Some particular points in projectile motion

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Abstract

This paper discusses some points about projectile motion. The usual assumption of neglecting air resistance is considered. We have obtained the equation of the path, range, the maximum height and the curvature of projectile motion in an especial way. We also found a particular point on the trajectory of projectile motion as this point has some considerable concepts in itself.

Keywords: Projectile motion- curvature of projectile- the area of the curve.

Resumen

Este artículo discute algunos puntos sobre el movimiento de proyectiles. El supuesto usual de dejar de lado la resistencia del aire es considerada. Hemos obtenido la ecuación de la trayectoria, el rango, la altura máxima y la curvatura del movimiento de un proyectil de una manera especial. También se encontró un punto determinado de la trayectoria del movimiento del proyectil como este punto tiene algunos conceptos considerables en sí mismos.

Palabras clave: Proyectil de movimiento- curvatura de proyectil-el área de curva.

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I. INTRODUCTION

Although many years have passed since the presentation of formulation of projectile motion, but there are still a lot of interesting points in this area to be mentioned. For example see to [1]. Perhaps deep physical concepts in combination with rigorous mathematical relations make projectile motion very significant. When projectile motion is studied in classical mechanics books or in multiple papers often starts with basic parameters of projectile. In this paper we got using the conception of impulse to the equation of the path, first. We investigate the radius of curvature by some especial states. Note that the radius of curvature is calculated with a different way to analytical mechanics in [2]. Then we obtain a point with this especial property. A point could be found by a same fractional of maximum height and range. In this part we have a detailed discussion about the point. We'll calculate the area under the projectile curve. Thereafter we'll compare it by the area of Archimedean triangle. In the last part of the paper we investigate an especial point so that this part consists on some debates. Finally we searched for the same point when the projectile has thrown from a height than horizontal.

II. THE EQUATION OF THE TRAJECTORY

Generally, there are articles that the equations and relations of projectile motion are concluded from concepts such as angular momentum and torque [3, 4]. However in this paper we tried to choose another way to study projectile motion. And we calculate the equation of the path by the concept of the impulse.

Suppose an object has thrown with angle of α to the horizontal with the initial velocity of V_0, at the moment of t we consider mg as an impulse to the object so that in a point in distance of x that the components of velocity are V_x and V_y. We can find out

$$mgt = \Delta p_y = m\left(V_y - V_{0y}\right). \tag{1}$$

Since $x = V_x\, t$ and by the equation of the velocity independent of time, *i.e.*

$$V_y = \sqrt{V_{0y}^2 - 2gy}.$$

We know

$$V_x = V_0 cos\alpha,$$
$$V_{0y} = V_0 sin\alpha.$$

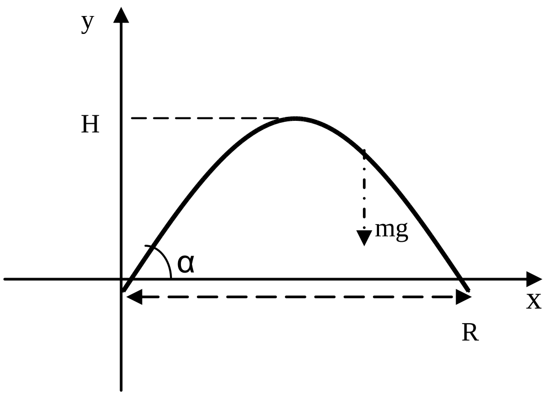

FIGURE 1. The force of weight inserting on the projectile at the instant t.

Then we obtain

$$y = -\frac{gx^2}{2{V_0}^2 cos^2\alpha} + xtan\ \alpha. \quad (2)$$

III. THE RADIUS OF CURVATURE OF PROJECTILE

Before starting a discussion about curvature let's take a short look at the area under projectile motion.

By the relation (2) the area under projectile curve yield

$$s = \int_0^R y\ dx = \frac{2}{3}\frac{V_0^4 sin^3\alpha\ cos\alpha}{g^2}. \quad (3)$$

That the area of the Archimedean triangle yield

$$s_\Delta = \frac{1}{2}\frac{V_0^4 sin^3\alpha \cos\alpha}{g^2}. \quad (4)$$

Detailed study of this issue, you can look for the reference [5]. Now we calculate the radius of curvature of projectile motion. As you know the velocity is

$$\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath}. \quad (5)$$

So, the unit vector of (5) is

$$\hat{V} = \frac{V_x}{\sqrt{V_x^2+V_y^2}}\ \hat{\imath} + \frac{V_y}{\sqrt{V_x^2+V_y^2}}\hat{\jmath}. \quad (6)$$

By the magnitude of derivative of last relation we conclude

$$\left|\hat{V}'\right| = \frac{gV_x}{(V_x^2+V_y^2)}. \quad (7)$$

We introduce the radius of curvature with following relation

$$\frac{1}{r} = \frac{|\hat{V}'|}{|\vec{V}|}, \quad (8)$$

r is the radius of curvature with substitute (7) in (8) we have

$$\frac{1}{r} = \frac{gV_x}{(V_x^2+V_y^2)^{\frac{3}{2}}}. \quad (9)$$

The relation of (9) is compatible with radius curvature in the analytical mechanics [2]. In the first especial state we search for the biggest osculating circle between the curve and horizontal axis.

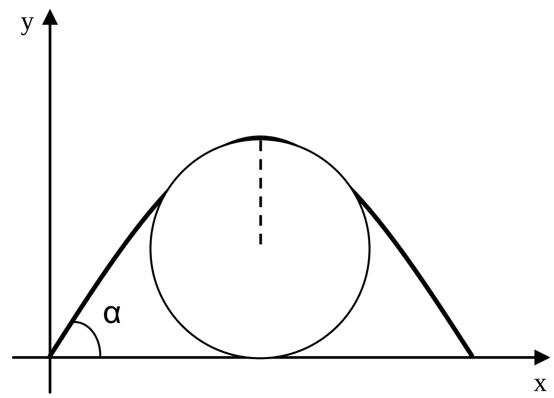

FIGURE 2. This figure shows the osculating circle with the radius (r) equal to half of the maximum height.

Based on the above figure:

$$r = \frac{H}{2}.$$

According to the above mention and the formula of radius of curvature we obtain $\alpha \approx 63$.

The ratio of area of this circle to the area of projectile curve is nearly 59 percent.

In the second especial sate we look for the osculating circle with the same area as the area under the projectile curve.

Based on the relations (9) and above condition, we obtain $\alpha \sim 31$.

IV. THE ESPECIAL POINT

Now we are looking for the especial point on the trajectory of projectile, as we said. In this point the ratio of horizontal distance to the range is equal to the ratio of height to the maximum height *i.e.*

$$M(x, y),\ \frac{x}{R} = \frac{y}{H}. \quad (10)$$

That M is the coordinates of this point, R is the range and H is the maximum height.

First we look for this point in the projectile curve when the projectile is projected from the origin of coordinate. Since the equality of (10) and the equations of motion and the values of R and H we can write

$$t = \frac{3V_0 \sin\alpha}{2g}. \quad (11)$$

And

$$\frac{x}{R} = \frac{3}{4}, \qquad \frac{y}{H} = \frac{3}{4}.$$

Now we are going to gain the area under projectile curve to this point, so

$$\acute{s} = \int_0^{\frac{3}{4}R} y\,dx = \left(\frac{3}{4}\right)^2 RH. \quad (12)$$

Again by the Archimedes theorem, and the mines of above relation and the triangle you can see in the figure we find it convenient to obtain the area of Archimedean triangle related to above equation

$$s'_{\Delta} = \frac{1}{2}\left(\frac{3}{4}\right)^3 RH. \quad (13)$$

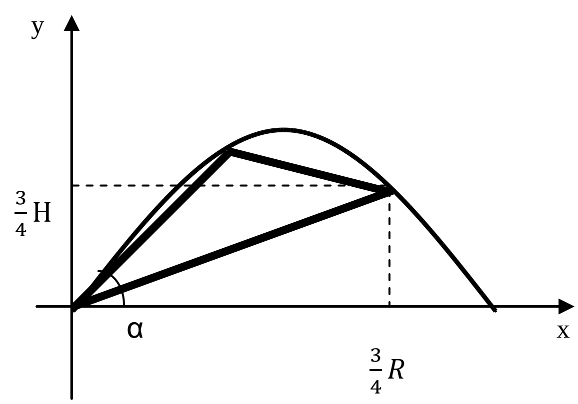

FIGURE 3. This figure shows the bold Archimedean triangle to the especial point.

Suppose a projectile is projected from the height of h than horizontal. This matter was investigated fully and exactly in [6]. We will find the same special point in this situation. As we know in this situation

$$R = \frac{V_0^2 \sin 2\alpha}{2g}\left[1 + \sqrt{1 + \frac{2gh}{V_0^2 sin^2\alpha}}\right]. \quad (14)$$

And again we use the Eq. (10) and equations of motion and values of R and H, so the special point will be found

$$T = \frac{2V_0 \sin\alpha}{g} - \frac{\frac{V_0 \sin\alpha}{g}}{1+\sqrt{1+\frac{2gh}{V_0^2 sin^2\alpha}}}. \quad (15)$$

By the limiting of the (15) we conclude the relation (11). And the last equation will be

$$\frac{x}{R} = \frac{y}{H} =$$

$$\frac{2}{1+\sqrt{1+\frac{2gh}{V_0^2 sin^2\alpha}}} - \frac{1}{\left(1+\sqrt{1+\frac{2gh}{V_0^2 sin^2\alpha}}\right)^2}. \quad (16)$$

Again by the limiting above relation we get proper conclusion.

V. CONCLUSIONS

In this paper we have discussed some section subjects of projectile motion, and of course the usual assumption of the constant acceleration of gravity and absence of air resistance have been considered. We began with the equation of the path and we had a new look at it. We invite the readers to develop the method we used. Then we had a brief look at the area under projectile curve. Thereafter unit vector of velocity and the magnitude of derivative of it will lead us to the radius of curvature of projectile. In the last part we found a point with an interesting property in the projectile motion and projectile motion from a height. We obtained the area under projectile curve and the Archimedean triangle to this point. As you see there are still many interesting points in the projectile motion that have not been found.

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