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Determination of Period and Frequency of Oscillatory Systems: A Non-Traditional Approach Through Energy

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Abstract

This paper proposes an alternative theoretical approach to analyzing oscillatory systems in situations involving translation or rotation, emphasizing energy as a methodological tool. Traditionally, the analysis of these systems is performed through the direct application of Newton's second law, which can pose conceptual challenges for high school students involving vector addition and decomposition. In this study, we discuss how elastic constants—the restoring characteristic—as well as the moment of inertia—the inertial characteristic—and, consequently, the period and frequency of oscillation, emerge naturally from the analysis of the system's mechanical energy, without using any approach explicitly involving rotational dynamics. The methodology presented aims to foster the development of critical thinking, as well as skills and strategies associated with solving complex problems, offering an effective alternative to traditional approaches, in a sort of introductory essay on analytical mechanics. We also present a proposal for a conceptual discussion associated with an experimental activity; the latter can be used by the teacher to spark students' curiosity and spark discussion on the topic in the classroom.

Keywords: Rotational dynamics, Oscillatory motion, period, frequency, Physics teaching.

Resumen

Este artículo propone un enfoque teórico alternativo para el análisis de sistemas oscilatorios en situaciones de traslación o rotación, haciendo hincapié en la energía como herramienta metodológica. Tradicionalmente, el análisis de estos sistemas se realiza mediante la aplicación directa de la segunda ley de Newton, lo que puede plantear desafíos conceptuales para estudiantes de secundaria que involucran la adición y descomposición de vectores. En este estudio, analizamos cómo las constantes elásticas (la característica restauradora), así como el momento de inercia (la característica inercial) y, en consecuencia, el período y la frecuencia de oscilación, surgen naturalmente del análisis de la energía mecánica del sistema, sin utilizar ningún enfoque que involucre explícitamente la dinámica rotacional. La metodología presentada busca fomentar el desarrollo del pensamiento crítico, así como las habilidades y estrategias asociadas con la resolución de problemas complejos, ofreciendo una alternativa efectiva a los enfoques tradicionales, en una especie de ensayo introductorio a la mecánica analítica. También presentamos una propuesta para una discusión conceptual asociada a una actividad experimental; esta última puede ser utilizada por el profesor para despertar la curiosidad de los estudiantes y generar debate sobre el tema en el aula.

Palabras clave: Dinámica rotacional, Movimiento oscilatorio, período, frecuencia, Enseñanza de la Física.

I. INTRODUCTION

In textbooks typically used in regular high school, it is stated, without formal proof, that a particle oscillating slightly around a stable equilibrium position (potential well) under the action of conservative forces performs a harmonic movement (HM). The usual practice is to seek examples (illustrations) that corroborate this assertion, showing that the resulting force on the particle in these cases is of the algebraic form $FR = -kx$ (x = algebraic displacement from the equilibrium position), the well-known Hooke's Law. Then, by comparison with the mass-spring system, we obtain what would become the "effective force constant" (k_{ef}) for a given physical situation. If m is the mass of the oscillating body, we obtain the angular frequency $\omega = \sqrt{\frac{kef}{m}}$ and the period $T = 2\pi\sqrt{\frac{m}{kef}}$. Examples commonly involve applications of SHM in various fields of physics, such as hydrostatics, gravitation and electrostatics [1, 2, 3] and [4].

When we have a system of discrete particles or even a rigid body, the associated difficulties become even greater. For these cases, we propose in this article a theoretical approach in which we choose to develop the theory through energy—along with the comparison (analogy) with the mass-spring system—with one of the main objectives being to obtain and discuss, both qualitatively and quantitatively, the following quantities: the "effective force constant" (k_{eff})—which, in oscillatory movements, relates to the restoring forces and torques—and also the system's "coefficient of inertia," which, in rotational movements around an axis, corresponds to the moment of inertia. These quantities determine the period and frequency of such movements.

We will discuss these concepts through a case study, analyzing a physical asymmetric pendulum without directly using the equations of rotational dynamics, as is done in higher education (e.g., Newton's Second Law in angular form, torque = moment of inertia x angular acceleration). Thus, one of the article's objectives is to arrive at the concept of moment of inertia in a different way from that traditionally used. Generally, the moment of inertia is quantitatively introduced when analyzing the kinetic energy of a system of solid particles (or rigid body) rotating around an axis [1]. If such an expression involves the inertial masses of the particles, in the authors' view, the moment of inertia as a physical—in fact, a coefficient of rotational inertia—is merely suggested; clarification would come through Newton's Second Law in angular form.

We understand that the methodological-didactic approach to theoretical development using energy broadens the range of situations that can be analyzed in a high school context. We can say that, theoretically, this article addresses problems involving oscillatory systems by subtly and fundamentally introducing some concepts of analytical mechanics, such as generalized coordinates and the elimination of binding forces. After all, as we know from mechanics, approaching some problems using the linear force-momentum approach can be quite laborious, even making it virtually impossible to solve certain problematic situations in some cases. In such cases, analytical methods involving energy can prove to be a fruitful option.

After the theoretical development of the physical pendulum, a didactic activity of discussion of concepts is proposed, involving some points that we consider important and interesting, with experimental results as auxiliary elements.

II. LITERATURE REVIEW

Regarding the theoretical development of HM via energy to determine the period and frequency of small oscillations, when researching some textbooks most commonly used in the basic cycle of undergraduate studies [1, 2, 3], and [4], we noticed that this method was addressed in only two solved exercises in reference [4]: one concerning the oscillation of a U-shaped liquid column and the other concerning the oscillation of a diatomic molecule. In turn, reference [1] follows the line used by us in this article, as its development falls within the scope of physics and mathematics commonly seen in high school. It is worth noting that in the oscillation of a liquid column, there is the particularity, for an incompressible fluid, that all the particles that make up the system have the same magnitude of velocity. In this article, we explore slightly more general situations in which the different parts of the system have their velocities related through the bonds respected by the system.

The other topic covered in this article, rotational dynamics, requires students to hone their critical thinking skills to understand abstract concepts such as moments of inertia, equilibrium, and torque. However, there are significant obstacles and difficulties in approaching this topic.

A study involving 70 11th-grade high school students from the SMA (Brazilian State School of Social Sciences) employed a descriptive quantitative methodology to investigate the topic [5]. Data was collected through a multiple-choice test based on five indicators of critical thinking: elementary clarification, basis for decision-making, inference, advanced clarification, and strategies and tactics. The results revealed that 51.4% of the students had low critical thinking skills, 34.3% moderate, and only 14.3% high. Performance varied across the indicators, with the best results in elementary clarification (61%) and the worst in advanced clarification (21.4%) and strategies and tactics (25.2%). This indicates that students have greater ease with basic understanding and recognition of fundamental concepts of rotational dynamics, but encounter difficulties in formulating inferences, making evidence-based decisions, and developing strategies to solve more complex problems.

These difficulties may be associated with traditional teaching approaches, which prioritize the memorization of concepts and formulas over the active exploration of physical principles and investigative problem-solving [5]. In this sense, a teaching model based on a problem-solving laboratory (PSL), as proposed by [6], highlights the importance of active and experimental learning. This model allows students to participate in the learning process through experiments, data analysis, and discussion of results, and has been shown to improve conceptual understanding and the development of scientific skills, such as hypothesis formulation and analysis of experimental errors.

The difficulties faced by students in physics can have several causes. The studies developed in [7] highlight the lack of connection between theory and practice, the difficulty in manipulating mathematical equations (especially formulas for moment of inertia and torque), the fragmented understanding of concepts, and the absence of structured problem-solving strategies. [8] add that most students have low skills in areas such as basic clarification, basic support, and advanced clarification, essential for analyzing problems, presenting evidence-based facts, and evaluating issues logically.

Conventional teaching methods, such as exclusively expository classes and traditional teaching materials, make it difficult to visualize and understand the concepts of rotational dynamics. The content is particularly challenging due to the need to visualize rotational motion, calculate the moment of inertia, and understand the relationships between complex physical quantities, which directly impacts students' ability to provide clear explanations or solve problems logically and systematically.

The work of [9] presents a systematic review on the teaching of oscillations, which offers a solid basis for developing teaching materials, technology-based instructional media and educational instruments related to oscillations, emphasizing the need to expand research on effective teaching methods on this topic, offering practical insights for educators and researchers.

Finally, the study by [10] shows that a significant portion of future science teachers have important conceptual difficulties on the topic of vibration and in this work, we seek to strengthen the fundamental concepts of basic physics that are essential for physics teachers.

III. METHODOLOGY AND RESULTS

We will analyze an asymmetrical physical pendulum that can rotate frictionlessly around a horizontal axis, where we will develop the concept through conservation of mechanical energy. In the authors' view, the analysis of such exercises not only provides examples of the fundamental points of the theoretical development we intend to present but also allows for a qualitative and quantitative discussion of important concepts involving physical pendulums, a discussion covered in the didactic proposal presented below.

A. Theoretical Development of the Case Study

Let the system consist of two point masses, m and M ($m < M$), rigidly connected by two rods of negligible mass of lengths ℓ and L ($\ell < L$), which form an angle β between them, this system capable of rotating with negligible friction around a horizontal axis, as shown in Figure 1.

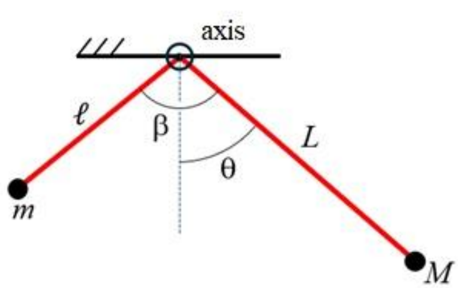

FIGURE 1. Asymmetrical physical pendulum that oscillates in a vertical plane.

Neglecting any friction or buoyancy forces, the two-mass system is acted upon by the connecting forces exerted by the rods and the weight forces. In this case, the system is conservative.

It is known from the study of statics that a system has two equilibrium points: the first, unstable, with the system's center of mass vertically above the suspension point, and the second, stable, with the system's center of mass vertically below the suspension point. We will analyze small oscillations of the system around the stable equilibrium position.

Equalizing the the intensities of the torques of the weights, to calculate the equilibrium positions:

$$m\,g\,.\,\ell\ \text{sen}\,(\beta - \alpha) = M\,g.\,L\ \text{sen}\,\alpha \qquad (1)$$

$$\alpha = \text{arc tg}\,[m\,\ell\ \text{sen}\,\beta\,/\,(m\,\ell\cos\beta + M\,L)] \qquad (2)$$

Considering the horizontal plane that passes through the axis as the reference level for the gravitational potential energy (height; $z = 0$), when the rod forms, at an instant t, an angle θ with the vertical direction, as shown in figure 1, we have the following expressions for the kinetic (Ek) and potential (E_p) energies of the system:

Equilibrium position α

$$E_{P1} = -\,m\,g\,\ell\cos(\beta - \alpha) - M\,g\,L\cos\alpha, \qquad (3)$$

$$E_{k1} = \frac{m\,u^2}{2} + \frac{M\,U^2}{2}. \qquad (4)$$

Since both masses have the same angular velocity (ω) due to the bond, the relationship between the linear velocities is (u / ℓ) = (U / L) = ω. Then putting the kinetic energy of the system in terms of the velocity U of the larger mass M:

$$E_{k1} = \frac{(m\,l^2 + M\,L^2)\,U^2}{2\,L^2}. \qquad (5)$$

Generic position θ

$$E_{P2} = -\,m\,g\,\ell\cos(\beta - \theta) - M\,g\,L\cos\theta. \qquad (6)$$

Similarly, with V being the velocity of the larger mass:

$$E_{k2} = \frac{(m\,l^2 + M\,L^2)\,V^2}{2\,L^2}. \qquad (7)$$

Expressing the generic angular coordinate (θ) in terms of the angular coordinate at equilibrium (α) and the angular displacement from the equilibrium position ($\Delta\theta$), we have:

$$\theta = \alpha + \Delta\,\theta \qquad (8)$$

From the conservation of mechanical energy:

$$E_{M1} = E_{M2} \;\rightarrow\; E_{P1} + E_{C1} = E_{P2} + E_{C2} \qquad (9).$$

From the previous equations:

$$\frac{(m\,l^2 + M\,L^2)\,U^2}{2\,L^2} = (1 - \cos\Delta\theta)\,[m\,g\,\ell\cos(\beta - \alpha) + M\,g\,L\cos\alpha]\; +$$

$$(\text{sen}\,\Delta\theta)\,[\,-\,m\,g\,\ell\,\text{sen}\,(\beta-\alpha)+M\,g\,L\,\text{sen}\,\alpha\,]\;+\;\frac{(m\,l^2+M\,L^2)\,V^2}{2\,L^2}. \quad (10)$$

Since the equilibrium equation (eq. 1a) is precisely $m\,g\,\ell\,\text{sen}\,(\beta-\alpha) = M\,g\,L\,\text{sen}\,\alpha$; it follows that

$$\frac{(m\,l^2+M\,L^2)\,U^2}{2\,L^2} = (1-\cos\Delta\theta)\,[m\,g\,\ell\cos(\beta-\alpha)+M\,g\,L\cos\alpha] + \frac{(m\,l^2+M\,L^2)\,V^2}{2\,L^2}. \quad (11)$$

Since these are small oscillations, we will consider approximations referring to small angles and binomial expansions.

$$\Delta\theta \approx \text{sen}\,\Delta\theta, \quad (12)$$

$$\text{and} \quad (1-\cos\Delta\theta) \approx (\Delta\theta)^2/2. \quad (13)$$

Geometrically, the linear displacement of the larger mass M along the arc of the circle can be approximated by a straightline segment of length X (approximation already known from the simple pendulum),

$$\Delta\theta \approx (X/L) \quad (14)$$

From equations 11 to 14, in equation 10, we have:

$$\frac{(m\,l^2+M\,L^2)\,U^2}{2} = \frac{g\,[m\,l\cos(\beta-\alpha)+M\,L\cos\alpha]\;X^2}{2} + \frac{(m\,l^2+M\,L^2)\,V^2}{2}. \quad (15)$$

The first term is the maximum kinetic energy of the system, which is reached at the stable equilibrium position. The equation corresponding to (15), in a mass-spring system of mass m' (inertial characteristic) and elastic force constant k' (restoring characteristic), taking the mechanical energy at the points of generic abscissa x and equilibrium ($x = 0$) is:

$$\frac{m'u'^2}{2} = \frac{m'v'^2}{2} + \frac{k'\,x^2}{2}. \quad (16)$$

Note that in the physical pendulum analyzed here, the movement of any of the masses depends, due to the geometric constraint, on the following characteristics of the system as a whole: i) the inertia associated with the rotation around the axis; ii) restoring characteristic.

It is concluded, then, comparing equations (15) and (16), that

$$m\,\ell^2 + M\,L^2 = \text{moment of inertia of the system}, \quad (17)$$

$$g\,[m\,\ell\cos(\beta-\alpha)+M\,L\cos\alpha] = \text{“effective force constant” of the system.} \quad (18)$$

The moment of inertia (I) and the "effective force constant"(k_{ef}) emerge, both qualitatively and quantitatively. The oscillation period is

$$T = 2\pi\sqrt{\frac{m\,l^2+M\,L^2}{g\,[m\,l\cos(\beta-\alpha)+M\,L\cos\alpha]}}. \quad (19)$$

Specific cases: i) $\beta = 0°$ (which implies $\alpha = 0°$, stable; or $\alpha = \pi$, unstable); masses "on the same side" (see Fig. 2), in that case:

$$T_1 = 2\pi\sqrt{\frac{m\,l^2+M\,L^2}{g\,(M\,L+m\,l)}}. \quad (20)$$

ii) $\beta = 180°$ (which implies $\alpha = 0°$, stable; or $\alpha = 180°$, unstable); diametrically opposite masses (see Fig. 3). In that case:

$$T_2 = 2\pi\sqrt{\frac{m\,l^2+M\,L^2}{g\,(M\,L-m\,l)}}. \quad (21)$$

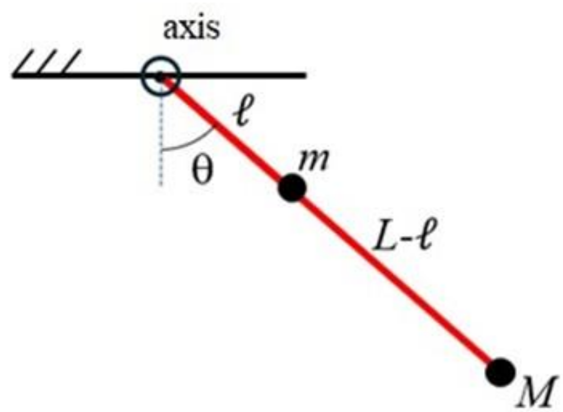

FIGURE 2. Asymmetrical physical pendulum that oscillates in a vertical plane for $\beta = 0°$.

For $\beta = 180°$, the torques due to gravitational forces have opposite directions (causing rotational effects in opposite directions); or, in terms of potential energy, while one mass rises, the other descends, reducing the effects of potential energy variation and, consequently, the gain of kinetic energy (and speeds).

In the case $\beta = 0°$, the torques due to gravitational forces have the same direction, causing overlapping rotational effects; or, in terms of potential energy, the two masses rise and fall together, accentuating the effects of potential energy variation and, consequently, the gain of kinetic energy (and speeds), resulting in a shorter period compared to the previously mentioned case.

B. Teaching Proposal

The authors understand the multiplicity of possibilities and objectives involved in each activity, whether theoretical or experimental. Here are just a few comments and suggestions regarding points to be discussed with the students: i) Highlight that the moment of inertia mathematically depends on the sum of products, where each term is directly proportional to the mass and the square of the distance of the terms to the axis of rotation; ii) Possibility of introducing the generalized expression for the moment of inertia for a system of n discrete particles $I = \sum_1^n m_i r_1^2$, where the derivation of the expression for rotational kinetic energy $E_k = \frac{I\,\omega^2}{2}$ occurs naturally from equations (5) and (7), and definition (17); iii) Discuss the dimensionality aspect of the formulas obtained; iv) Develop the scheme referring to $\beta = 180^o$ and ask students to investigate the changes that theoretically exist for the case of $\beta = 0^o$, regarding torques, energy variations, and their implications; v) As is known from the dynamics of rotation, when analyzing physical pendulums, it cannot be considered that all the mass of the physical pendulum is concentrated in the center of mass and treat it as simple pendulums, where the length of the "equivalent simple pendulum" is the distance between the center of mass (CM) and the axis of rotation [1, 2, 3] and [4].

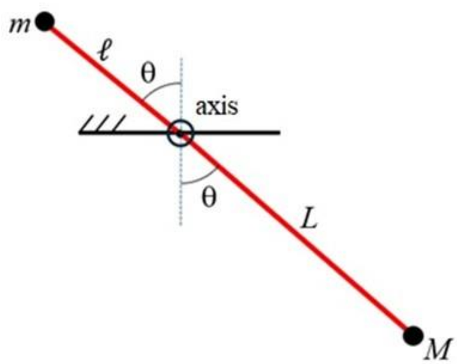

FIGURE 3. Asymmetrical physical pendulum that oscillates in a vertical plane for $\beta = 180^o$.

The following questions could be posed to students: could we replace the masses with a single mass located at the center of mass and think of the period of the physical pendulum in terms of an equivalent simple pendulum? In this case, it is possible to introduce the concept of radius of gyration.

Taking the axis of rotation as a reference point (origin), the centers of mass (CM) of the systems constituted by the two masses are at distances from the axis of rotation, for the cases $\beta = 0°$ and $\beta = 180°$ respectively:

$d_3 = (ML + m\,\ell) / (m + M)$ e $d_4 = (ML - m\,\ell) / (m + M)$,

$$T_3 = 2\pi\sqrt{\frac{(M\,L + m\,l)}{g\,(M + m)}}\,, \tag{22}$$

and

$$T_4 = 2\pi\sqrt{\frac{(M\,L - m\,l)}{g\,(M + m)}}\,. \tag{23}$$

In such predictions, T_3 and T_4, which are in themselves already wrong, lead us to $T_3 > T_4$, which is at odds with what is expected qualitatively, according to the previous conceptual argument (i and ii).

When considering masses and lengths with very close values $\gamma = (M / m) \approx 1$; $\sigma = (L / \ell) \approx 1$, the period T_4 (23) would approach zero and not "infinity", which shows a great mistake in the reasoning used to obtain such an expression.

Comparing the expressions for the periods of the physical pendulums (20) and (21) with the expression for the period of the simple pendulum, the true associated radii of gyration (v) would be:

$$d_1 = \frac{m\,l^2 + M\,L^2}{M\,L + m\,l} \text{ (for } \beta = 0^o). \tag{24}$$

$$d_2 = \frac{m\,l^2 + M\,L^2}{M\,L - m\,l} \text{ (for } \beta = 180^o). \tag{25}$$

For the case $\beta = 0^o$, it is possible to demonstrate that the period of the physical pendulum T_1 (20) is greater than that of a physical pendulum of length l and less than that of a simple pendulum of length L; or, in other words, that the distance d1 obtained previously is greater than l and less than L. Demonstrating:

$$2\pi\sqrt{\frac{l}{g}} < 2\pi\sqrt{\frac{m\,l^2 + M\,L^2}{g\,(M\,L + m\,l)}} \quad 2\pi\sqrt{\frac{L}{g}} \leftrightarrow \ell < \frac{m\,l^2 + M\,L^2}{(M\,L + m\,l)} < L.$$

The previous equation is true for any values of ℓ and L, with $\ell < L$. vi) Compare the theoretical predictions for the periods T_1 (20), T_2 (21), T_3 (22), and T_4 (23), and also for the associated radii of gyration d_1, d_2, d_3, and d_4, respectively, with the experimental results; vii) Include the influence of the rods, asking students to discuss how the expressions for the periods (and radii of gyration) would be altered.

Note that: representing M' and m' as the masses of the rods of greater and lesser lengths, respectively, and knowing that $I_{bars} = 1/12(M'L^2 + m'l^2)$, the general expression for the period T (19) takes the following form

$$T = 2\pi\sqrt{\frac{ml^2 + L^2 + Ibars}{g\,[(m + \frac{m'}{2})\,l\cos(\beta - \alpha) + \left(M + \frac{M'}{2}\right)L\cos\alpha]}}. \tag{26}$$

C. Apparatus and Experimental Results

The experiment was conducted with metal bars, cylinders, and fasteners, whose masses and characteristic dimensions were determined with appropriate measuring instruments, including their respective uncertainties. These procedures ensured the reliability of the data obtained and allowed comparison with the adopted theoretical model. Figures 3 and 4 illustrate the experimental setup and the main aspects observed. The set up specifications are:

$M_{\text{minor bar}} = (31{,}23 \mp 0{,}05)\ g$;

$M_{\text{bigger bar}} = (91{,}20 \mp 0{,}05)\ g$;

$M_{\text{fixing screw}} = (6{,}08 \mp 0{,}05)\ g$;

$M_{\text{cylinders}} = m_{\text{cylinders}} = (400{,}42 \mp 0{,}05)\ g$;

$L = (90 \pm 1)$ cm; $l = (30 \pm 1)$ cm.

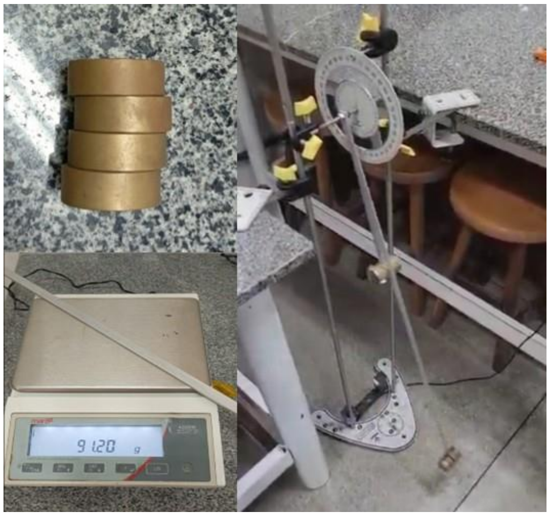

FIGURE 4. Set of materials used in the experiment: cylinders, metal bar and fixing elements. Regarding the measurement of periods, the sensor indicated the time interval of each oscillation.

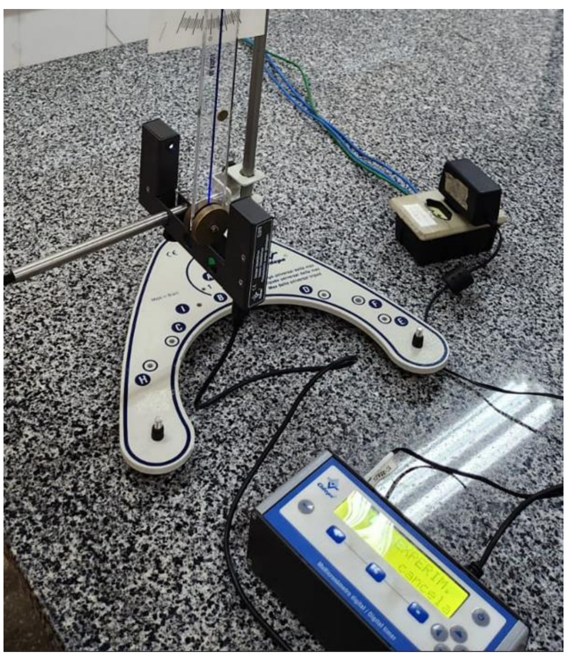

FIGURE 5. Experimental apparatus.

In equations (20) to (25), the values of M and m used will be the masses of the cylinders fixed to the bars added to the masses of the fixing screws; that is, $M = m = (406.5 \mp 0.1)$ g. Tables I and II present comparisons between experimental and theoretical values for the oscillation periods.

TABLE I. Comparison between the theoretical and experimental values for the oscilation periods for $\beta = 0^o$.

d_1 (cm)	T_1 (s) (theory)	d_3 (cm)	T_3 (s)	$T_{experiment}$ (s)*
57.5	1.74	46	1.36	(1.73 ∓ 0.01)

TABLE II. Comparison between the theoretical and experimental values for the oscilation periods for $\beta = 180^o$.

d_2 (cm)	T_1 (s) (theory)	d_4 (cm)	T_4 (s)	$T_{experiment}$(s)
115	2.46	23	0,96	(2.39 ∓ 001)

For $\beta = 5^o$
$\alpha = 1{,}62^o$; $T \approx T_1$

The two previous tables indicating a discrepancy between predictions based on a mistaken line of reasoning involving radii of gyration (d_3,T_3) and (d_4,T_4) and the experimental result, may constitute a trigger for a conceptual change.

IV. CONCLUSIONS

In this paper, we present an alternative approach to HM, regarding the quantities period and frequency, accessible physically and mathematically within the typical high school curriculum. While this approach is not necessarily the simplest (at least from an algebraic standpoint), when compared to the typical solution in the basic undergraduate cycle using Newton's Second Law in angular form (τR = Iα), it does provide high school students with a better understanding of the "power" of analytical methods. Equally important is addressing the concept of moment of inertia not only qualitatively, as is routinely done (both theoretically and experimentally), but also quantitatively; we hope, in this way, to contribute in some way to bridging the gaps identified.

In the authors' view, the transition from the discrete situation $I = \sum_1^n m_i r_1^2$ to the continuous one $\int r^2 dm$ becomes purely mathematical, and no longer physical (In this case, the results of the calculation could be presented to high school students, which would greatly expand the range of possible situations to be addressed.

The condition for approaching the problems along the path we followed, in the previous case study and also in general, is that the system of particles is subject to constraints such that it is possible to express both the total kinetic energy of the system and the total potential energy, as a function of the linear velocity and the linear displacement from the equilibrium position of a given (chosen) particle of the system.

Regarding the teaching activity, numerous topics can be discussed involving the proposed problem: extended-body equilibrium—along with stability and instability analysis—torque and moment of inertia, mechanical energy (potential and kinetic), center of mass, and radius of gyration. It is up to the teacher, depending on factors such as time and objectives, to decide which topics to cover and to what degree, as well as, of course, the methodology to be used.

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