Mathematical designing and short qualitative review of unconventional lasers based on photonic crystals[1]

Kamal Nain Chopra
Applied Physics Department, Maharaja Agrasen Institute of Technology, Rohini, GGSIP University, New Delhi - 110086, India.

E-mail: kchopra 2003@gmail.com

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Abstract
The unconventional lasers based on photonic crystals are very useful for certain applications like: data storage, biomedical applications, and display technologies. This is the reason for the sudden increase in the research efforts on the design and development of these lasers. The purpose of the present paper is to provide the technical analysis of the modeling and designing of these lasers, besides giving a brief review of the important recent investigations on the topic for the novel applications, and also discussing some of the experimental revelations in this field.

Keywords: Photonic crystal lasers, modeling of the photonic band structure of the photonic crystal lasers, spectral characterization.

Resumen
Los láseres convencionales basados en cristales fotónicos son muy útiles para algunas aplicaciones como: el almacenamiento de datos, aplicaciones biomédicas y tecnologías de visualización. Esta es la razón del aumento repentino de los esfuerzos de investigación en el diseño y desarrollo de estos láseres. El objetivo de este trabajo es proporcionar el análisis técnico de la modelización y diseño de los láseres, además de dar una breve reseña de las importantes investigaciones recientes sobre las nuevas aplicaciones, y discutir algunas de las revelaciones experimentales en este campo.

Palabras clave: Láseres de cristal fotónico, modelado de estructura de bandas fotónicas de los láseres de cristal fotónico, caracterización espectral.

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I. INTRODUCCIÓN

Photonic crystal lasers (PCLs) are an important class of unconventional lasers, and are based on nano-structures, for providing the mode confinement and the density of optical states (DOSs) structure, required for the feedback, having μm size and tunable on the bands of the photonic crystals.

A lot of interest [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] has been shown in studying the various aspects of novel unconventional lasers and their materials. Siriani [22] has done a very interesting discussion on Semiconductor Lasers, highlighting the fact that photonic crystals lasers are based on the evolving semiconductor laser technology, since the photonic crystals (PCs) may be considered as the ultimate manipulators of light, because of the fact that their design is flexible, and also they are able to influence the various photonic properties. Photonic crystals are capable of many manipulations like: altering the light emission, propagation, and matter interaction in the form of absorption/gain, and nonlinear effects. These on combining with the light-matter interactions produced by semiconductors, lead to many novel effects like: threshold less lasing and enhanced gain and absorption. Which in turn have helped the researchers in their work on the ultra-compact optical buffers, optical memory, and quantum photonic computing. All this scientific research has been the result of the continuous evolution of ideas and technologies which have revolutionized the ideas about the semiconductor lasers.

Classically, the semiconductor lasers are generally in the form of the diode laser; which consists of (i) an active region in the shape of the quantum wells, and (ii) the p-n junction,

[1] Some of the technical analysis presented in this paper is on the basis of the discussions with the researchers of the Photonics Group, Department of Physics, Indian Institute of Technology, Delhi, during the association of the author with the group, as Project Scientist in the Project on "Investigations on Optical correlators with high capacity holographic memory" DRDO, Govt. of India (From 17-06-2006 to 15-02-2009), sponsored by the Ministry of Defence, Government of India, New Delhi, India.

with the properties of the emitted light being determined mainly by: (a) the semiconductor material, whose band structure decides the energy of the interband transition producing light, and (b) to some extent by the physical structure *i.e.* the well width, and the waveguide geometry.

Interestingly, these quantum wells (Figure 1) are either inside a waveguide *i.e*. an edge-emitter; or between the distributed Bragg reflector stacks *i.e.* a vertical-cavity surface-emitting laser (VCSEL). With the advent of the successful efforts in combining the attractive and unique characteristics of semiconductors with novel ideas listed above, the envelope of the semiconductor emitters has broadened, with the establishment of the quantum cascade lasers, which though quite similar to the diode laser, mainly rely on the intersubband emission in a quantum well superstructure., and consequently enlarge the scope of the semiconductor photonics to regions of different wavelengths, smaller size, and stronger light-matter interaction.

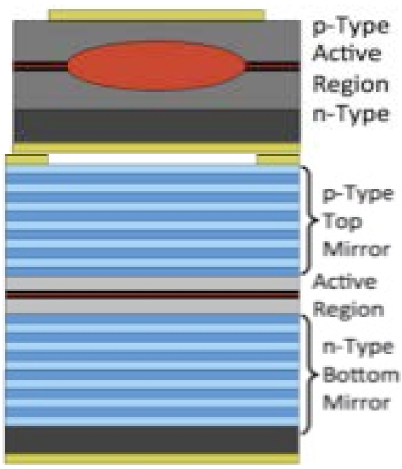

FIGURE 1. (Top) Conventional edge-emitter viewed from facet; and (Bottom) Conventional VCSEL viewed from side. Figure courtesy Siriani D., the new look of semiconductor lasers, CLEO Conference, Science and Innovations, http://blog.cleoconference.org/2013/02/the-new-look-of-semiconductor-lasers/.

Kurosaka *et al.* [23] have investigated the effects of non-lasing bands on the beam patterns in photonic-crystal lasers by evaluating the omni directional band structure both experimentally and theoretically, and have found that a new, weak dual-streak pattern is occasionally generated around the main lobe of the output beam, because of scattering of the lasing beam in the non-lasing bands, despite a wavenumber mismatch. On the basis of their results, they have emphasized that it is possible to design the high-quality devices without such a noise pattern; and this evaluation method should be useful for developing various high-functionality PC lasers. Kwon *et al*. [24] have discussed that the size of the smallest possible laser has been steadily approaching the physical limit of ~ (λ/2n)3, having already witnessed the series of evolution of the small lasers over a few decades, from the 1D VCSEL to the 2D photonic crystal laser, and back to the 1D ladder-type nanobeam laser. And that throughout these developments, the photonic crystals have helped the research efforts by providing the conceptual platform of effective photon confinement. The smallest mode volume achievable from the 1D photonic crystal laser is ~ 10(λ/n)3. The mode volume of the 2D photonic crystal is ~ 1(λ/n)3, about an order of magnitude smaller.

Busch *et al.* [25] have written a book on photonic crystals: Advances in design, fabrication, and characterization, in which the first part describes methods for the theoretical analysis of their optical properties as well as the results. Also a section provides the discussion on the fabrication, characterization and modeling of two and three-dimensional photonic crystals, while the final section presents a wide spectrum of applications: gas sensors, micro-lasers, and photonic crystal fibers.

This book is of interest not only to the advanced students and researchers in physics, electrical engineering, and material science, but also for departments Research and Development engaged in photonic crystal-related technological developments, in companies.

It is now widely accepted that the surface-emitting PC lasers have the potential of providing the next generation of compact and efficient light emitters, with useful applications in the important areas of the data storage, biomedical applications, and display technologies. However for the practical realization of this potential, these devices have to be developed for emission in a large wavelength region, especially in the violet to the visible wavelength range. Wright *et al*. [26] have described a monolithic GaN nanowire 2DPC based laser array, which is optically pumped with two distinct gain sections, fabricated by meeting the requirements like: the fabrication is scalable resulting in the tuneability over a large wavelength range possible to be performed, on a single substrate. The small area of the photonic crystal for making dense arrays; the simple fabrication process for nanowire structures, without removing the substrate or fabricating a distributed Bragg reflector (DBR) under the PC; and the light emission being vertical and single-mode. Interestingly, this design is made feasible by employing the low group velocity lasing, utilizing the lateral feedback, suitable for the incorporation of additional gain materials needed for different spectral regions. The combination of this technique with the wide gain bandwidth of semiconductor nitride materials has led to the interesting possibilities of designing and fabricating monolithic multiple-wavelength photonic devices.

Quite broad gain spectra are required for the fabrication of an array of lasers, capable of spanning a large spectral bandwidth, which can be done in a simple way, by choosing a material with gain, which is inhomogeneously broadened by an appropriate amount. However, this is accompanied by the reduced peak gain resulting in an increase of the laser threshold. The solution to this problem is provided by the rich dispersion characteristics of PCs. In case of the 2D and 3D photonic crystals, certain modes exhibit near-zero group velocity along certain lattice directions in certain bands, and in fact this can be utilized to reduce lasing threshold. Interestingly, these modes propagate slowly, and thus result in enabling increased interaction time of the electromagnetic fields with the gain material, and hence lowering the lasing threshold, which makes it possible to reach low lasing thresholds in material systems for exhibiting a small amount of gain or a reduced gain over a comparatively larger bandwidth.

Wright *et al*. [26] have exploited these characteristics of a 2DPC composed of an array of GaN nanowires in a hexagonal lattice, for achieving lasing in a broad gain bandwidth system. For their study, they have considered the nanowires having two gain sections (one composed of InGaN multiple quantum wells with an emission centred at 430 nm, and the other of a 150 nm thick InGaN under-layer with an emission centred at 385 nm) embedded axially for improving the quality of the subsequent multiple quantum well structures. The band structure for the case of a hexagonal lattice of GaN nanowires with a diameter 'd = 0.4a' and height 'h = 2a', where 'a' is the lattice constant, as calculated by Wright *et al*. (26) by using the plane-wave expansion method, has been reproduced below:

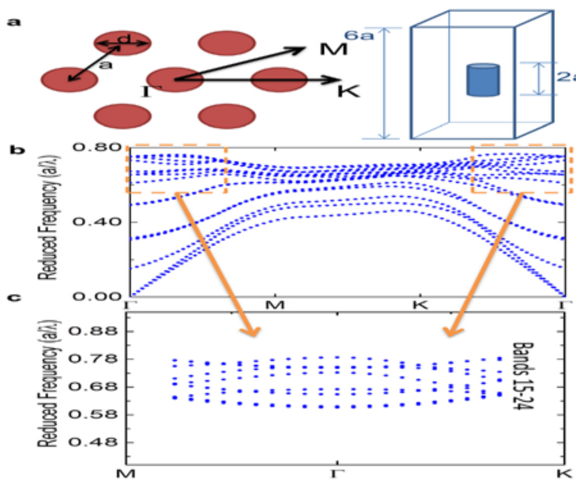

FIGURE 2. Modeling of the photonic band structure of the photonic crystal lasers.

It is clear that the lower band indices show large dispersion, and the higher band indices have much less dispersion with relatively flat bands. Interestingly, for certain bands, in the range of 20–23, a low dispersion is observed, and also, the slow group velocity modes are close to the Γ point making normal emission possible. It is to be understood that for utilizing the entire available gain bandwidth of the two gain regions, an array of 2DPCs with lattice constants ranging from 290 nm to 330 nm, has to be fabricated.

The 2DPC lasers have been analysed by using a micro-photoluminescence (μ-PL) setup, which enables optical excitation of a single pixel in the array or a group of pixels, by controlling the pump spot size. Also, the lasing threshold behaviour has been examined and illustrated by focusing on a pixel with a lattice constant of 320 nm and a nanowire diameter of ~140 nm. Very interesting results have been obtained: (i) at low pump intensity, a broad PL emission between 400–460 nm is observed; and (ii) as the pump intensity is increased, a sharp peak appears at the short wavelength edge of the PL spectrum; (iii) a further increase in the pump intensity results in the peak intensity of the narrow feature to rise rapidly in contrast to the slowly rising background PL. The peak intensity as a function of pump intensity or the so called "light-in/light-out" (LL), as obtained by them, has been reproduced below:

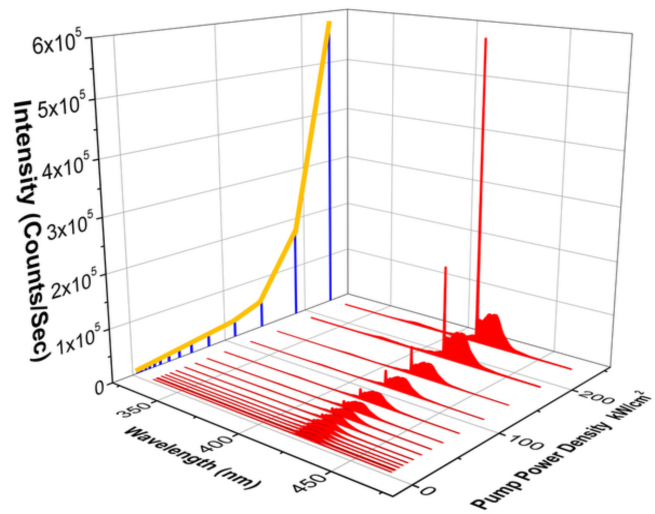

FIGURE 3. Spectral characterization of a representative photonic crystal laser.

A clear threshold behaviour is observed the lasing emission takes place at 415 nm with a threshold of ~130 kW/cm2; with a high degree of spatial coherence; and confined in narrow range (<0.2 nm). A great advantage of this approach is that, it is possible to fabricate densely packed micro-scale laser pixels by placing the 2DPC lasers very close to each other, which allows a single pump to optically excite multiple pixels simultaneously. The results obtained for the case of pumping 4 of the 2DPC lasers (the area of each laser = 10 μm × 10 μm; separated by ~5 um), as obtained by Wright *et al*. (26), have been reproduced below:

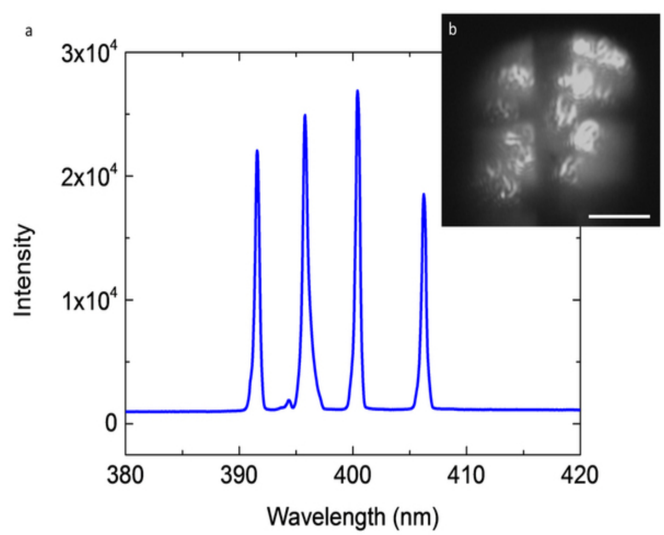

FIGURE 4. Four photonic crystal laser "pixels" operating together in close vicinity (The scale bar represents 10 μm.). Figure courtesy Wright *et al*., Multi-Colour Nanowire Photonic Crystal Laser Pixels,http://www.nature.com/srep/2013/131018/srep02982/full/srep02982.html.

An interesting point to be noted is that it is possible to achieve different colours and warmths by selectively exciting (optically or electrically) certain laser pixels each with a different emission wavelength, which is leading to the development of the wafer-scale laser arrays tunable in a wide range - the entire UV to visible region.

Shirvani-Mahdavi *et al*. [27] have demonstrated an amplified cholesteric liquid crystal (CLC) laser performance by utilizing a binary-dye mixture (with 62 wt% DCM and 38 wt% PM597) as the active medium, and an external stable resonator. On the basis of their measurements, they have shown that the laser efficiency is enhanced as compared to the highest efficiency of each individual dye. It has also been shown that by using such an active CLC in an external stable resonator leads to a ~92X improved efficiency over the single CLC laser, in which case, the binary-dye doped CLC simultaneously functions as laser oscillator and amplifier.

II. MATHEMATICAL ANALYSIS

The mathematical modeling and analysis is based on mainly two approaches: (i) 1-D Photonic crystals are considered as the optical nanostructures having periodic multi-layer dielectric stacks; and (ii) a classical system based on the Equations of Maxwell for studying the cholesteric liquid crystal materials. These treatments give an idea about the performance of the PCs with the given specifications, and also for modeling them for the required applications with the needed characteristics. The various parameters, on which the performance and the efficiency of the system depend for the particular case, are optimized to give the best possible result.

A. Optical nanostructures having periodic multi-layer dielectric stacks

1-D photonic crystals are the optical nanostructures having periodic multi-layer dielectric stacks (Figure1), which (i) establish a distributed feedback along the material; and (ii) define the allowed and forbidden photonic energy bands.

Cholesteric liquid crystal (CLC) materials are special types of quarter-wave stack photonic crystals in which refractive index, in a periodic helical structure with a pitch length (p), contrary to common photonic crystals, varies continuously from the extraordinary refractive index n_e to the ordinary refractive index n_o of the liquid crystal, resulting in their showing a selective photonic band gap (PBG) into which the circularly polarized incident light with the same handedness as the cholesteric helix is reflected, while the opposite handedness is transmitted. The photonic band edges (PBEs) occur at the wavelengths given by:

$$\lambda_s = n_o p, \tag{1}$$

and

$$\lambda_l = n_e p, \tag{2}$$

where s and l specify respectively the short wavelength and long wavelength of the PBG. As the density of photon states at the PBEs, against within the band, is very large, the group velocity approaches zero [28] and therefore, there is considerable possibility [29] of lasing in CLCs in the presence of a proper laser dye as an active material, without any external reflectors. This is really an interesting result, which classifies such lasers quite unconventional. The efficiency of dye-doped CLC lasers is crucially dependent on the order parameter of the transition dipole moment (TDM) of the dye S_{td} with respect to the local director (nˆ) of liquid crystal, which can be calculated [30] by the following equation:

$$S_{td} = [\frac{n_o I_{\parallel} - n_e I_{\perp}}{n_o I_{\parallel} + n_e I_{\perp}}], \tag{3}$$

where $I_{\parallel}$ and $I_{\perp}$ are the fluorescence intensities emitted from a dye-doped nematic film polarized parallel and perpendicular to nˆ respectively. The maximum possible value S_{td} =1 (for $I_{\perp}$=0) corresponds to the case of perfect alignment of the TDM parallel to nˆ, while S_{td} =0 implies an isotropic orientational distribution. On the other hand, the values $S_{td} < 0$ correspond to a preferred orientation perpendicular to nˆ.

B. Optical properties for a classical system based on the Maxwell's Equations

The optical properties for a classical system can be studied by neglecting the quantization of the electromagnetic field, and using the Equation of Maxwell. It is assumed that there are no free charges or currents present in the system, and the treatment is restricted to a purely dielectric media, in which the dispersion, and the non-linearity are neglected, and the refractive index is considered as independent of time, and the relative permeability is set to 1. Therefore, the relative permittivity $\varepsilon(r^{\rightarrow})$ can be written as a function of position only. Hence, the involved fields varying harmonically in time (any derivative in respect to time ∂/∂) can be replaced by —*iω*.

In this way, the Equation of Maxwell can be decoupled resulting in two independent equations for the electric field $E^{\rightarrow}(r^{\rightarrow})$ and for the magnetic field $H^{\rightarrow}(r^{\rightarrow})$ given by:

$$\nabla\times[\nabla E^{\rightarrow}(r^{\rightarrow})]=\{\frac{\omega}{c}\}^2\varepsilon_r(r^{\rightarrow})E^{\rightarrow}(r^{\rightarrow}), \tag{4}$$

$$\nabla\times[\frac{1}{\varepsilon^{\rightarrow}(r^{\rightarrow})}\nabla\times H^{\rightarrow}(r^{\rightarrow})]=(\frac{\omega}{c})^2 H^{\rightarrow}(r^{\rightarrow}), \tag{5}$$

It has to be kept in mind that for the sake of the mathematical convenience, it is normal to solve the equation for the magnetic field, and then to use it to calculate the electric field. The solution for the PC, which is a periodic system, can be obtained by using the Theorem of Bloch, according to which the solution must have the same periodicity as the crystal, and this provides the basic solution to this problem. The magnetic field $H^{\rightarrow}(r^{\rightarrow})$ can be written as:

$$H^{\rightarrow}_{k} = \exp(ik^{\rightarrow}.r^{\rightarrow})u^{\rightarrow}_{k}(r^{\rightarrow}). \quad (6)$$

Where $k\rightarrow$ is the Bloch wave vector, and $u\rightarrow k$ ($r\rightarrow$) is the Bloch function. The periodic function can be written as a sum of harmonic functions, and thus the basic form for the unknown Bloch function is given as:

$$u^{\rightarrow}_{k}(r^{\rightarrow}) = \sum_{G} c^{\rightarrow}G(k^{\rightarrow})\exp(iG^{\rightarrow}r^{\rightarrow})\cdot \quad (7)$$

Where $G\rightarrow$is the sum over all reciprocal lattice vectors; and $c\rightarrow G$ are the Fourier coefficients.

A set of partial differential calculations can be simplified [31] into a set of linear equations, by considering that the Fourier coefficients become accessible as:

$$\sum_{G} -\varepsilon^{-1}_{G\text{-}G} \times (k^{\rightarrow}+G^{\rightarrow}) \times (k^{\rightarrow}+G^{\rightarrow}) \times c^{\rightarrow}G = \frac{\omega^{2}}{c^{2}} c^{\rightarrow}G. \cdot \quad (8)$$

Finally, the Fourier transform of the inverse dielectric function ε^{-1}_{G} can be calculated by using the Fast-Fourier Transformation, which provides an easy tool for designing the PCs.

III. NOVEL RESEARCH AND CONCLUDING REMARKS

A lot of interest [32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44] has been shown in the last decade, in the studies of novel types of unconventional lasers and their applications.

Ota *et al.* [38] have made an interesting study of the frequency-doubled photonic crystal nanocavity lasers, and have reported that the photonic crystal nanocavity lasers with quantum dot gain are able to directly generate coherent visible light by nanocavity-enhanced nonlinear optical frequency doubling. As discussed by them, the monolithic fabrication of nonlinear optical frequency conversion lasers on the micron scale is not easily possible as it generally requires strong laser input, sufficiently large nonlinear optical crystals, and fulfillment of the phase-matching condition. An efficient method for reducing the nonlinear crystal size is the use of micro/nanoscale photonic media with strong photon confinement capability, like whispering-gallery-mode resonators, plasmonic structures, and photonic crystals. However, most of the experiments reported in literature are based on using the passive nonlinear optical media, which in fact require bulky external laser sources, in addition to the complicated optical setups for the introduction of photons into the structures.

Ota *et al.* [38] have studied the gallium arsenide based photonic crystal nanocavity lasers containing indium arsenide quantum dots, supporting an optical mode with a high quality factor, and a small mode volume, which implies that there is a strong temporal and spatial optical confinement. The quantum dots have a strong emission in the NIR, and produce the lasing oscillation under optical carrier injection; and the large second-order optical nonlinearity of GaAs results in the efficient intra-nanocavity frequency conversion processes, *e.g.* second-harmonic and sum-frequency generation, using the internally generated coherent NIR light, and hence it is clear that the monolithically fabricated nanoscale laser is able to generate frequency-converted light without requiring any additional optical elements.

Hecht [39] has made an interesting study, and has discussed that the tight confinement possible with photonic crystals can lead to the reduction in the threshold and the increase in the speed of nanocavity lasers, which has the potential for the development of the novel devices for applications from the optical interconnects to quantum computing. Altug and Vučković [40] have demonstrated a new type of laser composed of an array of coupled photonic crystal nanocavities that enables high differential quantum efficiency and output power, together with a low threshold power comparable to those of single photonic crystal cavity lasers, and have reported that the laser efficiency increases faster than the lasing threshold with an increase in the number of coupled cavities. A single mode lasing has been observed, and the output powers have been measured that are two orders of magnitude higher than in single nanocavity lasers. The laser behavior has been studied theoretically and it has been shown that the benefits resulting from the coupling of cavities are due to strong cavity effects such as the enhanced spontaneous emission rate.

Zhang *et al*. [41] have demonstrated a novel type of nanolaser, which combines the advantages of photonic crystal lasers and microdisk lasers, based on InAlGaAs/InGaAs quantum wells using pulsed optical-pumping at room temperature, and incorporates the properties of small footprint, small mode volume, sub-milliwatt threshold, and favours the vertical emission. It has been emphasized that this type of laser acts as a promising candidate for the highly-integrated on-chip nanolasers in applications for signal processing and index sensing.

Ogawa *et al*. [42] have demonstrated the efficient laser performance with Nd:$GdVO_4$ crystals grown by the floating zone method. They have achieved a slope efficiency of 67% with a 2-at. % Nd-doped crystal, by pumping at 808 nm.

They have also reported that pumping at 879 nm with a bandwidth of 1.8 nm is practical for laser diode pumping, at which pumping level, the slope efficiency reaches 78%.

Postigo *et al*. [43] have fabricated two-dimensional photonic crystal lasers on III–V semiconductor slabs; and have achieved Tuning of the spontaneous emission in micro and nanocavities by accurate control of the slab thickness.

They have fabricated the different structures like the Suzuki-phase or the coupled-cavity ring-like resonators, and have been able to obtain the Laser emission by pulsed optical pumping, and perform the optical characterization of the lasing, and thereby showing one or more laser peaks centred around 1.55 μm. In addition, far field characterization of the emission pattern has been realized showing different patterns depending on the geometrical shape of the structures, and it has been emphasized that these kinds of devices may be used as efficient nanolaser sources for optical communications or optical sensors.

By employing two-dimensional InGaAsP photonic band-edge lasers, Kim *et al.* [44] have experimentally demonstrated that a random mixture of two different PCs possesses a new band structure that is intermediate to that of the two host PCs. It has been reported that (i) the photonic band-edges shift monotonically, but with a strong bowing effect, as the mixed PC system is systematically transformed from one PC to the other, and (ii) the experimental observations are in excellent agreement with finite-difference time-domain simulations and model calculations based on virtual-crystal approximation with compositional disorder effect included. Kim *et al*. [44] have experimentally demonstrated that a random mixture of two different photonic crystals (PCs) possesses a new band structure that is intermediate to that of the two host PCs, by employing two-dimensional InGaAsP photonic band-edge lasers, They have reported that photonic band-edges shift monotonically, but with a strong bowing effect, as the mixed PC system is systematically transformed from one PC to the other, and that their experimental observations are in excellent agreement with finite-difference time-domain simulations and model calculations based on virtual-crystal approximation with compositional disorder effect included.

Thus, we see that many new ideas about the unconventional lasers and their applications have recently been reported. A conference [45] is also going to be held in June 2014, which is expected to throw light on some very new research results on the various kinds of lasers, and their applications.

Thus, it can be concluded that the use of unconventional lasers is going to increase in future because of the newer applications requiring new characteristics of laser emissions.

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