Noncommutative Dirac and Schrödinger equations in the background of the new combined Manning-Rosen and Yukawa tensor potentials using Bopp's shift method and perturbation theory

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ISSN 1870-9095

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(Received 20 November 2025, accepted 27 December 2025)

Abstract

Among the significant potential that showed its great importance in the literature is what is known as the Manning-Rosen potential with a Yukawa tensor coupling because it has wide applications to a wide variety of physical systems. In this work, new Bound-state solutions of the deformed Dirac equation with improved spin and pseudo-spin symmetries are investigated for the new combined Manning--Rosen and Yukawa tensor potentials (*NCMRYPs*) in the context of three-dimensional relativistic noncommutative quantum space (3D-RNCQS) symmetries. The new energy eigenvalues a are obtained using the parametric Bopp's shift method and the like Greene-Aldrich approximation for the centrifugal terms $\frac{z^2}{(1-z)^3}$, $\frac{z^{5/2}}{(1-z)^4}$, $\frac{z^{3/2}}{(1-z)^2}$, $\frac{z^{5/2}}{(1-z)^3}$, $\frac{z^3}{(1-z)^4}$, $\frac{z^{7/2}}{(1-z)^4}$ and $\frac{z^2}{(1-z)^4}$ to obtain the effective potentials of the *NCMRYPs* model in 3D-RNCQS symmetries. The new energy levels are sensitive depending on noncommutativity parameters (η, λ, γ), the potential depths (β, A, V_0) of the *NCMRYPs* model, the quantum numbers $(j, l/l_p, s/s_p, m/m_p)$ in addition to arbitrary spin-orbit coupling quantum number k, radial quantum numbers n, and screening parameter δ which are known in the literature. The non-relativistic limit is obtained and the composite systems such as molecules made of $N = 2$particles of masses $m_n (n = 1,2)$ in the frame of three-dimensional nonrelativistic noncommutative quantum space (3D-NRNCQS) symmetries are considered. After studying the relativistic and nonrelativistic solutions of the *NCMRYPs* model in 3D-RNCQS and 3D-NRNCQS symmetries, we examine some important cases that we see as useful to the reader and the researcher.

Keywords: Dirac equation; Schrödinger equation; Manning-Rosen potential; Pseudospin and spin symmetry; Yukawa tensor interaction; Noncommutative space; Bopp's shift method.

Resumen

Entre los potenciales significativos que han demostrado su gran importancia en la literatura se encuentra el denominado potencial de Manning-Rosen con acoplamiento tensorial de Yukawa, debido a sus amplias aplicaciones en una amplia variedad de sistemas físicos. En este trabajo, se investigan nuevas soluciones de estado ligado de la ecuación de Dirac deformada con simetrías de espín y pseudoespín mejoradas para los nuevos potenciales tensoriales combinados de Manning-Rosen y Yukawa (*NCMRYPs*) en el contexto de simetrías tridimensionales relativistas no conmutativas del espacio cuántico (3D-RNCQS). Los nuevos valores propios de energía a se obtienen utilizando el método paramétrico de desplazamiento de Bopp y la aproximación de Greene-Aldrich similar para los términos centrífugos $\frac{z^2}{(1-z)^3}$, $\frac{z^{5/2}}{(1-z)^4}$, $\frac{z^{3/2}}{(1-z)^2}$, $\frac{z^{5/2}}{(1-z)^3}$, $\frac{z^3}{(1-z)^4}$, $\frac{z^{7/2}}{(1-z)^4}$ y $\frac{z^2}{(1-z)^4}$ para obtener los potenciales efectivos del modelo NCMRYPs en simetrías 3D-RNCQS. Los nuevos niveles de energía son sensibles a los parámetros de no conmutatividad (η, λ, γ), las profundidades potenciales (β, A, V_0) del modelo *NCMRYPs*, los números cuánticos (j, l/l_p, s/s_p, m/m_p), además del número cuántico de acoplamiento espín-órbita arbitrario k, los números cuánticos radiales n y el parámetro de cribado δ, conocidos en la literatura. Se obtiene el límite no relativista y se consideran los sistemas compuestos, como moléculas formadas por $N = 2$ partículas de masas m_n ($n = 1, 2$), en el marco de simetrías tridimensionales del espacio cuántico no relativista no conmutativo (3D-NRNCQS). Tras estudiar las soluciones relativistas y no relativistas del modelo *NCMRYPs* en las simetrías 3D-RNCQS y 3D-NRNCQS, examinamos algunos casos importantes que consideramos útiles para el lector y el investigador.

Palabras clave: Ecuación de Dirac; Ecuación de Schrödinger; Potencial de Manning-Rosen; Pseudoespín y simetría de espín; Interacción del tensor de Yukawa; Espacio no conmutativo; Método de desplazamiento de Bopp.

I. INTRODUCTION

In 1933, Manning and Rosen (see Eq. (3.1)) proposed a potential function for diatomic molecules known as the Manning-Rosen (MR) potential model [1] which is used in different fields such as atomic, condensed matter, particle, and nuclear physics. In addition, this potential is used to describe the vibrations of diatomic molecules HCl, CH, LiH, CO, NO, O_2, I_2, N_2, H_2, and Ar_2 [2]. Wang *et al.* (2012), proposed a convenient form (see Eq. (3.2)) for the original expression of the MR potential function [3]. Wei and Dong carried out approximately analytical bound state solutions of the Dirac equation (DE) with the MR potential for arbitrary spin-orbit coupling quantum number k by taking a properly approximate expansion for the spin-orbit coupling term [4]. Chen et al; (2009) solved approximately the DE with the MR potential for the arbitrary spin-orbit quantum number k using the basic concept of the supersymmetric shape invariance formalism and the function analysis method [5]. Eshghi and Mehraban obtained analytically the approximate energy equation and the corresponding wave functions of the DE for the MR potential coupled with a Coulomb-like tensor under the condition of the pseudo-spin symmetry using the parametric generalization of the Nikiforov-Uvarov (NU) method [6]. Oktay and Sever obtained an approximate analytical solution of the DE for the Yukawa potential under the pseudospin symmetry condition using the asymptotic iteration method [7]. Aguda obtains the approximate analytical solutions of the DE for an improved expression of the Rosen-Morse potential energy model, including the Coulomb-like tensor under the condition of spin and pseudospin symmetry [8]. Jia *et al.* explored the analytical solutions of the DE with the spin symmetry for the improved MR potential energy model and presented the bound state energy equation and the corresponding upper and lower radial wave functions [9]. Yanar and Havare spin and pseudospin symmetry are obtained by solving the DE with centrifugal term Dirac spinors and energy relations with generalized MR potential using the NU method and also the Pekeris approximation to the centrifugal term [10]. Wei *et al.* (2008) studied approximately the bound state solutions of the Klein-Gordon equation with the MR potential [11]. Taskin [12] investigated approximately the bound state solutions of the DE with the MR potential within the framework of the spin symmetry and pseudo-spin symmetry concepts. Recently, Ahmadov *et al.* (2022) presented the bound state solutions of the DE with spin and pseudo-spin symmetries for the combined MR potential with Yukawa-like tensor interaction in the framework of supersymmetry quantum mechanics and NU methods [13]. It should be noted that there are other studies of MR that we mention [14, 15, 16, 17]. The objective of this work is to calculate the new relativistic and nonrelativistic energy eigenvalue for the combined MR and Yukawa tensor potentials using an unperturbed hypergeometric function with a centrifugal approximation factor within the framework of the extended symmetries of relativistic and non-relativistic quantum mechanics. For this, we develop a mathematical model using the unperturbed Dirac spinor to find the new energy eigenvalue. It is important to refer to previous studies that we have carried out in recent years related to MR potential, but in another context, we mention it. Very recently, we have studied the new generalized Schiöberg and MR potentials within the generalized tensor interactions in the framework of three-dimensional extended QM symmetries [18]. In 2021, we investigated the bound state of deformed KGE and SE under the modified equal vector and scalar MR and class Yukawa potential (CYP) in relativistic and nonrelativistic extended QM symmetries [19]. In the same context deformed (KGE and SE), we carried out a study on, modified MR [20] and modified MR plus quadratic Yukawa potential [21] in addition to the modified MR and Yukawa potential [22].To the best of my knowledge, no researcher has addressed the combined MR and Yukawa tensor potentials in the symmetries of deformed Dirac theory 3D-RNCQS. I hope that through this study we will discover more investigations at the sub-atomic scale and achieve more scientific knowledge of elementary particles in the field of Nanoscales. We aimed to shed more light on combined MR and Yukawa tensor potentials within the framework of an extended space that contains large symmetries based on the new postulate $\left[q_{\mu}^{(s,h,i)}{}^{*}, q_{\nu}^{(s,h,i)}\right] \neq 0and\left[\pi_{\mu}^{(s,h,i)}{}^{*}, \pi_{\nu}^{(s,h,i)}\right] \neq 0$ in addition to the generalized postulate $\left[q_{\mu}^{(s,h,i)}{}^{*}, \pi_{\nu}^{(s,h,i)}\right] \neq 0$ (see below equations). The wide interest of researchers in the field of noncommutativity came as a result of it being a strong candidate alternative to solve many of the problems that have emerged strongly such as quantum gravity, string theory, and the divergence problem of the standard model [22, 23, 24, 25, 26, 27, 28, 29, 30, 31]. NC of space-space and NC of phase-phase play an important role in changing the physical properties of a lot of quantum physical systems, and they have achieved interesting successes in recent years. The NC properties idea is not new but goes back decades and was suggested by Snyder [32, 33] in 1947, and its geometric analysis was introduced by Connes in 1991 and 1994 [34, 35]. Seiberg and Witten, extend earlier ideas about the appearance of NC geometry in string theory with a nonzero B-field and obtain a new version of gauge fields in noncommutative gauge theory [36]. Among the potential goals of NC deformation of space-space and phase-phase is the emergence of new quantum fluctuations capable of canceling the observed unwanted divergences or the infinities that appear to cause short-range effects in field theories that include gravitational theory [37]. The research reported in the present paper was motivated by the fact that the study of the new combined MR and Yukawa tensor potentials (NCMRYPs) in the 3D-RNCQS symmetries has not been reported in the available literature. In this work, the vector and scalar NCMRYPs model ($V_{mr}(d)$,$S_{mr}(d)$) to be employed is defined as:

$$\begin{cases} V_{mr}^{s}(d) = V_{mr}(r) - \frac{\partial V_{mr}(r)}{\partial r}\frac{\mathbf{L\Theta}}{2r} + O(\Theta^2), \\ S_{mr}^{s}(d) = S_{mr}(r) - \frac{\partial S_{mr}(r)}{\partial r}\frac{\mathbf{L\Theta}}{2r} + O(\Theta^2), \end{cases} \tag{1}$$

and

$$\begin{cases} V_{mr}^{p}(d) = V_{mr}(r) - \frac{\partial V_{mr}(r)}{\partial r}\frac{\mathbf{L_p\Theta}}{2r} + O(\Theta^2), \\ S_{mr}^{p}(d) = S_{mr}(r) - \frac{\partial S_{mr}(r)}{\partial r}\frac{\mathbf{L_p\Theta}}{2r} + O(\Theta^2), \end{cases} \tag{2}$$

where $\left(V_{mr}(r), S_{mr}(r)\right)$ are the vector and scalar potentials according to the view of 3D-RQM known in the literature [13,17]:

$$\begin{cases} V_{mr}(r) = \frac{\hbar}{2Mb^2}\left(\frac{\beta(\beta-1)\,exp(-4\delta r)}{(1-exp(-2\delta r))^2} - \frac{A\,exp(-2\delta r)}{1-exp(-2\delta r)}\right), \\ S_{mr}(r) = \frac{\hbar}{2Mb^2}\left(\frac{\beta(\beta-1)\,exp(-4\delta r)}{(1-exp(-2\delta r))^2} - \frac{A_s\,exp(-2\delta r)}{1-exp(-2\delta r)}\right), \end{cases} \tag{3.1}$$

Or a convenient form for the original expression [2]:

$$V_{mr}(r) = D_e\left(1 - \frac{exp(\alpha r_e)-1}{exp(\alpha r)-1}\right)^2, \tag{3.2}$$

where $\delta = \frac{1}{2b}$ is the screening parameter while A/A_s and β are parameters associated with the height of the potential, $(d \text{ and } r)$ are the distance between the two particles in the deformation of Dirac theory symmetries and QM symmetries, respectively. The two couplings ($\mathbf{L\Theta}$ and $\mathbf{L_p\Theta}$) are the scalar product of the usual components of the angular momentum operators $\mathbf{L}(L_x, L_y, L_z)/\mathbf{L_p}(L_p^x, L_p^y, L_p^z)$ and the modified noncommutativity vector $\mathbf{\Theta}(\theta_{12}, \theta_{23}, \theta_{13})/$ which present is the noncommutativity elements parameter. In the case of the NC-quantum group, the noncentral generators can be suitably realized as self-adjoint differential operators ($q_\mu^{(s,h,i)}, \pi_\nu^{(s,h,i)}$) appear in three varieties the first one is the canonical_structure (CS), the second is Lie structure (LS) and the last corresponds to the quantum plane (QP) in the representations of Schrödinger, Heisenberg, and interactions pictures, obeying the following set of commutation relations (we have used the natural units $\hbar = c = 1$) (see, e.g.; [38,39,40,41,42,43,44,45,46,47]):

$$\underbrace{\left[x_\mu^{(s,h,i)}, p_\nu^{(s,h,i)}\right] = i\hbar\delta_{\mu\nu}}_{In\ QM\_\text{sy.}} \Rightarrow \underbrace{\left[q_\mu^{(s,h,i)}, \pi_\nu^{(s,h,i)}\right]_* = i\hbar_{eff}\delta_{\mu\nu}}_{In\ DDT\_\text{sy.}} \tag{4.1}$$

and

$$\left[x_\mu^{(s,h,i)}, x_\nu^{(s,h,i)}\right] = 0 \Rightarrow \left[q_\mu^{(s,h,i)}, q_\nu^{(s,h,i)}\right]_* = \begin{cases} i\theta_{\mu\nu} : \eta_{\mu\nu} \in IC \text{ For CS}, \\ if_{\mu\nu}^{\eta} q_\theta^{(s,h,i)} : f_{\mu\nu}^{\eta} \in IC \text{ For LS}, \\ iC_{\mu\nu}^{\eta\delta} q_\theta^{(s,h,i)} q_\theta^{(s,h,i)} : C_{\mu\nu}^{\eta\delta} \in IC \text{ For QP}. \end{cases} \tag{4.2}$$

with $q_\mu^{(s,h,i)} = \left(q_\mu^s, q_\mu^h, q_\mu^i\right)$and $\pi_\mu^{(s,h,i)} = \left(\pi_\mu^s, \pi_\mu^h, \pi_\mu^i\right)$ are the generalized coordinates and the corresponding generalizing coordinates in the 3D-RNCQS and 3D-NRNCQS symmetries while IC denotes the complex number field while $x_\mu^{(s,h,i)} = \left(x_\mu^s, x_\mu^h, x_\mu^i\right)$ and $p_\mu^{(s,h,i)} = (p_\mu^s, p_\mu^h, p_\mu^i)$ are corresponding coordinates in the 3D-RQM and 3D-NRQM symmetries. Furthermore, the usual uncertainty relation corresponds to the LHS of Eq. (4.1) will be extended to become two uncertainties in the following formula in the new form symmetries as follows:

$$\left|\Delta x_\mu^{(s,h,i)} \Delta p_\nu^{(s,h,i)}\right| \geq \frac{\hbar\delta_{\mu\nu}}{2} \Rightarrow \left|\Delta q_\mu^{(s,h,i)} \Delta \pi_\nu^{(s,h,i)}\right| \geq \frac{\hbar_{eff}\delta_{\mu\nu}}{2}, \tag{5.1}$$

and

$$\left|\Delta q_\mu^{(s,h,i)} \Delta q_\nu^{(s,h,i)}\right| \geq \begin{cases} \left|\theta_{\mu\nu}\right|/2 \text{ For CS}, \\ f_{\mu\nu}/2 \text{ For LS}, \\ C_{\mu\nu}/2 \text{ For QP}. \end{cases} \tag{5.2}$$

Here $f_{\mu\nu}$and $C_{\mu\nu}$ are present the following average values, $\left|\sum_{\eta=1}^{3} f_{\mu\nu}^{\eta} q_\eta^{(s,h,i)}\right|$ and $\left|\sum_{\eta,\delta}^{3} C_{\mu\nu}^{\eta\delta} q_\eta^{(s,h,i)} q_\delta^{(s,h,i)}\right|$ respectively. The uncertainty relation in Eq. (5.1) is obtained as a result of the generalization of LHS Eq. (4.1) to RHS form while the second uncertainty relation in Eq. (5.2) is the result of the deformation of space-space that appears from RHS of Eq.(4.2) that is divided into three varieties. We extended the modified equal time noncommutative canonical commutation relations (METNCCCRs) to include the Heisenberg and interaction pictures in 3D-RNCQS and 3D-NRNCQS symmetries. Here $\hbar_{eff} \cong \hbar$ is the effective Planck constant, $\eta_{\mu\nu} = \varepsilon_{\mu\nu}\theta$ (θ is the non-commutative parameter and $\varepsilon_{\mu\nu}$ is just an antisymmetric number $\varepsilon_{\mu\nu} = -\varepsilon_{\nu\mu} = 1$ with $\mu \neq \nu$ and $\varepsilon_{\varepsilon\varepsilon} = 0$) which is an infinitesimal parameter if compared to the energy values and elements of antisymmetric (3×3) real matrices and $\delta_{\mu\nu}$ is the Kronecker symbol. The symbol $*$ denotes the Weyl-Moyal star product, which is generalized between two ordinary functions $f(x)g(x)$ to the new deformed form $f(q)g(q)$ is mapped onto the product of symbols of operators $f(x) * g(x)$, in the symmetries of deformation space-space symmetries, called the star-product determined by (see, e.g.; [48]):

$$f(x) * g(x) = \begin{cases} exp\left(i\varepsilon^{\mu\nu}\theta\partial_\mu^x\partial_\nu^x\right)(fg)(x) \text{ For CS}, \\ exp\left(\frac{i}{2}x_{nc\mu}^{(s,h,i)} g_k\left(i\partial_\mu^x, i\partial_\nu^x\right)\right)(fg)(x) \text{ For LS}, \\ \left[iq^{G(u,v,\partial_\mu^u,\partial_\nu^v)} f(u,v)g(u',v')\right]_{u'\to u}^{v'\to v} \text{ For QP}. \end{cases} \tag{6}$$

With

$$g_\eta(k,p) = -k_\mu p_\nu f_k^{\nu\nu} + \frac{1}{6}k_\mu p_\nu\left(p_\eta - k_\eta\right) f_l^{\nu\nu} f_m^{l\eta} + \ldots$$

In the current paper, we apply the MASCCCRs in the 3D-RNCQS and 3D-NRNCQS symmetries, which allows us to rewrite to the following simple form at the first order of noncommutativity parameter $\varepsilon^{\mu\nu}\theta$ as follows [49, 50, 51, 52, 53, 54, 55, 56, 57, 58]:

$$(f * g)(x) = exp\left(i\varepsilon^{\mu\nu}\theta\partial_\mu^x\partial_\nu^x\right)(fg)(x) \approx (fg)(x) - \frac{i\varepsilon^{\mu\nu}\theta}{2}\partial_\mu^x f \partial_\nu^x g ?_{x^\mu = x^\nu} + O(\theta^2)\,. \tag{7}$$

The indices $\mu, \nu = 1,2,3$ and $O(\theta^2)$ stand for the second and higher-order terms of the NC parameter. Physically, the second term in Eq. (7.1) presents the effects of space-space noncommutativity. The main aim of the paper is to investigate the (k, l)-states solutions of deformed DE and deformed Schrödinger equation (DSE) with the *NCMRYPs* model in the symmetries of 3D-RNCQS and 3D-NRNCQS symmetries, within the frame of parametric Bopp's shift method. The present paper is organized as follows. The first section includes the scope and purpose of our investigation, while the remaining parts of the paper are structured as follows. A review of the DE with the combined Manning-Rosen and Yukawa tensor potentials is presented in Sect. 2. Sect. 3 is devoted to studying the DDE by applying the usual well-known Bopp's shift method and the like Greene-Aldrich approximation for the centrifugal term to obtain the effective potentials of the *NCMRYPs* model in 3D-RNCQS symmetries. Furthermore, via standard perturbation theory, we find the expectation values of some radial terms to calculate the corrected relativistic energy generated by the effect of the perturbed effective potentials $\Sigma_{pert}^{mg}(r)$ and $\Delta_{pert}^{mg}(r)$ of the *NCMRYPs* model, we derive the global corrected energy with the NCMYPs model. In the next section, we obtain the nonrelativistic limit and consider the composite systems such as molecules made of $N=2$ particles of masses $m_n(n=1,2)$ in the frame of 3D-NRNCQS symmetries are considered. Sect. 5 is reserved to study the relativistic and nonrelativistic special cases that can be generated from the NCMYPs model. Finally, the conclusion is given in Sec.6

II. AN OVERVIEW OF DE UNDER COMBINED MANNING-ROSEN AND YUKAWA TENSOR POTENTIALS

For a deeper understanding of the relativistic interactions of fermion particles that interacted with NCMYP's model in extended Dirac theory, it is useful to recall the eigenvalues and the corresponding eigenfunctions that influenced this system within the framework of relativistic quantum mechanics known in the literature. In this case, the system is governed by the basic equation:

$$\begin{cases} H_D^{mr}\Psi_{nk}(r,\theta,\phi)=E_{nk}\Psi_{nk}(r,\theta,\phi) \\ H_D^{mr}=\alpha p+\beta\big(M+S_{mr}(r)\big)-i\beta dU(r)+V_{mr}(r), \end{cases} \quad (8)$$

here H_D^{mr} is the Dirac Hamiltonian operator, M is reduced rest mass, $\boldsymbol{p}=-i\hbar\boldsymbol{\nabla}$ is the momentum. The vector potential $V_{mr}(r)$ due to the four-vector linear momentum operator $A^{\mu}($ $V_{mr}(r),\mathbf{A}=\mathbf{0})$ and space-time scalar potential $S_{mr}(r)$ due to the mass, E_{nk} is the relativistic eigenvalues, (n,k) representing the principal and spin-orbit coupling terms, respectively. The tensor interaction $U(r)$ equal to $\left(-V_0\frac{exp(-\delta r)}{r}\right)$, V_0 denotes the strength of the interaction and δ is the screening parameter, $\alpha=anti\_diag(\tau_i,\tau_i)$, $\beta=diag(I_{2\times2},-I_{2\times2})$ and τ_i are the usual Pauli matrices. Since the combined Manning-Rosen and Yukawa tensor potentials have spherical symmetry, the solutions of the known form $\Psi_{nk}(r,\theta,\phi)=\begin{pmatrix}\frac{F_{nk}(r)}{r}Y_{jm}^{l}(\theta,\phi)\\ i\frac{G_{nk}(r)}{r}Y_{jm_p}^{l_p}(\theta,\phi)\end{pmatrix}$, $F_{nk}(r)$ and $G_{nk}(r)$ represent the upper and lower components of the Dirac spinors $\Psi_{nk}(r,\theta,\phi)$ while $Y_{jm}^{l}(\theta,\phi)$ and $Y_{jm_p}^{l_p}(\theta,\phi)$ are the spin and pseudospin spherical harmonics and (m,m_p) are the projections on the z-axis. The upper and lower components $F_{nk}^{s}(r)$ and $G_{nk}^{p}(r)$ for spin symmetry and pseudospin symmetry satisfy the two uncoupled differential equations as below:

$$\Big(\frac{d^2}{dr^2}-k(k+1)r^{-2}+U_{eff}^{yt-s}(r)-\left(M+E_{nk}^{s}-\Delta_{mr}^{p}(r)\right)\left(M-E_{nk}+\Sigma_{mr}^{s}(r)\right)+\frac{\frac{d\Delta_{mr}^{p}(r)}{dr}\left(\frac{d}{dr}+\frac{k}{r}-U(r)\right)}{M+E_{nk}-\Delta_{mr}^{p}(r)}\Big)F_{nk}^{s}(r)=0, \quad (9)$$

and

$$\Big(\frac{d^2}{dr^2}-k(k-1)r^{-2}+U_{eff}^{yt-p}(r)-\left(M+E_{nk}^{p}-\Delta_{mr}^{p}(r)\right)\left(M-E_{nk}+\Sigma_{mr}^{s}(r)\right)+\frac{\frac{d\Sigma_{mr}^{s}(r)}{dr}\left(\frac{d}{dr}-\frac{k}{r}+U(r)\right)}{M+E_{nk}+\Sigma_{mr}^{s}(r)}\Big)G_{nk}^{p}(r)=0. \quad (10)$$

Here

$$U_{eff}^{yt-s/p}(r)=\frac{2kU(r)}{r}\mp\frac{dU(r)}{dr}-U^2(r). \quad (11)$$

That can be expressed analytically as,

$$U_{eff}^{yt-s/p}(r)=B^{\mp}\frac{exp(-\delta r)}{r^2}\mp\delta V_0\frac{exp(-\delta r)}{r}-V_0^2\frac{exp(-2\delta r)}{r^2}, \quad (12)$$

with $B^{\mp}=(-2k\mp1)V_0$ while $\Sigma_{mr}^{s}(r)=V_{mr}(r)$ and $\Delta_{mr}^{p}(r)=V_{mr}(r)$ are determined by:

$$\begin{cases}\Sigma_{mr}^{s}(r)=\frac{\hbar}{2Mb^2}\left(\frac{\beta(\beta-1)\,exp(-4\delta r)}{(1-exp(-2\delta r))^2}-\frac{A\,exp(-2\delta r)}{1-exp(-2\delta r)}\right),\\ \text{and } \frac{d\Delta_{mr}^{p}(r)}{dr}=0\ \Rightarrow \Delta_{mr}^{p}=C_{SE}\text{ for spin sy.}\end{cases} \quad (13)$$

and

$$\begin{cases}\Delta_{mr}^{p}(r)=\frac{\hbar}{2Mb^2}\left(\frac{\beta(\beta-1)\,exp(-4\delta r)}{(1-exp(-2\delta r))^2}-\frac{A\,exp(-2\delta r)}{1-exp(-2\delta r)}\right)\\ \text{and } \frac{d\Sigma_{mr}^{s}(r)}{dr}=0\ \Rightarrow \Sigma_{mr}^{s}=C_{PS}\text{ for p-spin sy.}\end{cases} \quad (14)$$

We obtain the following second-order Schrödinger-like equation in 3D-RQM symmetries, respectively:

$$\left[\frac{d^2}{dr^2}-k(k+1)r^{-2}+U_{eff}^{yt-s}(r)-\Lambda_{nk}^{s}\left(M-E_{nk}^{s}+\Sigma_{mr}^{s}(r)\right)\right]F_{nk}^{s}(r)=0 \quad (15)$$

and

$$\left[\frac{d^2}{dr^2}-k(k-1)r^{-2}+U_{eff}^{yt-p}(r)-\left(M+E_{nk}^{p}-\Delta_{mr}^{p}(r)\right)\Lambda_{nk}^{p}\right]G_{nk}^{p}(r)=0 \quad (16)$$

with $k(k-1)$ and $k(k+1)$ are equals to $l_p(l_p-1)$ and $l(l+1)$, respectively. The authors of refs.[13,17] using both the Nikiforov-Uvarov method and Greene-Aldrich approximation for the centrifugal term to obtain the expressions for the upper and lower components $F_{nk}^{s}(r)$ and $G_{nk}^{p}(r)$ as hypergeometric polynomials $P_n^{(\nu_{nk}^1/2,\zeta_{nk}^1)}(1-2z)$ and $P_n^{(\nu_{pnk}^1/2,\zeta_{pnk}^1)}(1-2z)$ in 3D-RQM symmetries as,

$$F_{nk}^{s}(s)=C_{nk}^{s}s^{\nu_{nk}^{1}}(1-z)^{(1+\zeta_{nk}^{1})/2}P_{n}^{(2\nu_{nk}^{1},\zeta_{nk}^{1})}(1-2z), \quad (17)$$

and

$$G_{nk}^{p}(s)=C_{nk}^{p}z^{\nu_{pnk}^{1}}(1-z)^{\left(1+\zeta_{pnk}^{1}\right)/2}P_{n}^{\left(2\nu_{pnk}^{1},\zeta_{pnk}^{1}\right)}(1-2z), \quad (18)$$

here $z=exp(-2\delta r)$, ν_{nk}^{1}, ζ_{nk}^{1}, ν_{pnk}^{1} and ζ_{pnk}^{1} are given by:

$$\begin{cases}\nu_{nk}^{1}=\frac{1}{2\delta}\sqrt{M^{2}-E_{nk}^{s2}-C_{ES}(M-E_{nk}^{s})},\\ \zeta_{nk}^{1}=\sqrt{1+4k(k+1)+\Lambda_{nk}^{s}/\delta^{2}},\\ \nu_{pnk}^{1}=\frac{1}{2\delta}\sqrt{M^{2}-E_{nk}^{p2}-C_{PS}\left(M+E_{nk}^{ps}\right)},\\ \zeta_{pnk}^{1}=\sqrt{1+4k(k-1)+\Lambda_{nk}^{p}/\delta^{2}},\end{cases} \quad (19)$$

with $\Lambda_{nk}^{s}=M+E_{nk}^{s}-C_{SE}$, $\Lambda_{nk}^{p}=M-E_{nk}^{p}-C_{PS}$ while C_{nk}^{s} and C_{nk}^{p} are the normalization constants. For the spin symmetry and the p-spin symmetry, the equations of energy are given by [13,17]:

$$M^{2}-E_{nk}^{s2}-C_{ES}(M-E_{nk}^{s})=4\delta^{2}\left[\frac{\frac{A\Lambda_{nk}^{s}}{2M}-k(k+1)-n(n+1)\sqrt{\frac{1}{4}+k(k+1)+\frac{\beta(\beta-1)\Lambda_{nk}^{s}}{2M}}}{2n+1+2\sqrt{\frac{1}{4}+k(k+1)+\frac{A\Lambda_{nk}^{s}}{2M}}}\right]^{2}, \quad (20)$$

and

$$M^{2}-E_{nk}^{p2}+C_{PS}(M+E_{nk}^{p})=4\delta^{2}\left[\frac{\frac{A\Lambda_{nk}^{p}}{2M}-k(k-1)-n(n+1)\sqrt{\frac{1}{4}+k(k-1)+\frac{\beta(\beta-1)\Lambda_{nk}^{p}}{2M}}}{2n+1+2\sqrt{\frac{1}{4}+k(k-1)+\frac{\beta(\beta-1)\Lambda_{nk}^{p}}{2M}}}\right]^{2}. \quad (21)$$

Later, we will need another formula for each of the upper and lower components $F_{nk}^{s}(z)$ and $G_{nk}^{p}(z)$. We will use the transform expression of $P_{n}^{(a_{n},b_{n})}(1-2z)$ in the following form:

$$P_{n}^{(a_{n},b_{n})}(1-2z)=\frac{\Gamma(n+a_{n}+1)}{n!\Gamma(a_{n}+1)}\,{}_{2}F_{1}(-n,n+a_{n}+b_{n}+1;1+a_{n},z). \quad (22)$$

This allows us to reformulate them in terms of the generalized hypergeometric function ${}_{2}F_{1}(-n,n+2\nu_{nk}^{1}+\zeta_{nk}^{1}+1;1+2\nu_{nk}^{1},z)$ and ${}_{2}F_{1}\left(-n,n+2\nu_{pnk}^{1}+\zeta_{pnk}^{1}+1;1+2\nu_{pnk}^{1},z\right)$ as follows:

$$F_{nk}^{s}(s)=C_{nk}^{ns}s^{\nu_{nk}^{1}}(1-z)^{(1+\zeta_{nk}^{1})/2}\,{}_{2}F_{1}(-n,n+2\nu_{nk}^{1}+\zeta_{nk}^{1}+1;1+2\nu_{nk}^{1},z), \quad (23)$$

and

$$G_{nk}^{p}(s)=C_{nk}^{np}z^{\nu_{pnk}^{1}}(1-z)^{\left(1+\zeta_{pnk}^{1}\right)/2}\,{}_{2}F_{1}\left(-n,n+2\nu_{pnk}^{1}+\zeta_{pnk}^{1}+1;1+2\nu_{pnk}^{1},z\right), \quad (24)$$

here $C_{nk}^{ns}=C_{nk}^{s}\frac{\Gamma(n+2\nu_{nk}^{1}+1)}{n!\Gamma(2\nu_{nk}^{1}+1)}$, $C_{nk}^{np}=C_{nk}^{p}\frac{\Gamma\left(n+2\nu_{pnk}^{1}+1\right)}{n!\Gamma\left(2\nu_{pnk}^{1}+1\right)}$. The lower component $G_{nk}^{s}(s)$ of spin symmetry and the upper component $F_{nk}^{p}(s)$ of p-spin symmetry are obtained as:

$$\begin{cases}G_{nk}^{s}(z)=\frac{\left(\frac{d}{dr}+\frac{k}{r}-U(r)\right)F_{nk}^{s}(z)}{M+E_{nk}^{s}-C_{s}},\\ F_{nk}^{p}(z)=\frac{\left(\frac{d}{dr}-\frac{k}{r}-U(r)\right)G_{nk}^{p}(z)}{M-E_{nk}^{p}+C_{p}}.\end{cases} \quad (25)$$

III. THE NEW SOLUTIONS OF DDE UNDER NCMRYPs IN 3D-RNCQS SYMMETRIES

A. Review of Bopp's shift method

Let us begin in this subsection by finding the DDE in the symmetries of deformation Dirac theory under the NCMRYPs. Our objective is achieved by applying the new principles that we have seen in the introduction (Eqs. (4) and (7)), summarized in the new relationships between MASCCCRs and the notion of the Weyl-Moyal star product. Thus, these data allow us to rewrite the usual Dirac equation in Eq. (8) in the 3D-RNCQS symmetries as follows:

$$\begin{pmatrix}\alpha p+\beta\left(M+S_{mr}(r)\right)-i\alpha dU(r)\\ -\left(E_{nk}-V_{mr}(r)\right)\end{pmatrix}*\Psi_{nk}(r,\theta,\phi)=0. \quad (26)$$

In 3D-RNCQS symmetries, the upper and lower components $F_{nk}^{s}(r)$ and $G_{nk}^{p}(r)$ satisfying the following second-order differential equations:

$$\begin{bmatrix}\frac{d^{2}}{dr^{2}}-k(k+1)r^{-2}+U_{eff}^{yt-s}(r)\\ -\Lambda_{s}\left(M-E_{nk}^{s}+\Sigma_{mr}^{s}(r)\right)\end{bmatrix}*F_{nk}^{s}(r)=0, \quad (27)$$

and

$$\begin{bmatrix}\frac{d^{2}}{dr^{2}}-k(k-1)r^{-2}+U_{eff}^{yt-p}(r)\\ -\left(M+E_{nk}^{p}-\Delta_{mr}^{p}(r)\right)\Lambda_{p}\end{bmatrix}*G_{nk}^{p}(r)=0. \quad (28)$$

There are two approaches to including non-commutativity in quantum field theory: The first method is represented by rewriting the various NC physical fields such as the spinor $\boldsymbol{\Psi}_{nl}$, KG operator $\boldsymbol{\Phi}_{nl}$, antisymmetric bosonic tensor$\boldsymbol{F}_{\alpha\beta}$and tedrad fields $\boldsymbol{e}_{\mu}^{a}$ in terms of their corresponding fields $\left(\Psi_{nl},\Phi_{nl},e_{\mu}^{a},F_{\alpha\beta},\dots\right)$ in the known quantum space in the literature, in proportion to the non-commutative parameters $\boldsymbol{\Theta}(\theta_{12},\theta_{23},\theta_{13})/2$, which is similar to the Taylor development [24,59,60,61,62,63,64] while the second method depends on reformulating the non-commutative operator (q,π) with its view of the quantum operators (q,π) known in the literature and the properties of space associated with the non-commutative parameters $\boldsymbol{\Theta}(\theta_{12},\theta_{23},\theta_{13})/2$. It is normal for the physical results to be identical when using either of them. It is known to specialized researchers that Bopp had proposed a new quantization rule $(x,p) \to (q = x - \frac{i}{2}\partial_p, \pi = p + \frac{i}{2}\partial_x)$ instead of the usual correspondence $(x,p) \to (q = x, q = p + \frac{i}{2}\partial_x)$ which is called Bopp's shifts method (BSM) [64,65,66,67]. This quantization procedure is called Bopp quantization [68]. The Weyl-Moyal star product $f(x,p) * g(x,p)$ induces BSM in the respect that it is replaced by $f(x - \frac{i}{2}\partial_p, p + \frac{i}{2}\partial_x) * g(x,p)$ [69]. This, allows us to obtain

$$\begin{cases} \left[k(k+1)r^{-2} + U_{eff}^{yt-s}(r)\right] * F_{nk}^{s}(r) = \\ \left[k(k+1)d^{-2} + U_{eff}^{yt-s}(d)\right] F_{nk}^{s}(r), \\ \left(M - E_{nk}^{s} + \Sigma_{mr}^{s}(r)\right) * F_{nk}^{s}(r) = \\ \left(M - E_{nk}^{s} + \Sigma_{mr}^{s}(d)\right) F_{nk}^{s}(r), \\ \left[k(k-1)r^{-2} + U_{eff}^{yt-p}(r)\right] * G_{nk}^{p}(r) = \\ \left[k(k-1)d^{-2} + U_{eff}^{yt-p}(d)\right] G_{nk}^{p}(r), \\ \left(M + E_{nk}^{p} - \Delta_{mr}^{p}(r)\right) * G_{nk}^{p}(r) = \\ \left(M + E_{nk}^{p} - \Delta_{mr}^{p}(d)\right) * G_{nk}^{p}(r). \end{cases}$$

The BSM has achieved great success when applied by specialized researchers to the four basic equations that correspond to the relativistic Schrödinger equation (see, e.g.;[70,71,72]) and the other three relativistic equations represented by the Klein-Gordon equation (see, e.g.;[73,74,75,76,77,78,79,80]), Dirac equation (see, e.g.;[81,82,83,84,85,86,87,88,89,90]) and the Duffin-Kemmer-Petiau equation (see, e.g.;[90,91,92]). In addition to some recent related research (see, e.g.;[93,94,95, 96, 97,98]).

It is worth motioning that the BSM permutes us to reduce Eqs. (27) and (28) to the simplest form:

$$\begin{bmatrix} \frac{d^2}{dr^2} - k(k+1)d^{-2} + U_{eff}^{yt-s}(d) \\ -\Lambda_{nk}^{s}\left(M - E_{nk}^{s} + \Sigma_{mr}^{s}(d)\right) \end{bmatrix} F_{nk}^{s}(r) = 0, \quad (29)$$

and

$$\begin{bmatrix} \frac{d^2}{dr^2} - k(k-1)d^{-2} + U_{eff}^{yt-p}(d) \\ -\left(M + E_{nk}^{p} - \Delta_{mr}^{p}(d)\right)\Lambda_{nk}^{p} \end{bmatrix} G_{nk}^{p}(r) = 0. \quad (30)$$

The modified algebraic structure of noncommutative covariant canonical commutation relations with the notion of the Weyl-Moyal star product in Eqs. (4) becomes new METNCCCRs with ordinary known products in literature as follows (see, e.g.; [64,65,66,67]):

$$\begin{cases} \left[q_{\mu}^{(s,h,i)}, \pi_{\nu}^{(s,h,i)}\right] = i\hbar_{eff}\delta_{\mu\nu}, \\ \left[q_{\mu}^{(s,h,i)}, q_{\nu}^{(s,h,i)}\right] = i\eta_{\mu\nu}. \end{cases} \quad (31)$$

In the symmetries of 3D-RNCQS, the generalized positions and momentum coordinates $q_{\mu}^{(s,h,i)}$ and $\pi_{\mu}^{(s,h,i)}$ are defined as:

$$\begin{cases} q_{\mu}^{(s,h,i)} = x_{\mu}^{(s,h,i)} - \sum_{\nu=1}^{3} \frac{i\eta_{\mu\nu}}{2} p_{\nu}^{(s,h,i)}, \\ \pi_{\mu}^{(s,h,i)} = p_{\mu}^{(s,h,i)}. \end{cases} \quad (32)$$

This allows us to find the operator d^2 equal to [81, 82, 83, 84, 85, 86, 87, 88, 89, 90]:

$$d^2 = r^2 - \begin{cases} L\Theta \text{ for spin symmetry} \\ L_p\Theta \text{ for p-spin symmetry} \end{cases} + O(\Theta^2), \quad (33)$$

while the new operators $\Sigma_{mr}^{s}(d)$, $V_{mr}^{p}(d)$, $U_{eff}^{yt-s/p}(d)$, $k(k+1)d^{-2}$ and $k(k-1)d^{-2}$ in the 3D-RNCQS symmetries, are expressed as:

$$\begin{cases} \Sigma_{mr}^{s}(d) = \Sigma_{mr}^{s}(r) - \frac{\partial \Sigma_{mr}^{s}(r)}{\partial r}\frac{\mathbf{L\Theta}}{2r} + O(\Theta^2), \\ \Delta_{mr}^{p}(d) = \Delta_{mr}^{p}(r) - \frac{\partial \Delta_{mr}^{p}(r)}{\partial r}\frac{\mathbf{L_p\Theta}}{2r} + O(\Theta^2), \\ U_{eff}^{yt-s} = U_{eff}^{yt-s}(r) - \frac{\partial U_{eff}^{yt-s}(r)}{\partial r}\frac{\mathbf{L\Theta}}{2r} + O(\Theta^2), \\ U_{eff}^{yt-p}(d) = U_{eff}^{yt-p}(r) - \frac{\partial U_{eff}^{yt-p}(r)}{\partial r}\frac{\mathbf{L_p\Theta}}{2r} + O(\Theta^2), \\ k(k+1)d^{-2} = k(k+1)r^{-2} + k(k+1)r^{-4}\mathbf{L\Theta} + O(\Theta^2), \\ k(k-1)d^{-2} = k(k-1)r^{-2} + k(k-1)r^{-4}\mathbf{L_p\Theta} + O(\Theta^2). \end{cases} \quad (34)$$

Substituting Eqs. (34) into Eqs. (29) and (30), we find the following two like Shrodinger equations:

$$\left[\frac{d^2}{dr^2} - \frac{k(k+1)}{r^2} + U_{eff}^{yt-s}(r) - \Lambda_s\left(M - E_{nk}^{s} + \Sigma_{mr}^{s}(r)\right) - \Sigma_{mr}^{pert}(r)\right] F_{nk}^{s}(r) = 0, \quad (35)$$

and

$$\left[\frac{d^2}{dr^2} - \frac{k(k-1)}{r^2} + U_{eff}^{yt-p}(r) - \left(M + E_{nk}^{p} - \Delta_{mr}^{p}(r)\right)\Lambda_p - \Delta_{mr}^{pert}(r)\right] G_{nk}^{p}(r) = 0, \quad (36)$$

with

$$\Sigma_{mr}^{pert}(r) = \left(-\frac{\partial U_{eff}^{yt-s}(r)}{\partial r}\frac{1}{2r} + \frac{k(k+1)}{r^4} - \frac{\partial \Sigma_{mr}^{s}(r)(r)}{\partial r}\frac{\Lambda_{nk}^{s}}{2r}\right)\mathbf{L\Theta}, \quad (37)$$

and

$$\Delta_{mr}^{pert}(r) = \left(-\frac{\partial U_{eff}^{yt-p}(r)}{\partial r}\frac{1}{2r} + \frac{k(k-1)}{r^4} - \frac{\partial \Delta_{mr}^{p}(r)}{\partial r}\frac{\Lambda_{nk}^{p}}{2r}\right)\mathbf{L_p\Theta}. \quad (38)$$

By comparing (Eqs. (9) and (10)) and (Eqs. (35) and (36)), we observe two additive potentials $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$. Moreover, these terms are proportional to the infinitesimal noncommutativity parameter Θ. From a physical point of view, this means that these two spontaneously generated terms $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ as a result, the topological properties of the deformation space-space can be considered very small compared to the fundamental terms $\Sigma_{mr}^{s}(r)$ and $\Delta_{mr}^{p}(r)$, respectively. A direct calculation gives $\frac{\partial \Sigma_{mr}^{s}(r)}{\partial r}$ and $\frac{\partial U_{eff}^{yt-s/p}(r)}{\partial r}$ as follows:

$$\frac{\partial \Sigma_{mr}^{s}(r)}{\partial r} = -\frac{2\delta\beta(\beta-1)}{Mb^2}\frac{exp(-4\delta r)}{(1-exp(-2\delta r))^2} + \frac{2\beta\delta(\beta-1)}{2Mb^2}\frac{exp(-6\delta r)}{(1-exp(-2\delta r))^3} + \frac{\delta A}{Mb^2}\frac{exp(-2\delta r)}{1-exp(-2\delta r)} + \frac{\delta A}{Mb^2}\frac{exp(-4\delta r)}{(1-exp(-2\delta r))^2}, \quad (39)$$

and

$$\frac{\partial U_{eff}^{yt-\frac{s}{p}}}{\partial r} = -\delta\left(B^{\mp} \mp V_0\right)\frac{exp(-\delta r)}{r^2} - 2B^{\mp}\frac{exp(-\delta r)}{r^3} \mp (-\delta)\delta V_0\frac{exp(-\delta r)}{r} + 2\delta V_0^2\frac{exp(-2\delta r)}{r^2} + 2V_0^2\frac{exp(-2\delta r)}{r^3}. \quad (40)$$

Substituting Eqs. (39) and (40) into Eqs. (37) and (38), we obtain spontaneously generated terms $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ as follows:

$$\Sigma_{mr}^{pert}(r) = (\delta(B^- - V_0)\frac{exp(-\delta r)}{2r^3} + B^-\frac{exp(-\delta r)}{r^4} - \delta^2 V_0\frac{exp(-\delta r)}{2r^2} - \delta V_0^2\frac{exp(-2\delta r)}{r^3} - V_0^2\frac{exp(-2\delta r)}{r^4} + \frac{\delta\beta(\beta-1)\Lambda_{nk}^s}{Mb^2}\frac{exp(-4\delta r)}{r(1-exp(-2\delta r))^2} - \frac{\beta\delta(\beta-1)\Lambda_{nk}^s}{2Mb^2}\frac{exp(-6\delta r)}{r(1-exp(-2\delta r))^3} - \frac{\delta A\Lambda_{nk}^s}{2Mb^2}\frac{exp(-2\delta r)}{r(1-exp(-2\delta r))} - \frac{\delta A\Lambda_{nk}^s}{2Mb^2}\frac{exp(-4\delta r)}{r(1-exp(-2\delta r))^2} + \frac{k(k+1)}{r^4})\mathbf{L\Theta} + O(\Theta^2), \quad (41)$$

and

$$\Delta_{mr}^{pert}(r) = (\delta(B^+ + V_0)\frac{exp(-\delta r)}{2r^3} + +B^+\frac{exp(-\delta r)}{r^4} + +\delta^2 V_0\frac{exp(-\delta r)}{2r^2} - \delta V_0^2\frac{exp(-2\delta r)}{r^3} - V_0^2\frac{exp(-2\delta r)}{r^4} + \frac{\delta\beta(\beta-1)\Lambda_{nk}^p}{Mb^2}\frac{exp(-4\delta r)}{r(1-exp(-2\delta r))^2} - \frac{\delta\beta(\beta-1)\Lambda_{nk}^p}{2Mb^2}\frac{exp(-6\delta r)}{r(1-exp(-2\delta r))^3} - \frac{\delta A\Lambda_{nk}^p}{2Mb^2}\frac{exp(-2\delta r)}{r(1-exp(-2\delta r))} - \frac{\delta A\Lambda_{nk}^p}{2Mb^2}\frac{exp(-4\delta r)}{r(1-exp(-2\delta r))^2} + \frac{k(k-1)}{r^4})\mathbf{L_p\Theta} + O(\Theta^2). \quad (42)$$

For spin symmetry, we first consider Eq. (35), which contains the new combined Manning-Rosen and Yukawa tensor potentials in the deformation of Dirac theory symmetries. It can be solved exactly only for $k = 0$ and $k = -1$ in the absence of tensor interactions $V_0 = 0$, since the two centrifugal terms (proportional to $k(k+1)r^{-2}$ and $k(k+1)r^{-4}$) vanish. In the case of arbitrary k, an appropriate approximation needs to be employed on the centrifugal terms. We apply the following new approximation which was applied by Greene and Aldrich [99]:

$$\frac{1}{r^2} \approx \frac{4\delta^2 e^{-2\delta r}}{\left(1-e^{-2\delta r}\right)^2} = \frac{4\delta^2 z}{(1-z)^2} \Leftrightarrow \frac{1}{r} \approx \frac{2\delta e^{-\delta r}}{1-e^{-2\delta r}} = \frac{2\delta z^{\frac{1}{2}}}{1-z}. \quad (43)$$

For p-spin symmetry, we now consider Eq. (36) and will follow similar steps with the spin symmetry case in the deformation of Dirac theory symmetries. Same as before, Eq. (31) cannot be solved exactly for $k = 0$ and $k = 1$ without tensor interaction, since the two centrifugal terms (proportional to $k(k-1)r^{-2}$ and $k(k-1)r^{-4}$). Applying the approximations Eq. (43) to the centrifugal terms of Eqs. (41) and (42), the general form of the additive potentials $\Sigma_{mr}^{pert}(z)$ and $\Delta_{mr}^{pert}(z)$ will be as follows:

$$\Sigma_{mr}^{pert}(z) = (\chi_{nk}^{1s}\frac{z^2}{(1-z)^3} + \chi_{nk}^{2s}\frac{z^{5/2}}{(1-z)^4} + \chi_{nk}^{3s}\frac{z^{3/2}}{(1-z)^2} + \chi_{nk}^{4s}\frac{z^{5/2}}{(1-z)^3} + \chi_{nk}^{5s}\frac{z^3}{(1-z)^4} + \chi_{nk}^{6s}\frac{z^{7/2}}{(1-z)^4} + \chi_{nk}^{7s}\frac{z^2}{(1-z)^4})\mathbf{L\Theta} + O(\Theta^2), \quad (44)$$

and

$$\Delta_{mr}^{pert}(z) = (\chi_{nk}^{1p}\frac{z^2}{(1-z)^3} + \chi_{nk}^{2p}\frac{z^{5/2}}{(1-z)^4} + \chi_{nk}^{3p}\frac{z^{3/2}}{(1-z)^2} + \chi_{nk}^{4p}\frac{z^{5/2}}{(1-z)^3} + \chi_{nk}^{5p}\frac{z^3}{(1-z)^4} + \chi_{nk}^{6p}\frac{z^{7/2}}{(1-z)^4} + \chi_{nk}^{7p}\frac{z^2}{(1-z)^4})\mathbf{L_p\Theta} + O(\Theta^2), \quad (45)$$

with

$$\begin{cases} \chi_{nk}^{1s} = 4\delta^4(B^- - V_0), \chi_{nk}^{1p} = 4\delta^4(B^+ + V_0) \\ \chi_{nk}^{2s} = 16\delta^4 B^-, \beta_{nk}^{2p} = 16\delta^4 B^+, \\ \chi_{nk}^{3s} = -\left(2\delta^4 V_0 + \frac{\delta^2 A\Lambda_{nk}^s}{Mb^2}\right), \chi_{nk}^{3p} = 2\delta^4 V_0 - \frac{\delta^2 A\Lambda_{nk}^p}{Mb^2}, \\ \chi_{nk}^{4s} = \frac{2\delta^2\beta(\beta-1)\Lambda_{nk}^s}{Mb^2} - 8\delta^4 V_0^2 - \frac{\delta^2 A\Lambda_{nk}^s}{Mb^2}, \\ \chi_{nk}^{4p} = \frac{2\delta^2\beta(\beta-1)\Lambda_{nk}^p}{Mb^2} - 8\delta^4 V_0^2 - \frac{\delta^2 A\Lambda_{nk}^p}{Mb^2}, \\ \chi_{nk}^{5s} = \chi_{nk}^{5p} = -16\delta^4 V_0^2, \\ \chi\beta_{nk}^{6s} = -\frac{2\beta\delta^2(\beta-1)\Lambda_{nk}^s}{2Mb^2}, \chi_{nk}^{6p} = -\frac{2\beta\delta^2(\beta-1)\Lambda_{nk}^p}{2Mb^2}, \\ \chi_{nk}^{7s} = 16\delta^4 k(k+1), \chi_{nk}^{7p} = 16\delta^4 k(k-1). \end{cases} \quad (46)$$

Furthermore, using the unit step function (also known as the Heaviside step function $\theta(x)$ or simply the theta function) to rewrite the global induced two potentials $\Sigma_{t\_mr}^{pert}(r)$ and $\Delta_{t-mr}^{pert}(r)$ for spin and pseudospin symmetries corresponding to upper and lower components ($F_{nk}^s(s)$ and $G_{nk}^s(s)$) and ($F_{nk}^p(s)$ and $G_{nk}^p(s)$), respectively as:

$$\Sigma_{t\_mr}^{pert}(r) = \Sigma_{mr}^{pert}(r)\theta(|E_{nc}^{mr-s}|) - \Sigma_{mr}^{pert}(r)\theta(-|E_{nc}^{mr-s}|) = \begin{cases} \Sigma_{mr}^{pert}(r) \text{ for Uc of spin symmetry,} \\ -\Sigma_{mr}^{pert}(r) \text{ for Lc of spin symmetry,} \end{cases} \quad (47)$$

and

$$\Delta_{t\_mr}^{pert}(r) = \Delta_{ts}^{pert}(r)\theta\left(\left|E_{nc}^{mr-p}\right|\right) - \Delta_{mr}^{pert}(r)\theta\left(-\left|E_{nc}^{mr-p}\right|\right) = \begin{cases} \Delta_{mr}^{pert}(r) \text{ for Uc of p-pin symmetry ,} \\ -\Delta_{mr}^{pert}(r) \text{ for Lc of p-spin symmetry.} \end{cases} \quad (48)$$

Where the step function $\theta(x)$ is given by:

$$\theta(x) = \begin{cases} 1 \text{ for } x \geq 0, \\ 0 \text{ otherwise} \end{cases}. \quad (49)$$

Notably, the results yielded by the Greene and Aldrich approximation, for small values $\delta r << 1$, are in good agreement with those obtained using other methods. We have replaced the terms $k(k+1)r^{-4}$ and $k(k-1)r^{-4}$ with the approximation in Eq. (37). The combined Manning-Rosen and Yukawa tensor potentials are extended by including new additive potentials $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ expressed to the radial terms $\frac{z^2}{(1-z)^3}$, $\frac{z^{5/2}}{(1-z)^4}$, $\frac{z^{3/2}}{(1-z)^2}$, $\frac{z^{5/2}}{(1-z)^3}$, $\frac{z^3}{(1-z)^4}$, $\frac{z^{7/2}}{(1-z)^4}$ and $\frac{z^2}{(1-z)^4}$ to become the newly combined Manning-Rosen and Yukawa tensor potentials in 3D-RNCQS symmetries. The generated new two effective potentials $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ are also proportional to the infinitesimal vector $\mathbf{\Theta}$. This allows us to consider the new additive parts of the effective potential $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ as perturbation potentials compared with the main potentials $\Sigma_{mr}^s(r)$ and $\Delta_{mr}^p(r)$ which are also known with the parent potential operator in the symmetries of 3D-RNCQS, that is, the two inequalities $\Sigma_{mr}^{pert}(r) << \Sigma_{mr}^s(r)$ and $\Delta_{mr}^{pert}(r) << \Delta_{mr}^p(r)$ have become achieved. That is all physical justifications for applying the time-independent perturbation theory become satisfied to calculate the expectation values of previous radial terms. This allows us to give a complete prescription for determining the energy level of the generalized $\left(n,l,s,l_p,s_p,m,m_p\right)^{th}$ excited states.

B. The expectation values under the *NCMRYPs* in the 3D-RNCQS for spin symmetry

In this subsection, we want to apply the perturbative theory, in the case of deformation Dirac theory symmetries, we find the expectation values $M^{s-mr}_{1(nlms)}\equiv\left\langle\frac{z^2}{(1-z)^3}\right\rangle^{s-mr}_{(nlms)}$, $M^{s-mr}_{2(nlms)}\equiv\left\langle\frac{z^{5/2}}{(1-z)^4}\right\rangle^{s-mr}_{(nlms)}$, $M^{s-mr}_{3(nlms)}\equiv\left\langle\frac{z^{3/2}}{(1-z)^2}\right\rangle^{s-mr}_{(nlms)}$, $M^{s-mr}_{4(nlms)}\equiv\left\langle\frac{z^{5/2}}{(1-z)^3}\right\rangle^{s-mr}_{(nlms)}$, $M^{s-mr}_{5(nlms)}\equiv\left\langle\frac{z^3}{(1-z)^4}\right\rangle^{s-mr}_{(nlms)}$, $M^{s-mr}_{6(nlms)}\equiv\left\langle\frac{z^{7/2}}{(1-z)^4}\right\rangle^{s-mr}_{(nlms)}$ and $M^{s-mr}_{7(nlms)}\equiv\left\langle\frac{z^2}{(1-z)^4}\right\rangle^{s-mr}_{(nlms)}$ for the spin symmetry taking into account the unperturbed upper component $F^s_{nk}(r)$ which we have seen previously in Eq. (23). Thus after straightforward calculations, we obtain the following results:

$$M^{s-mr}_{1(nlms)}=C^{ns2}_{nk}\int_0^{+\infty}z^{2\nu^1_{nk}+2}(1-z)^{\zeta^1_{nk}-2}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dr, \quad (50.1)$$

$$M^{s-mr}_{2(nlms)}=C^{ns2}_{nk}\int_0^{+\infty}z^{2\nu^1_{nk}+5/2}(1-z)^{\zeta^1_{nk}-3}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dr, \quad (50.2)$$

$$M^{s-mr}_{3(nlms)}=C^{ns2}_{nk}\int_0^{+\infty}z^{2\nu^1_{nk}+3/2}(1-z)^{\zeta^1_{nk}-1}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dr, \quad (50.3)$$

$$M^{s-mr}_{4(nlms)}=C^{ns2}_{nk}\int_0^{+\infty}z^{2\nu^1_{nk}+5/2}(1-z)^{\zeta^1_{nk}-2}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dr, \quad (50.4)$$

$$M^{s-mr}_{5(nlms)}=C^{ns2}_{nk}\int_0^{+\infty}z^{2\nu^1_{nk}+3}(1-z)^{\zeta^1_{nk}-3}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dr, \quad (50.5)$$

$$M^{s-mr}_{6(nlms)}=C^{ns2}_{nk}\int_0^{+\infty}z^{2\nu^1_{nk}+7/2}(1-z)^{\zeta^1_{nk}-3}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dr, \quad (50.6)$$

and

$$M^{s-mr}_{7(nlms)}=C^{ns2}_{nk}\int_0^{+\infty}z^{2\nu^1_{nk}+2}(1-z)^{\zeta^1_{nk}-3}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dr. \quad (50.7)$$

We have used useful abbreviations $\langle R\rangle^{s-mr}_{(nlms)}$ instead to average values $\langle n,l,m|R|n,l,m\rangle$ to avoid the extra burden of writing equations. Furthermore, we have applied the property of the spherical harmonics, which has the form

$$\int Y^m_l(\theta,\phi)Y^{m\prime}_{l\prime}(\theta,\phi)\sin(\theta)\,d\theta d\phi=\delta_{ll\prime}\delta_{mm\prime}.$$

Introducing the change of variable $z=exp(-2\delta r)$. This maps the region $0\le r<\infty$ to $0\le s\le 1$ and allows us to obtain $r=-1/2\frac{dz}{\delta z}$, and transform Eqs. $(50,i=\overline{1,7})$ into the following form:

$$M^{s-mr}_{1(nlms)}=\frac{C^{ns2}_{nk}}{2\delta}\int_0^{+1}z^{2\nu^1_{nk}+2-1}(1-z)^{\zeta^1_{nk}-2}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dz, \quad (51.1)$$

$$M^{s-mr}_{2(nlms)}=\frac{C^{ns2}_{nk}}{2\delta}\int_0^{+1}z^{2\nu^1_{nk}+5/2-1}(1-z)^{\zeta^1_{nk}-3}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dz, \quad (51.2)$$

$$M^{s-mr}_{3(nlms)}=\frac{C^{ns2}_{nk}}{2\delta}\int_0^{+1}z^{2\nu^1_{nk}+3/2-1}(1-z)^{\zeta^1_{nk}-1}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dz, \quad (51.3)$$

$$M^{s-mr}_{4(nlms)}=\frac{C^{ns2}_{nk}}{2\delta}\int_0^{+1}z^{2\nu^1_{nk}+5/2-1}(1-z)^{\zeta^1_{nk}-2}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dz, \quad (51.4)$$

$$M^{s-mr}_{5(nlms)}=\frac{C^{ns2}_{nk}}{2\delta}\int_0^{+1}z^{2\nu^1_{nk}+3-1}(1-z)^{\zeta^1_{nk}-3}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dz, \quad (51.5)$$

$$M^{s-mr}_{6(nlms)}=\frac{C^{ns2}_{nk}}{2\delta}\int_0^{+1}z^{2\nu^1_{nk}+7/2-1}(1-z)^{\zeta^1_{nk}-3}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dz, \quad (51.6)$$

and

$$M^{s-mr}_{7(nlms)}=\frac{C^{ns2}_{nk}}{2\delta}\int_0^{+1}z^{2\nu^1_{nk}+2-1}(1-z)^{\zeta^1_{nk}-3}\left[2F_1(-n,n+2\nu^1_{nk}+\zeta^1_{nk}+1;1+2\nu^1_{nk},z)\right]^2dz\,. \quad (51.7)$$

We can evaluate the above integrals either in a recurrence way through the physical values of the principal quantum number $(n=0,1,\ldots)$ and then generalize the result to the general $\left(n,l,s,l_p,s_p,m,m_p\right)^{th}$ excited state or we use the method proposed by Dong *et al.* [100] and applied by Zhang [101], to obtain the general excited state directly. We calculate the integrals in Eqs. $(43,i=\overline{1,7})$ with the help of the special integral formula:

$$\int_0^{+1}z^{\alpha-1}(1-z)^{\beta}\left[{}_2F_1(-n,n+\beta+\alpha-2;2\alpha+1,z),\right]^2dz$$
$$=n\,!\,\Gamma(\alpha+1))\Gamma(\beta+1)\sum_{q=0}^{n}\frac{(-1)^q(n+\alpha+\beta)_q}{(q+\alpha)\Gamma(n-q)\,!q\,!\Gamma(q+\alpha+\beta+1)}$$
$${}_3F_2(-n,q+\alpha,n+\alpha+\beta;\alpha+1,q+\alpha+\beta;1), \quad (52)$$

here $3F_2(c_1,c_2,\zeta;c_3,\tau+\xi;1)$ equal to $\sum_{n=0}^{n}\frac{(c_1)_n(c_2)_n(\xi)_n}{(c_3)_n\,n\,!(\tau+\xi)}$, the symbol $(n+\alpha+\beta)_q$ denotes the rising factorial or Pochhammer symbol $\frac{\Gamma(n+\alpha+\beta+q)}{\Gamma(n+\alpha+\beta)}$ while $\Gamma(\xi)$ denoting the usual Gamma function. By identifying Eqs.$(51,i=\overline{1,7})$ with the integrals in Eqs. (52), we obtain the following results:

$$M^{s-mr}_{1(nlms)}=$$

$$\sum_{q=0}^{n}\frac{\beta_{nk}^{1}(-1)^q(n+K_{nk})_q}{(q+\nu_{nk}^1+2)(n-q)!\,q!\,\Gamma(q+K_{nk}+1)}$$
$$3F_2(-n,q+2\nu_{nk}^1+2,n+K_{nk},2\nu_{nk}^1+3;q+K_{nk}+1;1), \quad (53.1)$$

$$M_{2(nlms)}^{s-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{2}(-1)^q(n+K_{nk}-1/2)_q}{(q+2\nu_{nk}^1+5/2)(n-q)!\,q!\,\Gamma(q+K_{nk}+1/2)}$$
$${}_3F_2(-n,q+2\nu_{nk}^1+5/2,n+K_{nk}-1/2,2\nu_{nk}^1+7/2;q+K_{nk}+1/2;1)\,, \quad (53.2)$$

$$M_{3(nlms)}^{s-mr}=\sum_{q=0}^{n}\frac{\beta_{nl}^{3}(-1)^q(n+K_{nk}+1/2)_q}{(q+2\nu_{nk}^1+3/2)(n-q)!\,q!\,\Gamma(q+K_{nk}+3/2)}$$
$${}_3F_2(-n,q+2\nu_{nk}^1+3/2,n+K_{nk}+1/2,2\nu_{nk}^1+5/2;q+K_{nk}+3/2;1)\,, \quad (53.3)$$

$$M_{4(nlms)}^{s-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{4}(-1)^q(n+K_{nk}+1/2)_q}{(q+2\nu_{nk}^1+5/2)(n-q)!\,q!\,\Gamma(q+K_{nk}+3/2)}$$
$${}_3F_2(-n,q+2\nu_{nk}^1+5/2,n+K_{nk}+1/2,2\nu_{nk}^1+7/2;q+K_{nk}+3/2;1)\,, \quad (53.4)$$

$$M_{5(nlms)}^{s-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{5}(-1)^q(n+K_{nk})_q}{(q+2\nu_{nk}^1+3)(n-q)!\,q!\,\Gamma(q+K_{nk}+1)}$$
$${}_3F_2(-n,q+2\nu_{nk}^1+3,n+K_{nk},2\nu_{nk}^1+4;q+K_{nk}+1;1), \quad (53.5)$$

$$M_{6(nlms)}^{s-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{6}(-1)^q\left(n+K_{nk}+\frac{1}{2}\right)_q}{\left(q+2\nu_{nk}^1+\frac{7}{2}\right)(n-q)!\,q!\,\Gamma\left(q+K_{nk}+\frac{1}{2}\right)}$$
$${}_3F_2\left(-n,q+2\nu_{nk}^1+\frac{7}{2},n+K_{nk}+\frac{1}{2,2\nu_{nk}^1}+\frac{9}{2};q+K_{nk}+\frac{1}{2};1\right), \quad (53.6)$$

and

$$M_{7(nlms)}^{s-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{7}(-1)^q(n+K_{nk}-1)_q}{(q+2\nu_{nk}^1+2)(n-q)!\,q!\,\Gamma(q+K_{nk})}$$
$${}_3F_2(-n,q+2\nu_{nk}^1+2,n+K_{nk}-1,2\nu_{nk}^1+3;q+K_{nk};1), \quad (53.7)$$

with

$$\begin{cases} K_{nk}=2\nu_{nk}^1+\zeta_{nk}^1,\\ \beta_{nk}^1=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nk}^1+3)\Gamma(\zeta_{nk}^1-1),\\ \beta_{nk}^2=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nk}^1+7/2)\Gamma(\zeta_{nk}^1-2),\\ \beta_{nk}^3=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nk}^1+5/2)\Gamma(\zeta_{nk}^1),\\ \beta_{nk}^4=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nk}^1+7/2)\Gamma(\zeta_{nk}^1-1),\\ \beta_{nk}^5=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nk}^1+4)\Gamma(\zeta_{nk}^1-2),\\ \beta_{nk}^6=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nk}^1+9/2)\Gamma(\zeta_{nk}^1-2),\\ \beta_{nk}^7=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nk}^1+3)\Gamma(\zeta_{nk}^1-2). \end{cases} \quad (54)$$

and

$$\begin{cases} (n+K_{nk})_q=\frac{\Gamma(n+K_{nk}+q)}{\Gamma(n+K_{nk})},\\ (n+K_{nk}\pm 1/2)_q=\frac{\Gamma(n+K_{nk}\pm 1/2+q)}{\Gamma(n+K_{nk}+1/2)},\\ (n+K_{nk}-1)_q=\frac{\Gamma(n+K_{nk}-1+q)}{\Gamma(n+K_{nk}-1)}. \end{cases} \quad (55)$$

C. The expectation values under the *NCMRYPs* in the 3D-RNCQS for p-spin symmetry

In this subsection, we want to apply the perturbative theory, in the case of deformation Dirac theory symmetries, we find the expectation values $M_{1(nl_pm_ps_p)}^{p-mr}\equiv\left\langle\frac{z^2}{(1-z)^3}\right\rangle_{(nl_pm_ps_p)}^{p-mr}$, $M_{2(nl_pm_ps_p)}^{p-mr}\equiv\left\langle\frac{z^{5/2}}{(1-z)^4}\right\rangle_{(nl_pm_ps_p)}^{p-mr}$, $M_{3(nl_pm_ps_p)}^{p-mr}\equiv\left\langle\frac{z^{3/2}}{(1-z)^2}\right\rangle_{(nl_pm_ps_p)}^{p-mr}$, $M_{4(nl_pm_ps_p)}^{p-mr}\equiv\left\langle\frac{z^{5/2}}{(1-z)^3}\right\rangle_{(nl_pm_ps_p)}^{p-mr}$, $M_{5(nl_pm_ps_p)}^{p-mr}\equiv\left\langle\frac{z^3}{(1-z)^4}\right\rangle_{(nl_pm_ps_p)}^{p-mr}$, $M_{6(nl_pm_ps_p)}^{p-mr}\equiv\left\langle\frac{z^{7/2}}{(1-z)^4}\right\rangle_{(nl_pm_ps_p)}^{p-mr}$ and $M_{7(nl_pm_ps_p)}^{p-mr}\equiv\left\langle\frac{z^2}{(1-z)^4}\right\rangle_{(nl_pm_ps_p)}^{p-mr}$ for p-spin symmetry with tensor interaction taking into account the wave function which we have seen previously in Eq. (24). By examining the two expressions of the upper and lower components ($F_{nk}^s(r)$ and $G_{nk}^p(r)$) shown in Eqs. (23) and (24), we note that there is a possibility to move from the unperturbed upper component $F_{nk}^s(r)$ to the other lower component $G_{nk}^p(r)$ by making the following substitutions:

$$C_{nk}^{ns}\Leftrightarrow C_{nk}^{np},\,\nu_{nk}^1\Leftrightarrow{}^{\nu_{pnk}^1}\text{ and }\zeta_{nk}^1\Leftrightarrow\zeta_{pnk}^1\,. \quad (56)$$

This allows us to obtain the expectation values for p-spin symmetry from Eqs. $(45, i=\overline{1,7})$ without re-calculation, as follows:

$$M_{1(nl_pm_ps_p)}^{p-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{p1}(-1)^q\left(n+K_{nk}^p\right)_q}{(q+\nu_{pnk}^1+2)(n-q)!\,q!\,\Gamma\left(q+K_{nk}^p+1\right)}$$
$$3F_2\left(-n,q+2\nu_{pnk}^1+2,n+K_{nk}^p,2\nu_{pnk}^1+3;q+K_{nk}^p+1;1\right), \quad (57.1)$$

$$M_{2(nl_pm_ps_p)}^{p-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{p2}(-1)^q\left(n+K_{nk}^p-1/2\right)_q}{(q+2\nu_{pnk}^1+5/2)(n-q)!\,q!\,\Gamma\left(q+K_{nk}^p+1/2\right)}$$
$${}_3F_2\left(-n,q+2\nu_{pnk}^1+5/2,n+K_{nk}^p-1/2,2\nu_{pnk}^1+7/2;q+K_{nk}^p+1/2;1\right), \quad (57.2)$$

$$M_{3(nl_pm_ps_p)}^{p-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{p3}(-1)^q\left(n+K_{nk}^p+1/2\right)_q}{(q+2\nu_{pnk}^1+3/2)(n-q)!\,q!\,\Gamma\left(q+K_{nk}^p+3/2\right)}$$
$${}_3F_2\left(-n,q+2\nu_{pnk}^1+3/2,n+K_{nk}^p+1/2,2\nu_{pnk}^1+5/2;q+K_{nk}^p+3/2;1\right), \quad (57.3)$$

$$M_{4(nl_pm_ps_p)}^{p-mr}=$$

$$\sum_{q=0}^{n}\frac{\beta_{nk}^{p4}(-1)^q\left(n+K_{nk}^p+1/2\right)_q}{(q+2\nu_{pnk}^1+5/2)(n-q)!\,q!\,\Gamma\left(q+K_{nk}^p+3/2\right)}\,{}_3F_2\left(-n,q+2\nu_{pnk}^1+5/2,n+K_{nk}^p+1/2,2\nu_{pnk}^1+7/2;q+K_{nk}^p+3/2;1\right),\quad(57.4)$$

$$M_{5(nl_pm_ps_p)}^{p-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{p5}(-1)^q\left(n+K_{nk}^p\right)_q}{(q+2\nu_{pnk}^1+3)(n-q)!\,q!\,\Gamma\left(q+K_{nk}^p+1\right)}\,{}_3F_2\left(-n,q+2\nu_{pnk}^1+3,n+K_{nk}^p,2\nu_{pnk}^1+4;q+K_{nk}^p+1;1\right),\quad(57.5)$$

$$M_{6(nl_pm_ps_p)}^{p-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{p6}(-1)^q\left(n+K_{nk}^p+1/2\right)_q}{(q+2\nu_{pnk}^1+7/2)(n-q)!\,q!\,\Gamma\left(q+K_{nk}^p+1/2\right)}\,{}_3F_2\left(-n,q+2\nu_{pnk}^1+7/2,n+K_{nk}^p+1/2,2\nu_{pnk}^1+9/2;q+K_{nk}^p+1/2;1\right),\quad(57.6)$$

and

$$M_{7(nl_pm_ps_p)}^{p-mr}=\sum_{q=0}^{n}\frac{\beta_{nk}^{p7}(-1)^q\left(n+K_{nk}^p-1\right)_q}{(q+2\nu_{pnk}^1+2)(n-q)!\,q!\,\Gamma\left(q+K_{nk}^p\right)}\,{}_3F_2\left(-n,q+2\nu_{pnk}^1+2,n+K_{nk}^p-1,2\nu_{pnk}^1+3;q+K_{nk}^p;1\right),\quad(57.7)$$

with

$$\begin{cases}K_{nk}^p=2\nu_{pnk}^1+\zeta_{pnk}^1,\\ \beta_{nk}^{p1}=\frac{C_{nk}^{np2}}{2\delta}n!\,\Gamma\left(2\nu_{pnk}^1+3\right)\Gamma\left(\zeta_{pnk}^1-1\right),\\ \beta_{nk}^{p2}=\frac{C_{nk}^{np2}}{2\delta}n!\,\Gamma\left(2\nu_{pnk}^1+7/2\right)\Gamma\left(\zeta_{pnk}^1-2\right),\\ \beta_{nk}^{p3}=\frac{C_{nk}^{np2}}{2\delta}n!\,\Gamma\left(2\nu_{pnk}^1+5/2\right)\Gamma\left(\zeta_{pnk}^1\right),\\ \beta_{nk}^{p4}=\frac{C_{nk}^{np2}}{2\delta}n!\,\Gamma\left(2\nu_{pnk}^1+7/2\right)\Gamma\left(\zeta_{pnk}^1-1\right),\\ \beta_{nk}^{p5}=\frac{C_{nk}^{np2}}{2\delta}n!\,\Gamma\left(2\nu_{pnk}^1+4\right)\Gamma\left(\zeta_{pnk}^1-2\right),\\ \beta_{nk}^{p6}=\frac{C_{nk}^{np2}}{2\delta}n!\,\Gamma\left(2\nu_{pnk}^1+9/2\right)\Gamma\left(\zeta_{p_{nk}}^{\;1}-2\right),\\ \beta_{nk}^{p7}=\frac{C_{nk}^{np2}}{2\delta}n!\,\Gamma\left(2\nu_{pnk}^1+3\right)\Gamma\left(\zeta_{pnk}^1-2\right).\end{cases}\quad(58)$$

and

$$\begin{cases}\left(n+K_{nk}^p\right)_q=\frac{\Gamma(n+K_{nk}^p+q)}{\Gamma(n+K_{nk}^p)},\\ \left(n+K_{nk}^p\pm 1/2\right)_q=\frac{\Gamma(n+K_{nk}^p\pm 1/2+q)}{\Gamma(n+K_{nk}^p+1/2)},\\ \left(n+K_{nk}^p-1\right)_q=\frac{\Gamma(n+K_{nk}^p-1+q)}{\Gamma(n+K_{nk}^p-1)}.\end{cases}\quad(59)$$

D. New energy for *NCMRYPs* in 3D-RNCQS symmetries

The main objective underlined in this subsection is to find the contribution resulting from topological properties based on our strategy which we have successfully applied in previous works and which we try to develop in every new work. We can say that the global relativistic energy in the perspective of 3D-RNCQS symmetries produced with *NCMRYPs* model as a result of a major contribution to relativistic energy known in the literature under the combined Manning-Rosen and Yukawa tensor potentials model in usual Dirac theory and which we paved for through a quick look for the spin(p-spin)-symmetry in Eqs. (20) and (21), while the new contribution is produced from the topological properties under space-space deformation, which can be evaluated through several contributions, we will address three of them. The first one is generated from the effect of the perturbed spin-orbit effective potentials $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ corresponds to spin symmetry and pseudospin symmetry. These perturbed effective potentials are obtained by replacing the coupling of the angular momentum (L and L_p) operators and the NC vector $\mathbf{\Theta}$ with the new equivalent couplings ($\Theta\mathbf{L}\mathbf{S}$ and $\Theta\mathbf{L_p}\mathbf{S_p}$) for spin and p-spin-symmetry, respectively(with $\Theta^2=\Theta_{12}^2+\Theta_{23}^2+\Theta_{13}^2$). This degree of freedom comes considering that the infinitesimal NC vector $\mathbf{\Theta}$ is arbitrary. We have oriented the two spin-s and spin-s_p of the fermionic particles to become parallels to the vector $\mathbf{\Theta}$ which interacted with new combined Manning-Rosen and Yukawa tensor potentials. Additionally, we substitute the previous new spin-orbit couplings with the corresponding new physical form $(\Theta/2)\mathbf{G}^2$and $(\Theta/2)\mathbf{G}_p^2$, with $\mathbf{G^2}=\mathbf{J^2}-\mathbf{L^2}-\mathbf{S^2}$ and $\mathbf{G_p^2}=\mathbf{J^2}-\mathbf{L_p^2}-\mathbf{S_p^2}$ for a spin (p-spin)-symmetry, respectively. It is well known that the operators ($\mathbf{H_{rnc}^{mr}}$, $\mathbf{J^2}$, $\mathbf{L^2}$, $\mathbf{S^2}$ and $\mathbf{J_z}$) form a complete set of conserved physics quantities, the eigenvalues of the operators G^2 and G_p^2 are equal to the values:

$$F(j,l,s)=\frac{[j(j+1)-l(l+1)-s(s+1)]}{2},\quad\text{for } |l-s|\le j\le\ |l+s|$$

and

$$F\left(j,l_p,s_p\right)=\frac{\left[j(j+1)-l_p\left(l_p-1\right)-s_p\left(s_p+1\right)\right]}{2},\quad\text{for }\left|l_p-s_p\right|\le j\le\ \left|l_p+s_p\right|$$

that corresponding to the spin and p-spin-symmetry, respectively. Consequently, the partially corrected energies $\Delta E_{mr}^{so-s}(n,\delta,\beta,A,V_0,\eta,j,l,s)\equiv\Delta E_{mr}^{so-s}$ and $\Delta E_{mr}^{so-p}(n,\delta,\beta,A,V_0,\eta,j,l_p,s)\ \equiv\Delta E_{mr}^{so-p}$ due to the perturbed effective potentials $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ produced for the $\left(n,l,s,l_p,s_p\right)^{th}$ excited state, in 3D-RNCQS symmetries, as follows:

$$\begin{cases}\Delta E_{mr}^{so-s}=\Theta F(j,l,s)\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0),\\ \Delta E_{mr}^{so-p}=\Theta F\left(j,l_p,s_p\right)\left\langle X_p\right\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A,V_0).\end{cases}\quad(60)$$

The global two expectation values $\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0)$ and $\left\langle X_p\right\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A,V_0)$ for a spin(p-spin)-symmetry, respectively are determined from the following expressions:

$$\begin{cases}\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0)=\sum_{\mu=1}^{7}\beta_{nk}^{\mu s}M_{\mu(nlms)}^{s-mr},\\ \left\langle X_p\right\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A,V_0)=\sum_{\mu=1}^{7}\beta_{nk}^{\mu p}M_{\mu(nl_pm_ps_p)}^{p-mr},\end{cases}\quad(61)$$

where $\beta_{nk}^{\mu s}$ and $\beta_{nk}^{\mu p}\left(\mu=\overline{1,7}\right)$ are determined from Eqs. (46) while $M_{\mu(nlms)}^{s-mr}$ and $M_{\mu(nl_pm_ps_p)}^{p-mr}$ are determined from Eqs. ($53, i=\overline{1,7}$) and Eqs. ($57, i=\overline{1,7}$), respectively. The second main part is obtained from the magnetic effect of the perturbative effective potentials $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ under the *NCMRYPs* model in 3D-RNCQS symmetries. These effective potentials are achieved when we replace both ($\mathbf{L\Theta}$ and $\mathbf{L_p\Theta}$) by ($\lambda\aleph L_z$ and $\lambda\aleph L_z^p$), respectively, and Θ_{12} by $\lambda\aleph$, here ($\aleph$ and λ) are present the intensity of the magnetic field induced by the effect of the deformation of space-space geometry and a new infinitesimal noncommutativity parameter, so that the physical unit of the original noncommutativity parameter Θ_{12} (length)^2 is the same unit of $\lambda\aleph$, we have also need to apply $\langle n',l',m'|L_z|n,l,m\rangle=m\delta_{m'm}\delta_{l'l}\delta_{n'n}$ and $\langle n',l_p',m_p'|L_z^p|n,l_p,m_p\rangle=m_p\delta_{m_p'm_p}\delta_{l_p'l_p}\delta_{n'n}(-l_p'\leq m_p'\leq l_p$ and $-l\leq m\leq l)$ for spin(p-spin)-symmetry, respectively. All of these data allow for the discovery of the new energy shift $\Delta E_{mr}^{mg-s}(n,\delta,\beta,A,V_0,\lambda,j,l,m)\equiv\Delta E_{mr}^{mg-s}$ and $\Delta E_{mr}^{mg-p}(n,\delta,\beta,A,V_0,\lambda,j,l,m_p)\equiv\Delta E_{mr}^{mg-p}$ due to the perturbed Zeeman effect created by the influence of the *NCMRYPs* model for the $\left(n,l,s,l_p,s_p,m,m_p\right)^{th}$ excited state in 3D-RNCQS symmetries as follows:

$$\begin{cases}\Delta E_{mr}^{mg-s}=\lambda\aleph\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0)m,\\ \Delta E_{mr}^{mg-p}=\lambda\aleph\langle X_p\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A,V_0)m_p.\end{cases}\quad(62)$$

After we completed the self-energy additions resulting from the self-deformation generated by perturbed (spin/p-spin)-orbit interactions and the new modified Zeeman effect. We are now in the process of reviewing another addition that is no less important than the previous ones under the *NCMRYPs* model in 3D-RNCQS symmetries. This new physical phenomenon is generated automatically from the effect of perturbed effective potentials $\Sigma_{mr}^{pert}(r)$ and $\Delta_{mr}^{pert}(r)$ which we have seen in Eqs. (44) and (45). We consider the fermionic particles undergoing rotation with angular velocity $\boldsymbol{\omega}$. The features of this subjective phenomenon are determined by replacing the arbitrary vector $\boldsymbol{\Theta}$ with $\gamma\boldsymbol{\omega}$. Allowing us to replace the two couplings ($\mathbf{L\Theta}$ and $\mathbf{L_p\Theta}$) with ($\gamma\mathbf{L\omega}$ and $\mathbf{L_p\omega}$), respectively, as follows:

$$\begin{pmatrix}\mathbf{L}\\ \mathbf{L_p}\end{pmatrix}\boldsymbol{\Theta}\rightarrow\gamma\begin{pmatrix}\mathbf{L}\,\boldsymbol{\omega}\text{ for spin-sy}\\ \mathbf{L_p}\boldsymbol{\omega}\text{ for p-spin-sy}\end{pmatrix}.\quad(63)$$

Here, we consider γ is just an infinitesimal real proportional constant. We can express the effective potentials $\Sigma_{pert}^{mr-rot}(z)$ and $\Delta_{pert}^{mr-rot}(z)$ which induced the rotational movements of the fermionic particles as follows:

$$\Sigma_{pert}^{mr-rot}(z)=\gamma(\chi_{nk}^{1s}\frac{z^2}{(1-z)^3}+\chi_{nk}^{2s}\frac{z^{\frac{5}{2}}}{(1-z)^4}+$$
$$+\chi_{nk}^{3s}\frac{z^{3/2}}{(1-z)^2}+\chi_{nk}^{4s}\frac{z^{5/2}}{(1-z)^3}+\chi_{nk}^{5s}\frac{z^3}{(1-z)^4}$$
$$+\chi_{nk}^{6s}\frac{z^{7/2}}{(1-z)^4}+\chi_{nk}^{7s}\frac{z^2}{(1-z)^4})\boldsymbol{L\omega}+O(\Theta^2),\ (64.1)$$

and

$$\Delta_{pert}^{mr-rot}(z)=\gamma(\chi_{nk}^{1p}\frac{z^2}{(1-z)^3}+\chi_{nk}^{2p}\frac{z^{5/2}}{(1-z)^4}+$$
$$+\chi_{nk}^{3p}\frac{z^{3/2}}{(1-z)^2}+\chi_{nk}^{4p}\frac{z^{5/2}}{(1-z)^3}+\chi_{nk}^{5p}\frac{z^3}{(1-z)^4}+$$
$$+\chi_{nk}^{6p}\frac{z^{7/2}}{(1-z)^4}+\chi_{nk}^{7p}\frac{z^2}{(1-z)^4})\mathbf{L_p\omega}+O(\Theta^2).\ (64.2)$$

To simplify the calculations without changing the physical content, by applying the same principle that we examined a short while ago, we choose the rotational velocity $\boldsymbol{\omega}$ parallel to the (Oz) axis $(\boldsymbol{\omega}=\omega e_z)$. Thus, the above equation can be reduced to its simplified form as

$$\gamma\begin{pmatrix}\left(\sum_{\mu=1}^{7}\beta_{nk}^{\mu s}M_{\mu(nlms)}^{s-mr}\right)\mathbf{L\omega}\\ \left(\sum_{\mu=1}^{7}\beta_{nk}^{\mu p}M_{\mu(nl_pm_ps_p)}^{p-mr}\right)\mathbf{L_p\omega}\end{pmatrix}=\gamma\omega\begin{pmatrix}\left(\sum_{\mu=1}^{7}\beta_{nk}^{\mu s}M_{\mu(nlms)}^{s-mr}\right)L_z\\ \left(\sum_{\mu=1}^{7}\beta_{nk}^{\mu p}M_{\mu(nl_pm_ps_p)}^{p-mr}\right)L_z^p\end{pmatrix}.\quad(65)$$

All of this data permuted us to produce the corrected energies $\Delta E_{mr}^{rot-s}(n,\delta,\beta,A,V_0,\gamma,m)$ and $\Delta E_{mr}^{rot-p}(n,\delta,\beta,A,V_0,\gamma,m_p)$ due to the perturbed effective potentials $\Sigma_{pert}^{mr-rot}(z)$ and $\Delta_{pert}^{mr-rot}(z)$ which are generated automatically by the influence of the new combined Manning-Rosen and Yukawa tensor potentials for the $\left(n,l,l_p,m,m_p\right)^{th}$ excited state in 3D-RNCQS symmetries as follows:

$$\begin{pmatrix}\Delta E_{mr}^{rot-s}\\ \Delta E_{mr}^{rot-p}\end{pmatrix}=\gamma\omega\begin{pmatrix}\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0)m\\ \langle X_p\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A,V_0)m_p\end{pmatrix}.\ (66)$$

It is worth noting that the authors of reference [102] studied rotating isotropic and anisotropic harmonically confined ultra-cold Fermi gas in a two and three-dimensional space at zero temperature, but in this study, the rotational term was added to the Hamiltonian operator, in contrast to our case, where in our recent study, the two rotation operators $\Sigma_{pert}^{mr-rot}(z)\mathbf{L\omega}$ and $\Delta_{pert}^{mr-rot}(z)\mathbf{L_p\omega}$ automatically appear due to the augmented symmetries resulting from the deformation of space-space under the new combined Manning-Rosen and Yukawa tensor potentials. For fermionic particles with spin-1/2, the eigenvalues of the operators $\mathbf{G}^2$ and $\mathbf{G}_p^2$ are equal to the values:

$$\begin{cases} F(j,l,s)=\dfrac{\left[j(j+1)-l(l+1)-\frac{3}{4}\right]}{2}, \\ F(j,l_p,s_p)=\left[j(j+1)-l_p(l_p-1)-\frac{3}{4}\right]/2. \end{cases} \quad (67)$$

The possible values of $\{j\}$ that corresponding spin-1/2 can be taken ($l\pm 1/2$ and $l_p\pm 1/2$) for spin symmetry and pseudospin symmetry, this allows us to reformulate Eq. (67) as follows:

$$F(j=l\pm 1/2, s=1/2) = \frac{1}{2}\begin{cases} l \text{ Up polarity: } j=l+1/2, \\ -(l+1) \text{ Down polarity: } j=l-1/2. \end{cases} \quad (68)$$

and

$$F(j=l_p\pm 1/2, s_p=1/2) = \frac{1}{2}\begin{cases} l_p \text{ Up polarity: } j=l_p+1/2, \\ -(l_p+1) \text{ Down polarity: } j=l_p-1/2. \end{cases} \quad (69)$$

The global relativistic energy $E_{nc}^{mr-s}(n,\delta,\beta,A,V_0,\eta,\lambda,\gamma,j,l,s,m)$ and E_{nc}^{mr-p} $(n,\delta,\beta,A,V_0,\eta,\lambda,\gamma,j,l_p,s,m_p)$ for the case of spin-1/2 with new combined Manning-Rosen and Yukawa tensor potentials, in the framework of 3D-RNCQS symmetries, corresponding to the generalized $(n,l,s,l_p,s_p,m,m_p)^{th}$ excited with Up polarity (Up) with $j=l+1/2$ and down polarity (Dp) with $j=l-1/2$ as

$$E_{nc}^{mr-s}(n,\delta,\beta,A,V_0,\theta,\lambda,\gamma,j,l,s,m) = E_{nk}^{s}(n,\delta,\beta,A,V_0), +\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0)\left[(\lambda\aleph + \gamma\omega)m + \frac{\theta}{2}\begin{cases} l \text{ for Up with } j=l+\frac{1}{2} \\ -(l+1) \text{ for Dp with } j=l-\frac{1}{2} \end{cases}\right]. \quad (70)$$

and

$$E_{nc}^{mr-p}(n,\delta,\beta,A,V_0,\theta,\lambda,\gamma,j,l_p,s_p,m_p) = E_{nk}^{p}(n,\delta,\beta,A,V_0) + \langle X_p\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A,V_0) \left[(\lambda\aleph+\gamma\omega)m_p + \frac{\theta}{2}\begin{cases} l_p \text{ for Up with } j=l_p+\frac{1}{2} \\ -(l_p+1) \text{ for Dp with } j=l_p-\frac{1}{2} \end{cases}\right] \quad (71)$$

Where E_{nk}^{s} and E_{nk}^{p} are usual relativistic energies under combined Manning-Rosen and Yukawa tensor potentials obtained from equations of energy in Eqs.(20) and (21). We can now generalize our obtained energies E_{g-nc}^{mr-s} and E_{g-nc}^{mr-p} under the new combined Manning-Rosen and Yukawa tensor potentials which are produced with the globally induced two potentials $\Sigma_{t\_mr}^{pert}(r)$ and $\Delta_{t-mr}^{pert}(r)$ for spin and pseudospin symmetries corresponding to the upper and lower components ($F_{nk}^{s}(s)$ and $G_{nk}^{s}(s)$) and ($F_{nk}^{p}(s)$ and $G_{nk}^{p}(s)$), respectively as:

$$E_{g-nc}^{mr-s} = E_{nc}^{mr-s}\theta(|E_{nc}^{mr-s}|) - E_{nc}^{mr-s}\theta(-|E_{nc}^{mr-s}|) = \begin{cases} E_{nc}^{mr-s} \text{ for Uc of spin symmetry} \\ -E_{nc}^{mr-s} \text{ for Lc of spin symmetry} \end{cases} \quad (72)$$

and

$$E_{g-nc}^{mr-p} = E_{nc}^{mr-p}\theta(|E_{nc}^{mr-p}|) - E_{nc}^{mr-p}\theta(-|E_{nc}^{mr-p}|) = \begin{cases} E_{nc}^{mr-p} \text{ for UC of p-pin symmetry} \\ -E_{nc}^{mr-p} \text{ for Lc of p-pin symmetry} \end{cases} \quad (73)$$

I. IV. THE NEW COMBINED MANNING-ROSEN AND YUKAWA TENSOR INTERACTION IN 3D-NRNCQS SYMMETRIES

In order to study and analyze the nonrelativistic limit, in three-dimensional nonrelativistic noncommutative quantum mechanics (3D-3D-NRNCQS) symmetries of the new combined Manning-Rosen potential, two steps must be applied, the first step corresponds to the nonrelativistic limit, in usual nonrelativistic quantum energy. This is done by applying the following steps, we replace:

$(C_{ES},C_{PS},V_0)\to(0{,}0{,}0)$, $E_{nk}^{s}+M\to 2\mu$, $E_{nk}^{s}-M\to E_{nl}^{nr}$, $k(k+1)\to l(l+1)$.

This allows us to obtain the nonrelativistic energy levels as:

$$E_{nl}^{nr} = -\frac{1}{2\mu}\left[\delta\,\frac{2\delta^2 A - l(l+1) - n(n+1)\Lambda(l,\beta,\delta)}{n+\frac{1}{2}+\Lambda(l,\beta,\delta)}\right]^2. \quad (74)$$

Here $\Lambda(l,\beta,\delta)$ equal to $\sqrt{\frac{1}{4}+l(l+1)-2\delta^2\beta(\beta-1)}$. Now, the second step corresponds to the transformation of the relativistic coefficients $\chi_{nk}^{\mu s}(\mu=\overline{1,7})$ under the previous correspondence to the new nonrelativistic coefficients $\varepsilon_{nl}^{\mu}(\mu=\overline{1,7})$ of the nonrelativistic expectations values are given by:

$$\begin{cases} \varepsilon_{nl}^{1}=\varepsilon_{nl}^{2}=\varepsilon_{nl}^{5}=0, \\ \varepsilon_{nl}^{3}=-\frac{2\delta^2 A}{b^2}, \\ \varepsilon_{nl}^{4}=\frac{4\delta^2\beta(\beta-1)}{b^2}-\frac{2\delta^2 A}{b^2} \\ \varepsilon_{nl}^{6}=-\frac{2\beta\delta^2(\beta-1)\Lambda}{b^2}, \\ \varepsilon_{nl}^{7}=16\delta^4 l(l+1). \end{cases} \quad (75)$$

Allows us to reexport the relativistic expectation values $\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0$) of spin symmetry in Eq. (61) from the corresponding nonrelativistic expectation values $\langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A,V_0)$ as:

$$\langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A,V_0) = \sum_{\mu=3}^{7}\varepsilon_{nl}^{\mu}M_{\mu(nlms)}^{nr-mr} \quad (76)$$

with

$M_{3(nlms)}^{nr-mr} =$

$$\sum_{q=0}^{n} \frac{\beta_{nl}^{nr3}(-1)^q(n+K_{nl}^{nr}+1/2)_q}{(q+2\nu_{nl}^{nr1}+3/2)(n-q)!\,q!\,\Gamma(q+K_{nl}^{nr}+3/2)}\ {}_3F_2(-n,q+2\nu_{nl}^{nr1}+3/2,n+K_{nl}^{nr}+1/2,2\nu_{nl}^{nr1}+5/2;q+K_{nl}^{nr}+3/2;1), \quad (77.1)$$

$$M_{4(nlms)}^{nr-mr}=\sum_{q=0}^{n} \frac{\beta_{nl}^{nr4}(-1)^q(n+K_{nl}^{nr}+1/2)_q}{(q+2\nu_{nl}^{nr1}+5/2)(n-q)!\,q!\,\Gamma(q+K_{nl}^{nr}+3/2)}\ {}_3F_2(-n,q+2\nu_{nl}^{nr1}+5/2,n+K_{nl}^{nr}+1/2,2\nu_{nl}^{nr1}+7/2;q+K_{nl}^{nr}+3/2;1), \quad (77.2)$$

$$M_{6(nlms)}^{nr-mr}=\sum_{q=0}^{n} \frac{\beta_{nl}^{nr6}(-1)^q(n+K_{nl}^{nr}+1/2)_q}{(q+2\nu_{nl}^{nr1}+7/2)(n-q)!\,q!\,\Gamma(q+K_{nl}^{nr}+1/2)}\ {}_3F_2(-n,q+2\nu_{nl}^{nr1}+7/2,n+K_{nl}^{nr}+1/2,2\nu_{nl}^{nr1}+9/2;q+K_{nl}^{nr}+1/2;1), \quad (77.3)$$

and

$$M_{7(nlms)}^{nr-mr}=\sum_{q=0}^{n} \frac{\beta_{nl}^{nr7}(-1)^q(n+K_{nl}^{nr}-1)_q}{(q+2\nu_{nl}^{nr1}+2)(n-q)!\,q!\,\Gamma(q+K_{nl}^{nr})}\ {}_3F_2(-n,q+2\nu_{nl}^{nr1}+2,n+K_{nl}^{nr}-1,2\nu_{nl}^{nr1}+3;q+K_{nl}^{nr};1), \quad (77.4)$$

with

$$\begin{cases} K_{nl}^{nr}=2\nu_{nl}^{nr1}+\zeta_{nl}^{nl1}, \\ \beta_{nl}^{nr3}=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(\nu_{nl}^{nr1}+5/2)\Gamma(\zeta_{nl}^{nl1}), \\ \beta_{nl}^{nr4}=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nl}^{nr1}+7/2)\Gamma(\zeta_{nl}^{nl1}-1), \\ \beta_{nl}^{nr6}=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nl}^{nr1}+9/2)\Gamma(\zeta_{nl}^{nl1}-2), \\ \beta_{nl}^{nr7}=\frac{C_{nk}^{ns2}}{2\delta}n!\,\Gamma(2\nu_{nl}^{nr1}+3)\Gamma(\zeta_{nl}^{nl1}-2). \end{cases} \quad (78)$$

and

$$\begin{cases} (n+K_{nl}^{nr}+1/2)_q=\frac{\Gamma(n+K_{nl}^{nr}+1/2+q)}{\Gamma(n+K_{nl}^{nr}+1/2)}, \\ (n+K_{nl}^{nr}-1)_q=\frac{\Gamma(n+K_{nl}^{nr}-1+q)}{\Gamma(n+K_{nl}^{nr}-1-1)}. \end{cases} \quad (79)$$

This permuted expressing the nonrelativistic correction energy $\Delta E_{nc-nr}^{mr}(n,\delta,V_0,A,\eta,\lambda,\gamma,j,l,s,m)$ produced by the new combined Manning-Rosen and Yukawa tensor potentials as

$$\Delta E_{nc-nr}^{mr}(n,\delta,V_0,A,\eta,\lambda,\gamma,j,l,s,m)=\langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A,V_0)(\lambda\aleph+\gamma\omega)m+\langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A,V_0)\frac{\theta}{2}\begin{cases} l & \text{Up}: j=l+1/2, \\ -(l+1) & \text{Dp}: j=l-1/2. \end{cases} \quad (80)$$

The global nonrelativistic energy $E_{nc-nr}^{mr}(n,\delta,V_0,A,\theta,\lambda,\gamma,j,l,s,m)$ produced with the new Manning-Rosen potential in 3D-NRNCQS symmetries as a result the topological properties of the deformation space-space is the sum of usual energy E_{nl}^{mr} in Eq. (74) under combined Manning-Rosen and Yukawa tensor potentials in 3D-NRNCQS symmetries and the obtained correction $\Delta E_{nc-nr}^{mr}(n,\delta,V_0,A,\theta,\lambda,\gamma,j,l,s,m)$ in Eq. (80) as follows:

$$E_{nc-nr}^{mr}(n,\delta,V_0,A,\theta,\lambda,\gamma,j,l,s,m)=-\frac{1}{2\mu}\left[\delta\frac{2\delta^2A-l(l+1)-n(n+1)\Lambda(l,\beta,\delta)}{n+\frac{1}{2}+\Lambda(l,\beta,\delta)}\right]^2+\langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A,V_0)\left[(\lambda\aleph+\gamma\omega)m+\frac{\theta}{2}\begin{cases} l & \text{Up}: j=l+1/2 \\ -(l+1) & \text{Dp}: j=l-1/2 \end{cases}\right]. \quad (81)$$

Now, considering composite systems such as molecules made of $N=2$ particles of masses $m_n(n=1,2)$ in the frame of NC algebra, it is worth taking into account the features of the descriptions of the systems in the nonrelativistic case, it was obtained those composite systems with different masses are described with different NC parameters [48, 49, 50]:

$$\left[q_\mu^{(s,h,i)},q_\nu^{(s,h,i)}\right]_*=i\theta_{\mu\nu}^c, \quad (82)$$

where the noncommutativity parameter $\theta_{\mu\nu}^c$ is determined from:

$$\theta_{\mu\nu}^c=\sum_{n=1}^{2}\mu_n^2\delta_{\mu\nu}^{(n)}, \quad (83)$$

with $\mu_1=\frac{\mu_1}{\mu_1+\mu_2}$ and $\mu_2=\frac{\mu_2}{\mu_1+\mu_2}$, and $\delta_{\mu\nu}^{(n)}$ is the parameter of non-commutativity, corresponding to the mass particle of mass μ_n. Note that in the case of a physical system composed of two identical particles $\mu_1=\mu_2$ such as the diatomic O_2, I_2, N_2, H_2, and Ar_2 molecules under the effect of the new Manning-Rosen potential, the parameter $\delta_{\mu\nu}^{(n)}=\delta_{\mu\nu}$. Thus, the three parameters η, λ, and γ which appear in Eq. (81) are changed to become as follows:

$$\Xi^{c2}=\left(\sum_{n=1}^{2}\mu_n^2\Xi_{12}^{(n)}\right)^2+\left(\sum_{n=1}^{2}\mu_n^2\Xi_{23}^{(n)}\right)^2+\left(\sum_{n=1}^{2}\mu_n^2\Xi_{13}^{(n)}\right)^2, \quad (84)$$

with $\Xi^c=(\theta^c,\lambda^c,\gamma^c)$. As mentioned above, in the case of a system of two particles with the same mass $\mu_1=\mu_2$, we have $\eta_{\mu\nu}^{(n)}=\eta_{\mu\nu}$, $\lambda_{\mu\nu}^{(n)}=\lambda_{\mu\nu}$ and $\gamma_{\mu\nu}^{(n)}=\gamma_{\mu\nu}$. Finally, we can generalize our obtained nonrelativistic total energy $E_{nc-nr}^{s}(n,\delta,\eta,A,\eta^c,\lambda^c,\gamma^c,j,l,s,m)$ under the new Manning-Rosen potential considering that composite systems with different masses are described with different NC parameters for the HCl, CH, LiH, CO, and NO diatomic molecules as:

$$E_{nc-nr}^{mr}=-\frac{1}{2\mu}\left[\delta\frac{2\delta^2A-l(l+1)-n(n+1)\Lambda(l,\beta,\delta)}{n+\frac{1}{2}+\Lambda(l,\beta,\delta)}\right]^2+\langle X\rangle_{(nlms)}^{mr-nr}\left[(\lambda^c\aleph+\gamma^c\omega)m+\frac{\theta^c}{2}\begin{cases} l & \text{Up } j=l+1/2 \\ -(l+1) & \text{Dp}: j=l-1/2 \end{cases}\right]. \quad (85)$$

V. STUDY OF IMPORTANT RELATIVISTIC AND NON-RELATIVISTIC CASES IN THE CONTEXT OF 3D-NRNCQS SYMMETRIES

In this section, we are about to examine some particular cases regarding the new relativistic bound state energy eigenvalues in Eqs. (70) and (71) and the nonrelativistic bound state energy eigenvalues in Eq. (81). We could derive some particular potentials, useful for other physical systems, by adjusting relevant parameters of the NCMRYPs model in 3D-RNCQS and 3D-NRNCQS symmetries, such as the new s-wave cases and both the new Dirac and Schrödinger-Manning-Rosen problems in 3D-RNCQS and 3D-NRNCQS symmetries.

A. New s-wave under deformed (Dirac-Schrödinger) equations with *NCMRYPs* model and Manning-Rosen problem

If we consider $l=0$ and $l_p=0$($k=-1$ and $k=+1$ for spin and p-spin symmetry, respectively), we obtain directly the *s*-wave. The new corresponding relativistic energy eigenvalue equations in 3D-RNCQS symmetries reduce to:

$$E_{nc}^{mr-s}(n,\delta,\beta,A,V_0,\theta,\lambda,\gamma,j,l=0,s,m) = E_{n(-1)}^{s}+\langle X\rangle_{(n0ms)}^{mr}(n,\delta,\beta,A,V_0) \left[(\lambda\aleph+\gamma\omega)m+\frac{\theta}{2}\begin{cases}0 & \text{for Up with } j=1/2\\ -1 & \text{for Dp with } j=-1/2\end{cases}\right], \quad (86)$$

and

$$E_{nc}^{mr-p}(n,\delta,\beta,A,V_0,\theta,\lambda,\gamma,j,l_p=0,s_p,m_p) = E_{n1}^{p}+\langle X_p\rangle_{(n0m_ps_p)}^{mr}(n,\delta,\beta,A,V_0) \left[(\lambda\aleph+\gamma\omega)m_p+\frac{\theta}{2}\begin{cases}0 & \text{for Up with } j=1/2\\ -1 & \text{for Dp with } j=1/2\end{cases}\right], \quad (87)$$

with $E_{n(-1)}^{s}$ and E_{n1}^{p} are given by[13]:

$$M^2-E_{n,-1}^{s2}-C_{ES}\left(M-E_{n,-1}^{s}\right)=4\delta^2\left[\frac{\frac{A\left(M+E_{n,-1}^{s}-C_{SE}\right)}{2M}-n(n+1)\Lambda_{-1}^{s}}{2n+1+2\Lambda_{-1}^{s}}\right]^2, \quad (88)$$

and

$$M^2-E_{n1}^{p2}+C_{PS}\left(M+E_{n1}^{p}\right)=4\delta^2\left[\frac{\frac{A\left(M-E_{n1}^{p}-C_{PS}\right)}{2M}-n(n+1)\Lambda_{1}^{s}}{2n+1+2\Lambda_{1}^{s}}\right]^2. \quad (89)$$

While the new corresponding nonrelativistic energy eigenvalue in Eq. (81) reduces to:

$$E_{nc-nr}^{mr}(n,\delta,V_0,A,\theta,\lambda,\gamma,j,l=0,s,m) = -\frac{1}{2\mu}\left[\delta\frac{2\delta^2A-n(n+1)\Lambda(0,\beta,\delta)}{n+\frac{1}{2}+\Lambda(0,\beta,\delta)}\right]^2 +\langle X\rangle_{(n0ms)}^{mr-nr}(n,\delta,\beta,A,V_0)\left[(\lambda\aleph+\gamma\omega)m+\frac{\theta}{2}\begin{cases}0 & \text{Up: } j=\frac{1}{2}\\ \mp 1 & \text{Dp: } j=-\frac{1}{2}\end{cases}\right]. \quad (90)$$

Here Λ_{-1}^{s}, Λ_{1}^{s} and $\Lambda(0,\beta,\delta)$ are equals to $\sqrt{\frac{1}{4}+\frac{\beta(\beta-1)\left(M+E_{n,-1}^{s}-C_{SE}\right)}{2M}}$, $\sqrt{\frac{1}{4}+\frac{\beta(\beta-1)\left(M-E_{n1}^{p}-C_{PS}\right)}{2M}}$ and $\sqrt{\frac{1}{4}-2\delta^2\beta(\beta-1)}$ respectively. The new expectations values $\langle X\rangle_{(n0ms)}^{mr}$, $\langle X_p\rangle_{(n0m_ps_p)}^{mr}$ and $\langle X\rangle_{(n0ms)}^{mr-nr}(n,\delta,\beta,A,V_0)$ are determined from:

$$\begin{cases}\langle X\rangle_{(n0ms)}^{mr}(n,\delta,\beta,A,V_0)=\lim_{l\to 0}\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0),\\ \langle X_p\rangle_{(n0m_ps_p)}^{mr}(n,\delta,\beta,A,V_0)=\lim_{l_p\to 0}\langle X_p\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A,V_0),\\ \langle X\rangle_{(n0ms)}^{mr-nr}(n,\delta,\beta,A,V_0)=\lim_{l\to 0}\langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A,V_0).\end{cases} \quad (91)$$

B. Deformed (Dirac-Schrödinger) new Manning-Rosen problems:

If we consider $V_0=0$, our studied potential turns to the new Manning-Rosen potential, and the new energy eigenvalue, in 3D-RNCQS symmetries, for the spin and p-spin symmetry becomes as:

$$E_{nc}^{m-s}(n,\delta,\beta,A,\theta,\lambda,\gamma,j,l,s,m) = E_{nk}^{ms}+\langle X\rangle_{(nlms)}^{m}(n,\delta,\beta,A) \left[(\lambda\aleph+\gamma\omega)m+\frac{\theta}{2}\begin{cases}l & \text{for Up with } j=l+\frac{1}{2}\\ -(l+1) & \text{for Dp with } j=l-\frac{1}{2}\end{cases}\right], \quad (92)$$

and

$$E_{nc}^{m-p}\left(n,\delta,\beta,A,\theta,\lambda,\gamma,j,l_p,s_p,m_p\right) = E_{nk}^{mp}+\langle X_p\rangle_{(nl_pm_ps_p)}^{m}(n,\delta,\beta,A) \left[(\lambda\aleph+\gamma\omega)m_p+\frac{\theta}{2}\begin{cases}l_p & \text{for Up with } j=l_p+\frac{1}{2}\\ -\left(l_p+1\right) & \text{for Dp with } j=l_p-\frac{1}{2}\end{cases}\right], \quad (93)$$

with E_{nk}^{ms} and E_{nk}^{mp} are given by [13]:

$$M^2-E_{nk}^{ms2}-C_{ES}(M-E_{nk}^{ms})=4\delta^2\left[\frac{\frac{A\Lambda_{nk}^{ms}}{2M}-k(k+1)-n(n+1)\sqrt{\frac{1}{4}+k(k+1)+\frac{\beta(\beta-1)\Lambda_{nk}^{ms}}{2M}}}{2n+1+2\sqrt{\frac{1}{4}+k(k+1)+\frac{A\Lambda_{nk}^{ms}}{2M}}}\right]^2 \quad (94)$$

and

$$M^2-E_{nk}^{mp2}+C_{PS}\left(M+E_{nk}^{mp}\right)=4\delta^2$$

$$\left[\frac{\frac{A\Lambda_{nk}^{mp}}{2M}-k(k-1)-n(n+1)\sqrt{\frac{1}{4}+k(k-1)+\frac{\beta(\beta-1)\Lambda_{nk}^{mp}}{2M}}}{2n+1+2\sqrt{\frac{1}{4}+k(k-1)+\frac{\beta(\beta-1)\Lambda_{nk}^{mp}}{2M}}}\right]^2 \quad (95)$$

While the new corresponding nonrelativistic energy eigenvalue in Eq. (81) reduces to:

$$E_{nc-nr}^{m}=-\frac{1}{2\mu}\left[\delta\frac{2\delta^2 A-l(l+1)-n(n+1)\Lambda(l,\beta,\delta)}{n+\frac{1}{2}+\Lambda(l,\beta,\delta)}\right]^2 +\langle X\rangle_{(nlms)}^{m-nr}\left[(\lambda\aleph+\gamma\omega)m+\frac{\theta}{2}\begin{cases} l \text{ Up } j=l+\frac{1}{2}\\ -(l+1)\text{ Dp: } j=\ l-\frac{1}{2}\end{cases}\right], \quad (96)$$

here Λ_{nk}^{ms} and Λ_{nk}^{mp} are equals to $(M+E_{nk}^{ms}-C_{SE})$ and $(M-E_{nk}^{p}-C_{PS})$ respectively. The new expectations values $\langle X\rangle_{(nlms)}^{m}(n,\delta,\beta,A)$, $\langle X_p\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A)$ and $\langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A)$ are determined from:

$$\begin{cases} \langle X\rangle_{(nlms)}^{m}(n,\delta,\beta,A)=\lim_{V_0\to 0}\langle X\rangle_{(nlms)}^{mr}(n,\delta,\beta,A,V_0),\\ \langle X_p\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A)=\lim_{V_0\to 0}\langle X_p\rangle_{(nl_pm_ps_p)}^{mr}(n,\delta,\beta,A,V_0),\\ \langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A)=\lim_{V_0\to 0}\langle X\rangle_{(nlms)}^{mr-nr}(n,\delta,\beta,A,V_0).\end{cases} \quad (97)$$

It is crucial to highlight that using perturbation theory to find second-order corrections under the new combined MR and Yukawa tensor potentials is ineffective because we have only used first-order corrections of infinitesimal parameters (θ,λ,γ). Therefore, all the energetic corrections resulting from the deformation of space-space are of the first order of (θ,λ,γ) according to the postulates we adopted in our current research in Eqs. (4.2) and (7), this is one of the most important new results of this research. Worthwhile it is better to mention that for the three- simultaneous limits $(\theta,\lambda,\gamma)\to(0{,}0{,}0)$, we recover the equations of energy for the spin symmetry and the p-spin symmetry, under the combined Manning-Rosen and Yukawa tensor potentials which are treated in Refs. [13, 17]. Through our theoretical study of the new MR potential including Yukawa-like tensor interactions in 3D-RNCQS symmetries based on the study of researchers, (Ahmadov *et al.* and Ortakaya *et al.*) who clearly showed that shown that tensor interaction removes the degeneracy between two states in the pseudospin and spin doublets in usual 3D-RNCQS and 3D-NRNCQS symmetries, and through our current study, we found that the effect of deformation of space-space on energy is proportional to three infinitesimal parameters (θ,λ,γ). This means that the new energy is slightly offset from its counterpart in the literature. This confirms the conclusion reached by the researchers in Refs. [13, 17] remains valid and confirmed in our current research.

VI. SUMMARY AND CONCLUSIONS

In this paper, we have obtained the new approximate solutions of the deformed Dirac equation in three-dimensional relativistic noncommutative quantum mechanics, for the new Manning-Rosen potential including a tensor Yukawa interaction within the framework of pseudospin and spin symmetry limits. Bopp's shift and perturbation theory methods were used to solve the deformed Dirac equation analytically. We have obtained the global energy eigenvalues in terms of the quantum numbers $(j,k,l/l_p,s/s_p,m/m_p)$, the potential depths (β,A,V_0) of the studied potentials, the range of the potentials δ, and noncommutativity parameters (θ,λ,γ). We have analyzed the nonrelativistic solutions of the Manning-Rosen potential. Furthermore, we have applied our results to the composite systems such as diatomic molecules HCl, CH, LiH, CO, NO, O_2, I_2, N_2, H_2, and Ar_2. By altering parameters (β,A,V_0), we have obtained specific potentials which is helpful for other physical systems such as the s-wave of the new combined Manning-Rosen and Yukawa tensor potentials and the new Manning-Rosen problem in 3D-RNCQS and 3D-NRNCQS symmetries. It is worth mentioning that, in all cases, to make the three simultaneous limits $(\theta,\lambda,\gamma)\to(0{,}0{,}0)$, the ordinary physical quantities are recovered in Refs. [13, 17].

ACKNOWLEDGEMENTS

This work has been partly supported by the Algerian Ministry of Higher Education and Scientific Research and DGRST and by the Laboratory of Physics and Material Chemistry at the University of M'sila. The author is grateful to the reviewers.

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