Geometry and laboratory students

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Abstract
Students in General Physics laboratories have a particular way of plotting graphs, evaluating variables, using equations, and so forth, which pretends to show what they learned in their math courses. That is evident in drawings and analyses that show an unusual treatment of experimental data. Interviews with the students concerned confirmed a lack of physical interpretations of the data to be plotted, as well as non existent connections between the objective of the experiment and the graph of the data. Some of them expected to follow an algebraic procedure to get an acceptable and "mathematically correct" solution. The students' responses to a brief test on the uses of graphs showed that the emphasis on graph plotting in engineering courses is far below the needs of engineering practice in research labs or industry.

Keywords: Laboratory, Geometry, Graphs, Kinematics.

Resumen
Los estudiantes de laboratorio tienen una forma particular de graficar, evaluar variables, utilizar ecuaciones, etc. que muestran lo que han aprendido en los cursos de matemáticas. Es evidente en a partir de los dibujos, gráficas y análisis que tienen un tratamiento poco usual de los datos. Para comprender esto, realizamos entrevistas con los estudiantes que confirmaron una falta de interpretación física en los datos que se grafican, así como una falta de conexión entre el objetivo del experimento y la gráfica de los datos, algunos de ellos pensaban que se tenía que hacer un procedimiento especial para llegar a la "solución matemáticamente correcta". Las respuestas de los estudiantes a un pequeño cuestionario sobre el uso de las gráficas, mostraron que el énfasis en la graficación en los cursos de ingeniería está muy lejos de las necesidades de la investigación o la industria.

Palabras clave: Laboratorio, Geometría, Gráficas, Cinemática.

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I. INTRODUCTION

Students in the first General Physics Laboratory courses have a peculiar way of plotting graphs, interpreting variables, formulating equations, and so on, that show what they have learned in their Mathematics courses. The gap between math and its application to lab courses is evident in their analysis and graphs, where we observe a forced, and many times wrong, treatment of experimental data. To extrapolate the math to the analysis of graphs and equations obtained from experiments, is one of the skills that a student should acquire in his lab courses.

It is a well-known fact that context is crucial for problem solving and comprehension [1, 2, 3, 4]. The laboratory poses a learning situation where mathematical knowledge is applied beyond its original context, which implies that students face a transference of knowledge, that is not fulfilled in a direct way, and before arriving at a full transference, they develop what we use to call a "lab geometry" that is part of the "lab math". In this paper we intend to show that many of the mistakes are due to an incorrect extrapolation that happens when changing context, but once the student manages to do a correct "translation" of mathematics to physics, the mathematical treatment is done as we would expect from an engineering student.

In our work we analyzed the lab reports that students hand in every week, and the in-depth interviews that are scheduled with our students in the personalized instruction labs the authors have taught for the last five years. We work in a semi- presencial environment, and in our courses from 2008 to 2010 we designed this experimental research on 86 students. In their interviews we identified some cognitive obstacles [5, 6] related to the transference of math to the analysis of graphs plotted in the lab. We have classified those obstacles into four main sections, which we discuss in what follows.

II. GEOMETRY

A. Horizontal or Vertical?

The first difficulty a student faces when asked to plot a graph from a data table, is to assign one variable to the horizontal axis and another to the vertical one. We recommend using the horizontal axis for the independent variable, but if he doesn't know for sure what the right variable is, it is not an easy task for the student to choose one for the horizontal axis. This comes from the math course experience, in which a function is given for the graph to be plotted. The inverse problem, given a graph to obtain the function it represents, is seldom presented.

On the other hand, in experimental work, the process is often traveled in the opposite sense: you start with a graph, and try to obtain an empirical equation, or at least a proportionality constant for a given range of values. For instance: to obtain the elastic constant of a coil spring (Hooke's law) the student performs the experiment and obtains a collection of data on applied force and spring elongation. The laboratory instructions not always identify the independent variable and the student will make an arbitrary choice to plot force in the horizontal or vertical axis. Only when he thinks over what was asked of him, he may find a physical reason to assign force to the horizontal axis. Incidentally, we observe that with engineering students, it makes more sense to use the "systems approach": the independent variable is the *input variable*, and the dependent variable, the *system's response*.

We do not want to engage in the discussion of the intercept, because it would be leading us astray from the objective of this work. It is enough to add that the identification of the slope with a "systematic error" causes some confusion to the students, which will then try to make the intercept of their graph equal to zero, which will obviously be a mistake.

B. Location of the origin

The second problem a student has to solve is to assign a coordinate origin, because not every experiment starts in the point (0, 0); in some of them either one or both variables are not zero at the beginning of the experiment. This is the only difficulty the students can overcome with common sense: some assign a null value as origin, and after that make a "cut" on the axis, as they see in many technical illustrations. Others prefer to use as origin the first data point in their table, and finally others make a change of variable such that they have a zero on the origin. Each one of these solutions has its advantages, and they have to be discussed with the students to identify which one is the best for their particular case. Besides, in many experiments, such as Hooke's law or the simple pendulum, even if you have no data near the origin, you must extrapolate the graph up to that point, to have an idea of how the function behaves near the origin, to avoid adjusting a straight line to what is actually a segment of a curve.

C. Scales on each axis

Once the student gets the data from an experiment, he has to plot a graph, but first he has to decide what scale he is going to use. A surprising finding is that in the majority of cases analyzed in the interviews (84%), the student try to use the same scale in both axis, that is to say that, if in the horizontal scale a given interval between two lines represents, say, one second, then in the vertical scale the same interval must represent one meter (or maybe one centimeter). Thus, the student thinks, he gives "corresponding" scales to both axis, and the second distance can *never* represent 2m or 2.5m, let alone 5Km.

We think that this attitude arises because, when you plot values in the math class, units are not taken into account, or if ever, it is implied that the scale on both axis is always the same; this is not necessarily so, but is never discussed in class. That would be immaterial, if it were not for the fact that in some cases it will avoid to give the correct proportions to the graph, and the student will plot a deformed graph to describe his experiment, and may not arrive to a correct physical interpretation.

D. Identifying the variables

As it happens in kinematics problems solving [7], students have an obstacle to match the geometry learned in the math class to the geometry of the graphs that appear in the lab. The most evident example of this is the straight line equation. Almost none of the students interviewed could match the variables used in the experiments to the variables x, y in the standard slope-intercept form of the straight line equation. Thus, in the Hooke's law experiment, or in finding the average speed of a moving body, or any other linear process, we found a kind of reserve, a lack of credibility, or even a rejection of the idea that you can identify the variables as describing a straight line, even if they are not named x and y. The authors some times insist that you must use as names of the variables in the experiment the initials of the physical magnitudes (l for length, t for time, F for force, and so forth). In that way, the need to specify the *units* is evident, which is not true for x and y. This is another cognitive obstacle that, as happens with the other two, has not been sufficiently discussed in the literature.

III. A COMPLICATED EXPERIMENT

Students in this study performed a low cost experiment, outside the university and with their own means, which is unlike the traditional laboratory environment. This is an actual experiment, without a predetermined outcome; in it, the student will obtain his average speed, going from his home to the campus. For that purpose, he must choose at least six points on his path, and with a stopwatch, record the time at which he passes over them. His home is the first point, and the entrance to the campus the last one. He is instructed to make a table, in order to plot a graph, to identify the kind of movement he does in going through his path, and from the graph obtain the best estimate of his average speed. He is also asked to use as units meters and seconds. The novel feature in this case is that *he* is the "moving particle". A more practical interpretation is that the experiment is about obtaining the average speed of public transportation in a particular area of Mexico City.

From those simple instructions, the student will make an interpretation of the text, as well as of the experiment itself, and he is interviewed by the instructor, to get feedback on those aspects he did not quite understand. Results are diverse, and can be analyzed from two different approaches: From the point of view of the influence of the context [8], as appears in the research on Latin-American college students by authors like Benegas [1] and Buteler [4], and from the point of view of the "individual geometry" of each student, which has received little attention in the literature.

Our work is centered on the above mentioned geometry, which the student will use to try finding some regularity in his experimental data. There are, of course, technical aspects like determining the scale of a map (although, in recent times, we have more students using Google Earth, or even a car fitted with a GPS), or reading a stopwatch, which are of marginal importance in this paper.

The first problem they face is of course to assign an origin to the time axis, if it does not start at zero. Most students present the solutions discussed above; in the interviews, we find that, not having a clear objective (obtaining an empirical equation), some will make a "cut" in the time axis, others will set at zero the time they started to measure, and only a few make a change of variable to get a better description, intended to provide the equation from which the average speed will be calculated.

The students present in the interview session their data tables and a sketch of the graphs they try to plot from them. A typical table looks like Table I (actual student data).

The tabulation required must have positions measured from home in a column and times to reach that position on the other, the times being taken from home to the current position. The names in the interval column designate places in northern Mexico City.

After obtaining the scale of the map, or the measuring device, they present a table in which the position is given in one of the following formats:

a) Position in mm, on the map.
b) Distance in mm, on the map.
c) Point-to point distances, in m, without a common origin
d) Distance from the origin (home) in m.

As for the time, the options are similar:

a) Time in h, min and sec.
b) Point-to point time differences, in min and sec, or in minutes, seconds and hundredths (from a digital stopwatch or cell phone).
c) Time differences, converted to seconds
d) Time in seconds from the origin to the current position.

TABLE I. Point-to point time and distance data.

point	Inter-val	Time (s)	d (m)	Speed (m/s)
1	Home (Tlal)	0	0	0
2	Tlal Pte Vig	480	2333.25	4.86
3	Pte Vig CCH	660	4088.25	6.19
4	CCH Ros	780	4493.25	5.76
5	Ros Bus stn	420	4664.2	11.11
6	Bus stn UAM	840	6547.5	7.79
	Sum	3180	22126	35.72
			Average	7.14

In both instances, cases c) and d) are the ones with physical meaning, but only d) gives useful values to plot a meaningful graph to show the trajectory of the moving "body" (the student himself). The illustrated student data are of type "c", and this brings out another conceptual problem: opposed to his intuition, plotting this data may give a re-entrant graph, because distances are arbitrary and the speed in each interval is variable (See Fig. 1 below).

When a student produces a table like the example, it is easy to extract from it the values referred to the origin. We only have to add the values in each row to the preceding one, as shown in Table II.

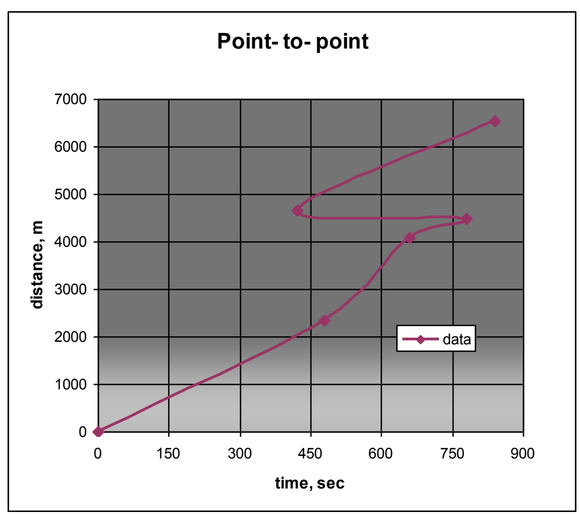

FIGURE 1. Graph of the data in Table I.

If, on the contrary, the student presents a type d) table like Table II, the point-to-point times and distances are obtained from it by the inverse process of taking differences from each row and the one above. We encourage the student to present both tables, because the former allow us to calculate the point-to- point speeds (last column). The average speed is obtained averaging over the column.

TABLE II. Accumulative times and distances.

Σt, sec	Σd, meters	speed, m/s	slope, m/s
0	0	0	0
480	2333.25	4.86	4.86
1140	6421.50	5.63	6.19
1920	10914.75	5.68	5.76
2340	15579.00	6.66	11.11
3180	22126.50	6.96	7.79
	Average =	**5.96**	**7.14**

It is, as we said, Table II which allows us to plot a significant graph (Fig. 2). In it we get the average speed by fitting a straight line and calculating its slope. In both tables, the first row is a dummy (time and distance from home to the origin), which is included to have the student include point (0, 0) in its graphs.

In Table II, point to-point slopes are calculated, to compare with the average speeds of Table I, and also to stress the relation between average speed and slope of the segments. In this particular case, a good agreement is obtained by both methods. The student also must obtain uncertainties, using the uncertainty in the measurement of distance, and the reaction time in starting and stopping the stopwatch. Those data were not included, because we considered them irrelevant to our discussion. To the most advanced students, we also suggest fitting a straight line by least squares. The least-squares fit is included in the spreadsheet graph, to compare the student's result with it. In this way, we complete the learning process.

Coming back to the contextual comprehension difficulty, to begin with, the student does not, as other authors have pointed out, use the previous knowledge on graphs acquired in the same course [9, 10]; i.e., he makes a sharp distinction between reality and the classroom; he seems not to respond to any objective, but rather responds to what the teacher requested, ignoring if the shape of the graph will help him understand the motion of the body.

In the interviews we bring face to face the student with his geometrical knowledge, to make him realize what we want to graph. But hey show a poor knowledge of geometry and basic algebra, as well as of what information you can get out of a graph. Our students, in spite of having time and position data, could not plot the graph in 83% of the cases (n=71).

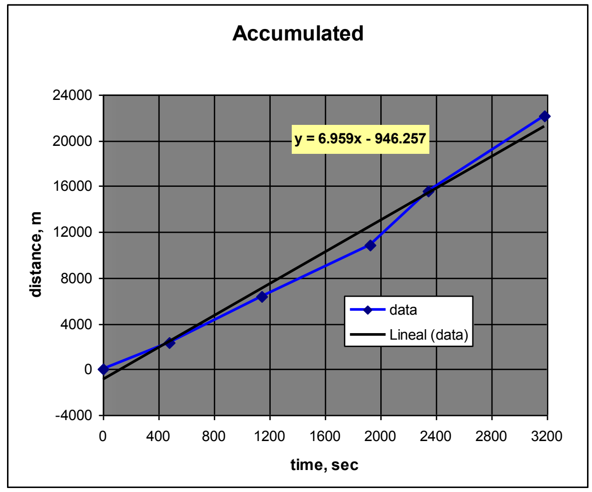

FIGURE 2. Graph of the average speed by fitting a straight line and calculating its slope.

When the discussion with the student allows him to plot an adequate graph of x vs. t, he should be able to propose a function, but again a cognitive obstacle comes out, and the student is unable to formulate an equation for the graph. We have to discuss the subject further, to make the student write an equation, and get the average speed in his trajectory from his home to the campus.

IV. CONCLUSIONS

Most students are flabbergasted by the mathematical analysis of simple experiments, and they don't know how to plot their data, in spite of having a table of values The analysis of their difficulties in handling algebra and geometry, shows us that they are not simple mistakes; we are dealing with a situation linked to what is called a "distant transference" of knowledge [9] and the change of context to which the knowledge is applied; when facing this situation, the student develops an *ad hoc* geometry for the problem in question.

Another aspect we did not detect in the revision of lab reports, but came out in the interviews, was that in the "individual geometry" of the student it is not realized that we can extract from a graph any information beyond "whether it represent a curve or a straight line".

The graph plotting process is seen as useless work, having a sense only in lab courses, but not in real life, in the professional activity of an engineer. We have no doubt that this is one of the reasons why experimental work has so little success in some universities: its usefulness is not foreseen in professional life.

Those interviews confirmed a lack of physical interpretation of the data to be plotted, as well as an inexistent connection between the graph of the data and the objective pursued in the experiment, because some of them were expecting an elaborate algebraic treatment to arrive to an acceptable, "mathematically correct" solution.

In personalized interviews we have verified that it is the discussion of the experiment from the points of view of physics and engineering which allows the students to make a good transference of their mathematical knowledge, and forsake the "mathematics of the laboratory".

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