of gravity and relativity are revisited describing the main concepts of those theories, where it is showed the possibility to apply some of the known equivalences to consider another possible results and properties from the classical theories.

ACKNOWLEDGEMENTS

The author would like to thank Professor Sergio S. Cornejo for the review and comments for this paper.

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