Measuring g: An inexhaustible source of instruction and creativity

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Abstract

Swinging a simple pendulum is a common experiment to measure gravity acceleration in high school and college physics courses. However, the instructiveness of this lab work is essentially increased when students exercise other methods to measure g in class or as research projects. In particular, conical pendulum proves to be a more versatile experimental technique: students do not face experimental limitations inherent to the oscillating pendulum, like small angles of swinging. Furthermore, mathematics of the conical pendulum is much simpler. Video clips recorded with cell phones, digital cameras together with trackers or alike software, proved being practical to reduce experimental errors. In college physics labs, the physical pendulum is a "must" apparatus to measure g. It is instructive to diversify the pendulums' designs, up to the real oddly shaped plates scaling maps of the countries, states and cities. Finding their center of gravity and moments of inertia is an additional experimental challenge. Circular and rectangular cards suggest nice exercises to deduce physics formulas; among the most amazing physical pendulums is the "partial ring" apparatus. A quite different method to measure g in a simple hydrostatic experiment is a perfect application of students' knowledge of gas laws. They lower a straight test tube open end down into a tank of water. Due to the pressure of the captured air, water level inside the tube never reaches that of the tank. To determine g, students must measure the depth of the tube's lowering and water level inside the tube; current value of the atmospheric pressure must be known. We suggest attaching the tube to a bigger air-filled container, which dramatically increases accuracy of measurements and opens a variety of experimental possibilities. All the discussed activities were practiced in real educational conditions. Didactical outcomes, students' and in-service teachers' feedback are analyzed.

Keywords: Gravity acceleration, conical pendulum, physical pendulum, gas laws, atmospheric pressure.

Resumen

Un experimento muy usado en las escuelas secundarias, bachilleratos y universidades para medir la aceleración de la gravedad es el péndulo simple. Sin embargo, la enseñanza obtenida con este experimento de laboratorio se incrementa notablemente cuando los estudiantes ejercitan otros métodos para la medición de g, ya sea en clases o como un proyecto de investigación. En particular, el péndulo cónico es una técnica versátil para este fin: los estudiantes no se encuentran con las limitaciones experimentales inherentes al péndulo simple, como los ángulos pequeños de oscilación. Además, las matemáticas involucradas en el péndulo cónico son mucho más sencillas. Los errores experimentales se reducen grandemente si se usan cámaras digitales de video o teléfonos celulares juntamente con software como *tracker* o similares. En los laboratorios universitarios, el péndulo físico es un dispositivo obligatorio para medir *g*. Es instructivo diversificar los diseños de los péndulos, por ejemplo en forma de mapa de los países, estados y ciudades. Encontrar el centro de gravedad y momentos de inercia es un reto experimental adicional. Las formas circulares y rectangulares sugieren ejercicios para deducir formulas; entre los péndulos físicos más asombrosos está el de "anillos parciales". Un método muy diferente para medir *g* en un experimento hidrostático sencillo es una aplicación perfecta del conocimiento de los estudiantes de las leyes de los gases ideales. Los estudiantes sumergen un tubo recto abierto de un lado en un recipiente con agua. Debido a la presión del aire capturado, el nivel de agua dentro del tubo nunca alcanza el nivel del tanque. Para determinar *g*, los estudiantes deben medir la altura del tubo sumergido y el nivel de agua dentro de este; se supone conocida la presión atmosférica local. Sugerimos unir el tubo a un contenedor más grande lleno de aire, el cual incrementa dramáticamente la precisión de las mediciones y aumenta las posibilidades experimentales. Todas las actividades mencionadas fueron realizadas por estudiantes en condiciones reales. Se analizan los resultados didácticos y retroalimentación de maestros.

Palabras clave: Aceleración de la gravedad, péndulo cónico, péndulo físico, leyes de los gases, presión atmosférica.

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I. INTRODUCTION

Definitely teaching Science is different than teaching other subjects. The entire process of doing science is different. Also the means by which knowledge is acquired is not the same in Science than it is in History or Mathematics or Poetry. On the other hand, teaching Physics at the basic school levels is a real challenge for educators. One of the main teachers' problems is to maintain student's attention during the class. In fact, there is a worldwide decrease in learners' interest towards the scientific subjects, in particular Physics [5, 6, 7, and 8]. In order to maintain the students´ interest, it is necessary that teachers make more attractive the topic presented to the class, otherwise they get the idea that Physics is only a set of formulas good only for solution of some pretty abstract problems. Many teachers still use the simple didactical scheme of mostly giving the class data to substitute into the formulas and find some values. That drives students to an idea of Physics being just another branch of mathematics or an algebra practice.

In a Physics class, laboratory work is essential. Experimental projects are an opportunity for educators to make Physics more interesting to students. Experiments that involve knowledge of many parts of physics are important to develop creativity and to make knowledge more significative for them [8]. The laboratory of Physics is also the place where students get the opportunity to *do* physics. It is in the laboratory that students learn to practice the activities of scientists - asking questions, performing procedures, collecting and analyzing data, proving models, and finding the new questions to explore. Physics teachers should exploit the laboratory projects to teach in a more interesting manner this subject. It is also important to do the experimental work with materials easy to find, otherwise students get the impression that real science can be performed only with sophisticated apparatuses.

Getting students involved in hands-on, minds-on learning is pedagogically beneficial for a significative learning without the need of a constructivist approach which may be frustrating for most students [9].

In what follows, we propose modifications to the traditional methods of measuring the acceleration of gravity to arouse students' interest and develop their creativity by proposing new ideas and identifying sources of errors in the suggested experiments. Projects were performed by students both in Mexico and Ukraine.

II. DIDACTIC EXPERIMENTS TO DETERMINE THE ACCELERATION OF GRAVITY

Measuring gravity is an important activity because didactically can be done from many fronts: for example, in high school physics courses, the simple pendulum oscillations and the free fall of objects from certain height are common experiments to determine *g*. Students and teachers have a well known study of these procedures giving the feeling that teaching physics is a static activity and that the same experiments are carried out over and over every year. This feeling causes students to lose their interest in this important subject.

We present here a few experiments to determine the acceleration of gravity, *g*, starting from the classical simple pendulum and free fall for comparison. The experiments we suggest are the the physical pendulum in a variety of shapes like an oscillating playing card and also the use of a tube introduced into a tank of water and measurement of the height of the water inside the tube. With the use of the gas laws, students can determine the value of *g*. These experiments are barely used by physics educators, missing their didactic potential. The aim of this diversification of experiments to measure the same important constant is both, to give students the opportunity to practice previous knowledge like the mathematics involved in determining the moments of inertia of the objects and to make physics education more appealing to both teachers and students. In this series of experiments students made the set up guided by the instructor. The analysis of results was performed by students using basic statistics like linear regression in the *Excel* spreadsheet and compared with those obtained in the experiments involving more accurate software like tracker and a cell telephone camera. However, some of the experiments were left as out-of-class projects for the students and results were discussed in a session held at the school. The experiments were also discussed with physics teachers to motivate them to use modern and simple technology to analyze experiments in the laboratory without the need of expensive equipment.

Results of the activities are discussed in the next section.
Students in both countries found these activities interesting and motivating.

The reference value

Students were asked to calculate the value of *g* from the mean radius and mass of the Earth in order to compare their experimental results with this theoretical value [10]. Using $R=6371km$, $M=5.9736\times10^{24}kg$, and the formula

$$g = G\frac{M}{R^2}, \qquad (1)$$

where *G* is the Universal Constant of gravitation, $G=6.67\times10^{-11}Nm^2/kg^2$, they were able to get a value of $g=981cm/s^2$.

It is very important to have a reference value from which students can judge their results.

We describe briefly the experiments performed by students from Mexico and Ukraine, taking this theoretical value of *g* as a reference.

A. Free Fall Experiment

The classical experiment of an object falling freely from a certain height h was performed by the students, but instead of measuring the time with a chronometer or with a switching devise, they used the software *Tracker* from the University of Cabrillo [11], giving an accuracy in the measurement of the time of *1/30s* (which is the speed of the video: 30fps).

Students plotted h *vs* t to get g from the coefficient c of the equation

$$h = ct^2 + dt + e. \qquad (2)$$

The object was dropped freely from a height h of 1m and its motion recorded with a cell phone to be analyzed with Tracker as shown in Fig. 1.

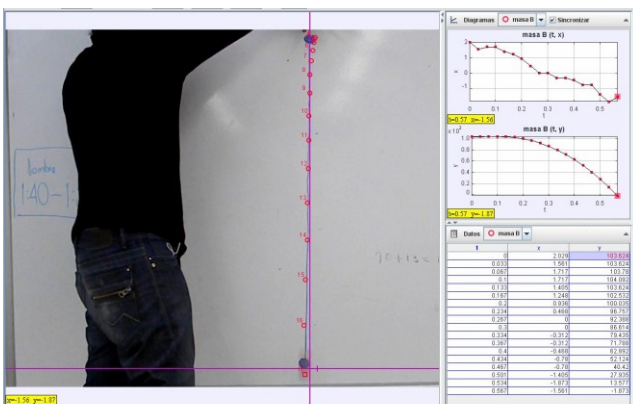

FIGURE 1. Analysis of the free fall experiment with Tracker.

B. Simple Pendulum

Students performed also the classical experiment of the simple pendulum oscillations. The suspended mass used for this experiment was a plasticine ball of 50g. For the pendulum, they repeated the experiment with different lengths of the string and plotted T^2 *vs* l and used linear regression to adjust a line to the data with the *Excel* spreadsheet. First they measured the period counting ten oscillations, and then they recorded the experiment and analyzed it with Tracker.

From the equation of the simple pendulum, for small oscillations:

$$T^2 = \frac{4\pi^2}{g} l. \qquad (3)$$

Students were able to calculate the value of g from the slope of the line obtained.

C. Conical Pendulum

Conical pendulum is not used by most teachers in the measurement of the acceleration of gravity g, missing their pedagogical value. Furthermore, the mathematics involved in the conical pendulum is easier than that of the simple pendulum and it is an excellent physics and mathematical exercise for students to deduce the exact equation for the period of rotation [1]:

$$T = 2\pi \sqrt{\frac{h}{g}}. \qquad (4)$$

Here h is the height of the cone circumscribed by the string when the pendulum rotates, as it is shown in Fig. 2.

It is relatively easy to perform the experiment marking a circle on the floor or on the table and then holding the upper end of the string directly above the centre of the circle and twirling the plasticine ball in a horizontal plane.

The experiment was performed using a string of 1 m length and a plasticine ball of 50g. Students analyzed the motion of the ball using a cell phone camera to record the motion and then using Tracker and the *Excel* spreadsheet to determine the period of the pendulum. Results are shown in Table I.

It was a new experience for the students to use in this form the pendulum to determine g.

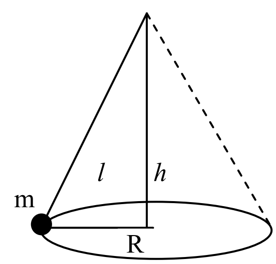

FIGURE 2. Conical Pendulum.

D. Physical Pendulum

The physical pendulum is a good didactic and pedagogical instrument for educators; students are able to foster their creativity suggesting different shapes for the oscillating object. The oscillations of the pendulums were analyzed again using Tracker. Calculations of the moments of inertia are good mathematical exercise for students. The first physical pendulum they analyzed was a card with a pinhole [2] made along the main diagonal at a distance L from the center, as shown in Fig. 3.

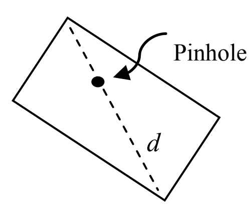

FIGURE 3. Physical pendulum made from an index card.

Measuring the period T of the pendulum oscillations, it can be shown that the acceleration of gravity, for small oscillations, is given by

$$g = \frac{\pi^2 (d^2 + 12L^2)}{3LT^2}. \qquad (5)$$

Here L is the distance from the center of the card to the pinhole, d is the length of the main diagonal [2].

The second physical pendulum students used to calculate g was a ring made of wood of 6.16g of mass and of 6.1cm radius, as shown in Fig. 4.

A surprising fact for students is that the period of this oscillating pendulum does not change if one part of the ring is cut [3], as can easily be shown. Students can prove that the period is given by

$$T = 2\pi \sqrt{\frac{2R}{g}}. \qquad (6)$$

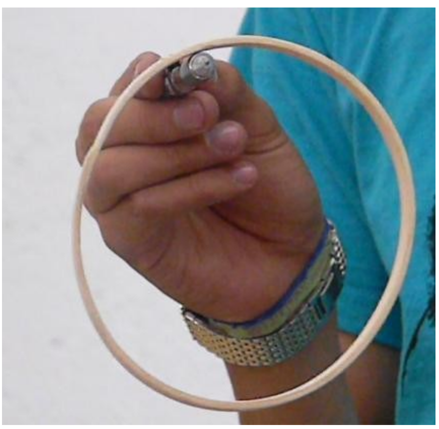

FIGURE 4. Student holding a ring whose oscillations were analyzed with Tracker.

Measuring the radius R of the ring and the period T of its oscillations, students were able to calculate g.

Again, it is a good physical and mathematical exercise for the students to verify that no matter the length of the circumference of the partial ring, the period is still the same.

FIGURE 5. Student performing the experiment with a partial ring.

Another physical pendulum proposed to determine g was a rod made of wood oscillating through a pin in one end. If the rod has length l and radius r, then students can easily show that the acceleration of gravity is given by

$$g = \frac{2\pi^2 (4l^2 + 3r^2)}{3lT^2}. \qquad (7)$$

For the rod used by the students, $l = 32cm$ and $r=3.5cm$.

Limitless educational resources of physical pendulums may be taken advantage of in this experiment. It was a challenge for students the calculation of moments of inertia and to deduce the expression for g. This experience helps also students to understand the physical meaning of the moment of inertia and apply their mathematical knowledge.

Among the suggested physical pendulums are the plates shaped as maps of countries. Students must determine the moment of inertia of the given pendulum, a challenge by itself. Fig. 7 shows an apparatus to measure the moment of inertia of a given object of irregular shape.

Measuring the periods T and T_o of torsional oscillations with and without the object on the disc, it is possible to determine the moment of inertia of the unknown object (the map in our case) [13].

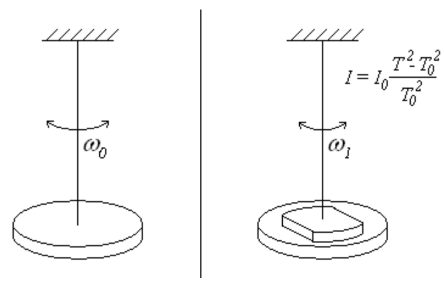

FIGURE 6. Apparatus to determine the moment of inertia of a given object with respect to an axis which coincides with string holding the two objects.

We report here the experiment performed by a team of Mexican students using a map of Mexico made of cardboard with a mass of 30g and moment of inertia determined by this method: I=1605 ± 85gcm^2.

The period for the oscillations of the physical pendulum is given by [10]

$$T = 2\pi \sqrt{\frac{I}{mgL}}. \qquad (8)$$

Where I is the moment of inertia, m the mass and L is the distance from the centre of gravity of the map to the axis of rotation. The centre of gravity was determined by the students using the standard method of suspending the object with a *weightless* string from two different points and checking the point where the traces of the string intersect.

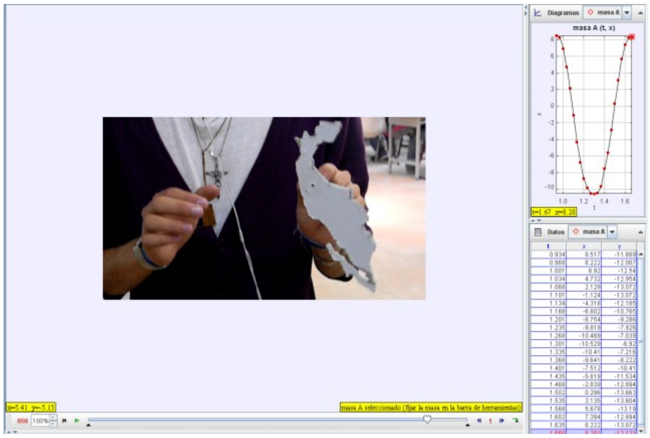

FIGURE 7. A map of Mexico oscillating around an axis passing through the "Mexico City". Tracker analysis is shown.

E. Alexander´s Diving Bell

Finally, students determined the acceleration of gravity using the Alexander´s diving bell [4]. The didactical advantage of this experiment is that students employ, besides the hydrostatic equations, also the ideal gas laws. In this experiment, they pushed a long tube corked at the top end straight down into a water tank with transparent walls. Due to the extra pressure of surrounding water, the level of water inside the immersed tube goes up though remains lower than that in the tank because of the resistance of the air captured inside the tube.

Figs. 8 and 9 show the experimental set up required for this experiment [4].

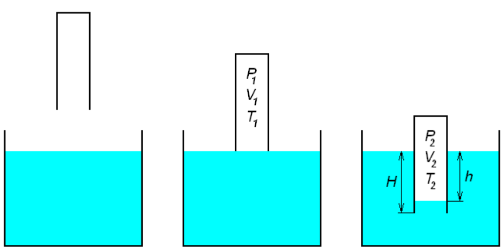

FIGURE 8. The Alexander´s diving bell experiment to measure the acceleration of gravity. The test tube is submerged open end down into a water tank with transparent walls. By measuring *h*, it is possible to determine *g*.

From the Clapeyron-Mendeleev equation, we get

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}. \qquad (9)$$

If the water temperatures equals to that of the air (indeed they differ very little [4]), we can assume $T_1 = T_2$ and then

$$P_1V_1 = P_2V_2. \qquad (10)$$

And also $P_1 = P_{atm}$, the atmospheric pressure, while $P_2 = P_{atm} + \rho gh$, according to Fig. 8. Substituted to Eq. (10) that gives:

$$\rho gh = \frac{P_{atm}(V_1 - V_2)}{V_2} = \frac{P_{atm}\Delta V}{V_2}. \qquad (11)$$

Since it is possible to know the P_{atm} value from on-line sources [13], then *g* can be determined by immersing the tube into the water at different depths *h*, and plotting $\frac{P_{atm}\Delta V}{V_2}$ vs *h*. The graph is a straight line with slope ρg.

The major problem here is that for the realistic lengths of the immersed tubes the difference *H-h* is too small to be measured accurately. We recommend increasing the amount of the captured air since that results in bigger (and thus easier to measure) *H-h* differences. In fact, for the given cross-section of the tube, there is an ideal volume for which *H-h* can be measured with the highest accuracy. To increase the volume of the compressed air, another container was attached to the tube with a hose, as shown in Fig. 9.

Fig. 10 shows students performing this experiment as the Introductory Physics labwork at Kharkiv Kharazin University.

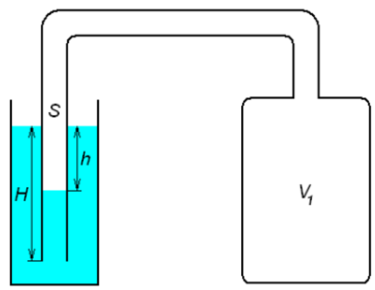

FIGURE 9. The volume of the tube was increased attaching a recipient with a hose. The distance *H−h* is dramatically increased with this adaptation.

FIGURE 10. Students performing the Alexander´s diving bell experiment to measure the acceleration of gravity.

Table I summarizes student´s results obtained with the different experiments described.

TABLE I. Values of *g* obtained by students using the different methods described.

Method	g (cm/s^2)
Free fall	976 ± 14
Simple Pendulum with Tracker	985 ± 33
Simple pendulum without Tracker	1125 ± 6.3
Conical Pendulum	930 ± 31
Physical Pendulum (card)	920 ± 45
Physical Pendulum (Ring)	980 ± 23
Physical Pendulum (Rod)	988 ± 15
Physical Pendulum (Map)	969 ± 38
Alexander´s diving bell	985 ± 63

III. CONCLUSIONS

Measuring the acceleration of gravity *g* is a rich source of didactic possibilities for the physics instructor and a good opportunity to rekindle students' creativity. It helps students to view things from a different perspective. The mathematics involved is quite simple and students can deduce formulas easily to calculate *g,* giving them the opportunity to experience scientific work.

Values obtained from different methods were also different; The difference in results fosters also students' creativity when they suggest new ways to improve them; students also think about possible sources of errors when performing the experiments.

With the simple pendulum and free fall experiment, students realize that the value of *g* is independent of the size and mass of the object; this fact still surprises them and motivates them to repeat the experiment several times with different masses and materials. The aim of these activities is to lead the student to the process of scientific inquiry. The idea is to let the students perform the experiment by themselves and that they discover the sources of errors in the measurements and to suggest how they can improve the accuracy. With Tracker that is accomplished using a fast video camera (of more than 30fps), since in the free fall experiment, for instance, the falling object appears fuzzy after falling 1m (4.4m/s approx.).

On the other hand, telling the student what and how to do since the beginning of the experiment, leads to a decrease in his interest [8]. It is important to let him discover how to perform the experiment to reduce the errors in the measurements.

There are still many experiments to measure *g*, which students searched in books and on the web, motivated by what they did in the laboratory. One of the suggestions was to drop a magnet and measure the time taken to travel between two points using the fact that a magnet moving through a coil of wire induces a voltage. The description of this experiment can be found in [14]. Other experiments suggested by high school teachers were the Atwood machine and a vibrating mass attached to a spring analyzed with Tracker.

On the other hand, many events in Nature and human activities are related to *g* and students can use this fact to calculate it. For instance, the Mexican ritual of the *Papantla flyers* could be recorded and analyzed with tracker. Deducing the equations of the dynamics of the motion (crescent-length conical pendulum), is another good mathematical exercise for the students. This analysis gives students the opportunity to study also American prehispanic cultures.

As additional information, the setting of Papantla flyers comprises an 80 or 90 feet pole and five people, one of them an Indian priest. The 4 flyers represent air, fire, water and earth elements, and also the 4 cardinal points. The dancers and the priest climb the pole one by one. Once on top the priest starts playing and dancing on a 9 inch platform, with a flute and drum, and the dancers throw themselves backwards tied to the pole by the ankles. They slowly start their descent, making exactly 13 revolutions around the pole, thus representing the 52 year time span of the pre-Hispanic cosmic cycle (4 dancers X 13 revolutions = 52), after which a new sun is born and life begins again.

This rite is believed to be started 1,500 years ago in central Mexico, as a prayer to the god of Sun, for fertility and good harvest. After the conquest, the ceremonial rite was disguised as a game to avoid the Christian priests to interfere and allow the Indians to continue practice their religion.

It is hoped that the experiments and activities described in this work will be helpful for educators to enrich their didactic repertory.

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