Eratosthenes' measurement of the Earth's radius in a middle school lab session

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Abstract

We describe a middle school lab session designed to explore and understand the Eratosthenes' method for determining the Earth radius. The lab session is divided into six lab stations completely independent though related, each one with different apparatuses / materials. The stations are diversified and range from simple tasks, like simulations or simple measurements, to tasks that involve a higher cognitive ability. The lab session includes several physical contents, such as the Earth movements (rotation, translation and precession) and its consequences: variation of temperature and shadows length during the day, Sun rays inclination, time zones and seasons.

Keywords: Earth movements (rotation, translation and precession), shadows length, Sun rays inclination, time zones, seasons, Eratosthenes, Earth radius.

Resumen

En este artículo se describe una sesión de laboratorio para enseñanza secundaria, concebida para explotar didácticamente el método de determinación del radio de la Tierra de Eratóstenes. La sesión se compone de seis estaciones de laboratorio independientes. Aunque están relacionadas, cada una cuenta con materiales propios y diferentes. El contenido de estas estaciones de laboratorio es diversificado, incluyendo por un lado, tareas simples, como es el caso de simulaciones o mediciones sencillas; y por otro, tareas que implican una capacidad cognitiva superior. La sesión de laboratorio aborda diversos tópicos de la física, tales como los movimientos de la Tierra (rotación, traslación y precesión) y sus consecuencias: variación de la temperatura y de la longitud de las sombras durante el día, inclinación de los rayos solares, husos horarios y estaciones del año.

Palabras clave: Movimientos de la Tierra (rotación, traslación y precesión), longitud de las, inclinación de los rayos solares, husos horarios, estaciones del año, Eratóstenes, radio de la Tierra.

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I. INTRODUCTION

Eratosthenes' measurement of Earth's radius, around 230 BCE, is one of the most popular and amazing experiences in the history of Science. A poll among readers of Physics World magazine placed it in the top ten of the most beautiful experiments in Science[1]. Over the centuries, it has been admired for its apparent simplicity, for the economy of means employed in the measurement of fundamental parameter of our planet, and also for bringing together different domains of knowledge: Geography, Mathematics and Physics.

In spite of the inaccuracies of some of the measurements involved in this experience, the final result turned out to be remarkably close to the correct one. Regrettably, students are not always aware of this historical treasure, possibly because teachers have difficulties in adapting it to the level they teach.

Since Eratosthenes' original writings were lost, historians rely on the testimony of others, like Cleomedes, Strabo and Ptolemy. According to these reports, Eratosthenes knew that in the summer solstice (the longest day of the year), at noon, the Sun cast no shadow on the walls of deep well in the town of Syene (modern Aswan), from which he drew the implication that the Sun rays were vertical. In Alexandria, north of Syene, where Eratosthenes lived, he observed that, on the same day, at the same time, the Sun did cast a shadow of a vertical pole. From this observation, he was able to measure the angle of the Sun rays in Alexandria, and, according to Cleomedes, obtained 7º 12' ($1/50^{th}$ of a circle)[2].

A map of ancient Egypt shows that Syene is close to the Tropic of Cancer and that Alexandria and Syene lie almost on a direct north-south line. At the time, the distance from Alexandria to Syene was estimated to be about 5000 stades. Historians argue on the manner by which these distances were measured, but some believe that Eratosthenes used professional walkers. It is hard to know for sure what corresponds exactly the unit stade. Despite the controversy surrounding ancient units[3], it is more or less consensual that one stade would correspond to one-tenth statute mile[1]. In this case, 5000 stades would correspond to 500 miles (about 804.6km).

The basic geometrical reasoning of Eratosthenes is illustrated in Fig. 1. Since opposite interior angles of two parallel lines are equal, the angle subtended by the arc between Syene and Alexandria at the earth's center is the same as the angle of the Sun rays inclination at Alexandria (angular deviation of the Sun from the vertical direction in the Alexandria surface).

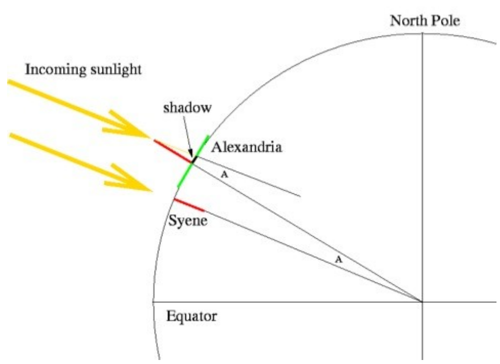

FIGURE 1. The calculation of the earth radius by Eratosthenes[4].

Taking the distance between Alexandria and Syene as 1/50 of the circumference of the Earth, we can readily derive:

$$r \approx \frac{804600 \cdot 50}{2\pi} \approx 6402803\, m. \quad (1)$$

Eratosthenes reasoning assumes that the Earth is spherical, that the Sun is so far away that its rays can be taken as parallel, that Alexandria and Syene lie on the same meridian, and that Syene lies on the Tropic of Cancer, so that vertical objects cast no shadows on the summer solstice. We know today that Syene was in fact 37 miles north of the tropic of Cancer and 3º 30' east of Alexandria. The distance between these two locations is 453 miles, rather than 500 estimated by Eratosthenes, and the difference in latitude is 7º 7', instead of 7º 12'. These inaccuracies in no way diminish Eratosthenes remarkable achievement, given the means at his disposal.

II. PROCEDURE

We designed a middle school classroom activity, specifically for the 7th grade, to explore and understand this historical experiment. The activity occupied two lessons, each lasting 90min. The lessons were divided into a lecture session and a lab session. The contents for each lesson are in Tables I and II.

The lab sessions were designed around an interactive model of laboratory classes that combine hands-on-activities in an active learning environment: the lab stations.

TABLE I. Contents and lab stations for the first lesson.

Lesson	Contents	Lab stations
1	Earth movements Variation of temperature Time zones	1 2

TABLE II. Contents and lab stations for the second lesson.

Lesson	Contents	Lab stations
2	Sun rays inclination	1
	Shadows length during the day	2
	Seasons (introduction)	3
	The radius of the Earth by Eratosthenes	4

Altogether the activity contained six lab stations that we will describe in detail, following a brief introduction to the concept of lab stations.

III. LAB STATIONS

Laboratory activities play a central role in the learning of physics regardless of the educational level[5]. However, despite their proven advantages, teachers avoid them (preferring demonstrations) because they require a significant amount of material, and take a long time to prepare and to implement.

In this model, the classroom/laboratory is divided, usually, into few experimental stations, each one with different apparatuses/materials; students, divided in groups of three or four, travel from station to station with a pre-defined time frame. The lab stations must be independent because each group performs them in a different sequence.

The stations may be diversified and can range from simple tasks (such as simple measurements) to tasks that involve a higher cognitive ability, like testing hypotheses, constructing explanatory models from observations or solving open-ended problems.

At every station, students also have to answer some theoretical questions from a lab worksheet. The teacher serves as a supervisor, being aware of the students' difficulties, monitoring the discussions within groups and thus providing a personalized learning.

These lab stations have several advantages when compared with typical lab sessions; to start, a reduction of the material requirements, a formative assessment, a better work flow from students and a higher motivation. The variety of experiments and the possibility to develop different types of abilities, like conceptual and quantitative understandings of physics principles and science process abilities also contribute for the advantages of this type of sessions. They are adaptable to any syllabus and can provide a personalized learning environment.

All the material is stored in kits properly identified to facilitate the subsequent use, so teachers spend less time in preparing and storing the material. In each kit, the material is organized by lab station (Fig. 2).

FIGURE 2. Kit with the material required, organized by lab station.

The lab stations presented here can be easily constructed by teachers with few and low-cost materials. Students spend 10 min at each lab station.

A. Lab Station 1 (1st Lesson)

The first two lab stations were implemented in the first lesson, after the lecture session. An overhead projector was used to simulate Sun; a thermometer, a globe and some flags indicating several countries (Fig. 3) constituted the remaining material.

FIGURE 3. Picture of the Lab station 1 (1st lesson).

The questions and instructions related to this station were as follows:

1. *Look at the flags in the globe. In which countries is day time?*
2. *In which country did the sunset occurred first?*
3. *In which country did the sunrise occurred first?*
4. *Indicate the cardinal points (north, south, east, west) of each country represented, in relation to Portugal.*
5. *Measure the temperature of a country where it is still day time and of another country where it is night. What conclusions can you draw?*

B. Lab Station 2 (1st Lesson)

This station is related with the Earth movements and time zones. The students had at their disposal many small size globes (Fig. 4) were asked to simulate the three movements already learned (rotation, translation and precession) with their peers. Afterwards, they simulated the three movements in the teacher's presence, for assessment. In the second part of the lab, the students made a prediction of time in some international locations (taking Lisbon time as reference) with the help of a normal size globe and a map of time zones.

FIGURE 4. Picture of the Lab station 2 (1st lesson).

Here are the questions and instructions:

1. *Indicate the Earth movements and simulate them for your teacher.*
2. *If it is the teatime in London, the famous Big Ben strikes 5 p.m.. What time is it in Paris?*
3. *If it is 10 a.m. in Lisbon, what time is it in...*

a) *... London?*
b) *... Azores?*
c) *... Brussels?*

4. *In 2008, the Olympic Games were held in Beijing. The Portuguese triathlon champion and five-times champion of Europe, Vanessa Fernandes, held its race at 10 a.m. (local time). What time was it in Lisbon then?*

C. Lab Station 1 (2nd Lesson)

This lab station was designed to study in a real situation context the shadows length during the day. The overhead projector represented the Sun and a small stick (made by rubber) was placed on the globe, in Portugal (Fig. 5).

More specifically, the questions and instructions given were:

1. *Place the small stick (made by rubber) in Portugal. Turn on the overhead projector to simulate the sunrise in Portugal.*
2. *How does shadow length changes during the day? (Simulate with the globe)*
3. *If you are facing your shadow when its length is minimal, which is the cardinal point that you see ...*

a) *In front of you?*
b) *Behind you?*
c) *On your right?*

d) *On your left?*

FIGURE 5. Picture of the Lab station 1 (2nd lesson).

D. Lab Station 2 (2nd Lesson)

The concept of shadow and its main features is studied in this station. Here, the students explored the size of a shadow by moving an object closer or farther to a light bulb and, as the source has a none-negligible size, they could see the two distinct regions in a shadow; one of full-shadow, called the umbra, the other of half-shadow, called the penumbra (the penumbra has a varying, not uniform, shadow density).

To conclude, the students must draw a geometrical sketch where they have to identify the two regions. This station was designed with the next lesson in mind, where eclipses were discussed. Here are the questions and instructions in the lab worksheet:

1. *Why do shadows appear?*
2. *Explore the size of a shadow by moving an object closer or farther to the light bulb. What conclusions can you draw?*
3. *Make a geometrical sketch where you can identify the umbra and the penumbra.*

FIGURE 6. Picture of the Lab station 2 (2nd lesson).

E. Lab Station 3 (2nd Lesson)

The second lesson preceded the study of the seasons. It was intended that students study the Sun rays inclination before receiving any formal instruction on the subject. To explore this concept, a flashlight and a paper sheet with a square drawn on it were available. The main goal was to let students recognize, from their own experience, that increasing the angle of flashlight with the normal of the surface results in a distribution of the same energy in a larger area, *i.e.* a lower energy per unit area.

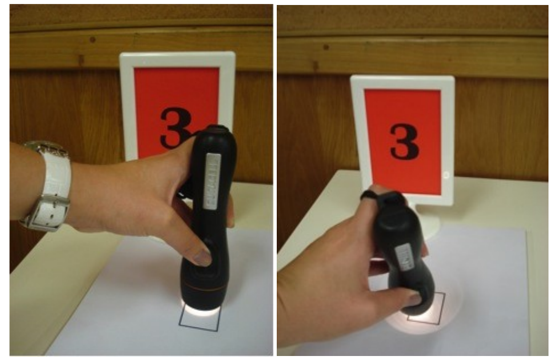

FIGURE 7. Picture of the Lab station 3 (2nd lesson).

More specifically the questions and instructions were:

1. *In which situation does the square take more energy?*
2. *In which situation does the sheet take more energy?*
3. *Which situation corresponds to a higher inclination of the rays emitted by the flashlight? Justify.*
4. *Which situation can represent the summer? Justify.*
5. *You have two globes in two situations (A and B) and a flashlight in a stand. Turn on the flashlight and see the illuminated area in each case. Focus your attention in the same region, for instance, Portugal.*
   a) *In which case is there less energy per area?*
   b) *Which situation can represent the summer?*

FIGURE 8. This picture shows the globe representing the summer in the north hemisphere.

F. Lab Station 4 (2nd Lesson)

Eratosthenes' method for determining the Earth radius is finally approached in this lab station. It is advisable to talk about this historical experiment in the lecture session, preferably following a discussion of the concept of Sun rays inclination. Fig. 1 is presented and explored in the lecture part.

In this workstation, students had a globe, an ancient Egypt map and a protractor.

Students start by reading a text with a description of the experiment; they proceed to answer the following questions and carry out some instructions:

FIGURE 9. Picture of the Lab station 4 (2nd lesson).

1. *Explain the absence of shadow in Syene in that specific day.*
2. *With a flashlight illuminate the city of Syene in order to demonstrate the summer solstice.*

FIGURE 10. Picture of a student answering the question 2.

3. *In a regular day (not in summer solstice), Syene and Alexandria have shadows. Will they be of the same size?*
4. *If the Earth were flat, the shadows length in Syene and Alexandria, at the same time, be the same?*
5. *In the following diagram there is a representation of the experience of Eratosthenes. In this diagram...*

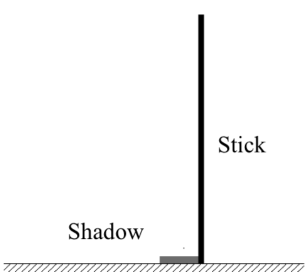

FIGURE 11. Diagram from the question 5.

a) *… draw the Sun ray.*

b) *… find the angle between the Sun and the vertical direction.*

6. *Calculate the Earth radius through this experience.*

IV. CONCLUSIONS

The methodology implemented in this activity provided students with an active learning experience and a reasoned understanding of some important physical concepts. The lab stations allow students not just to understand Eratosthenes' method for determining the Earth radius, but also to deal with problems and questions by combining reasoning with observation, manipulation, and critical attention to detail.

Undoubtedly, there is a gap between seeing and understanding, but the manner in which the laboratory lesson is staged and intellectual activity is organized around the experiment can help to bridge it[6]. The lab stations were not independent hands-on activities; all of the steps were important ingredients designed in a reasoning line of questioning.

The presented pedagogical approach to the Eratosthenes' measurement of the Earth's radius helps students to construct gradually their knowledge by themselves, but, it is necessary that this type of teaching strategy continue with other subjects during the rest of the year, in order to be effective[5].

The assessment was an important tool of this lab session. The teacher was able to accompany in the lab all the discussions within groups and intervene with more questions, in order to promote conceptual change and a reasoned learning. The lab worksheet was also evaluated, so the teacher could be made aware of the students' main difficulties.

The students enjoyed this session and felt more motivated. However, they argued that the time allotted for each lab station was too short. In some cases, they asked more time to perform the experimental tasks and to discuss within the group; the answers, according to them, would be more complete and better consolidated.

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