"Equat causa effectum" and the Kepler orbits

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Note

The Newtonian field of gravity of a spherically symmetric mass distribution is spherically symmetric. In contrast, the Kepler orbits are *not* spherically symmetric. Hence, their symmetry - the symmetry of the 'effect' - is smaller than that of the field causing them.[1] Does this contradict the old rule "equat causa effectum"?

For a given field, the actual orbit is determined by the initial values of position and velocity of the orbiting body (planet). The question posed above discards the initial values. What about their symmetry?[2]

Kinematically, the direction of the velocity vector represents a distinguished vector in space. For the field, all directions towards or away from the center of the mass distribution are distinguished. Consequently, orbits towards or away from the center exhibit the highest possible spatial symmetry for orbits: straight lines.

Dynamically, Newton's Law 2 states the equilibrium between the field force (centripetal force) and the inertial force, $-md\mathbf{v}/dt$ (*m* - constant mass of the planet, **v** - its velocity). The latter one can be decomposed in the centrifugal force, which is related to the change of the direction of motion, and the tangential component, which is related to the change of the modulus of the velocity. There are two distinguished cases.

1. The centrifugal force vanishes. This is the case of motion towards or away from the center discussed above.
2. The tangential component of the inertial force vanishes. This is the case of circular orbits.

In both cases, the symmetries of *both* components of the inertial force are in harmony with the symmetry of the field.

The set of all possible Kepler orbits forms a spherically symmetric figure. Here, no single initial condition plays a role. The same holds true for the set of all possible Kepler orbits under the condition of

- given energy, or
- given modulus of the angular momentum, or
- given energy *and* modulus of the angular momentum.

Each of these conditions endows a classification on the set of all possible Kepler orbits.

If the angular momentum vector is fixed, the set of all possible Kepler orbits forms a rotationally symmetric figure.[3] (This endows a classification, too.)

Alternatively, one may argue, that, according to Newton's Law 2, the appropriate, because immediate 'effect' of a force, **F**, is not the orbit, $\mathbf{r}(t)$, but the 'momentum orbit', $\mathbf{p}(t)$, hence - at constant mass - the hodograph, $\mathbf{v}(t)$.[4] As a matter of fact, the hodograph of a Kepler ellipse is a circle.[5] This will be explored elsewhere.

In summary, the rule "equat causa effectum" applies to certain sets of possible effects, not to each single effect, if the cause under consideration (here, the gravitation field) is not the only influence. If the additional influences (here, the initial conditions) "equal" the cause, the single effect does as well. If not, deviations between cause and effect occur, and the rule "equat causa effectum" applies only partially. Alternatively, these additional influences (here, the inital conditions) are included into the 'cause'. This restores that rule completely.

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[1]According to A. Zee (*Fearful Symmetry*, 1999, p. 14), it was an outstanding result by Newton to have admitted to think this difference.

[2]It is known from electrons in quantum wells, that the complexity of their stationary states (band structure) depends on the angle between the direction of motion and the symmetry axis of the well.

[3]I propose to explore, whether the Laplace-Runge-Lenz and related vectors classify the Kepler orbits in an analogous manner, *cf* http://en.wikipedia.org/wiki/Laplace%E2%80%93Runge%E2%80%93Lenz_vector.

[4]W. R. Hamilton: *The hodograph or a new method of expressing in symbolic language the Newtonian law of attraction*, Proc. Roy Irish Acad 3 (1846), 344-353. The literal meaning of the word 'hodograph' is path describer.

[5]J. C. Maxwell, *Matter and Motion*, New York: Dover 1991, § 133; D. L. Goodstein & J. R. Goodstein, *Feynman's Lost Lecture. The Motion of Planets Around the Sun*, New York, London: Norton 1996