# Precession and nutation visualized

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**Abstract**

The use of visualization techniques has been a valuable instrument for explaining physical phenomena, especially when they are difficult to observe directly in the laboratory. We present a didactic experiment that allows the visualization of precession and nutation patterns generated by a rotating rigid body. The *light painting* technique used in photography is applied to a gyroscope properly adapted with LEDs. The precession and nutation of the gyroscope are registered as light patterns in space that clearly show the characteristic features of this important physical phenomenon, such as the cusp-like motion as well as the looping trajectories. The aim of this work is to provide with new elements that contribute to facilitate the teaching of this important physical phenomenon for all-level physics students.

**Keywords:** Nutation, Precession, Gyroscope.

**Resumen**

El uso de técnicas de visualización para explicar fenómenos físicos ha resultado una herramienta valiosa en la enseñanza de la física, en especial cuando se trata de fenómenos que son difíciles de observar en forma directa. En este trabajo se diseña un experimento didáctico que permite visualizar los patrones de precesión y nutación generados por un cuerpo rígido en rotación. Se utilizan técnicas de fotografía de larga exposición y un giróscopo con adaptaciones apropiadas que consisten en la utilización de LEDs y un entorno con condiciones de iluminación controlada. Se presentan y discuten los resultados correspondientes a diversas configuraciones y condiciones iniciales, obteniendo patrones de precesión y nutación con rizos y cúspides con distintas orientaciones y amplitudes. La aportación de este trabajo está enfocada a la facilitación de la enseñanza de un importante fenómeno cuya descripción matemática no es sencilla.

**Palabras clave:** Nutación, Precesión, Giróscopo.

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## I.INTRODUCTION

Gyroscopic motion is undoubtedly one of the most fascinating phenomena of classical physics. Its significance has been enormous from the theoretical, practical and pedagogical point of view. It is a phenomenon that is familiar to most people since their first experience with the top, a popular toy that most children have had in their hands, and hence have an intuitive understanding of their fascinating motions. The gyroscope is a mechanical system of great importance to physicists and engineers. Its unique movements that seem to defy gravity actually allow us to understand it better, showing us the beauty of the underlying physical laws, such as the conservation of angular momentum and the conservation of energy.

The rich physics involved in the gyroscopic motion is one of the reasons by which its study is an important part in the professional training of a physicist. However, its mathematical description is not simple and its study is considered one of the advanced topics of theoretical physics. Moreover, students are faced in the laboratory with the practical difficulty of observing directly the most interesting details of the gyroscopic motion. This is why visualization techniques applied to this system become valuable teaching tools, not only for the better understanding by the advanced student, but also to make it accessible to students who begin their career in physics as well as in demonstrations to students of precollege grades.

In this work, we present a didactic experiment that allows the visualization of the precession and nutation patterns generated by the gyroscope. We use the so called *light painting* technique [1] applied to a gyroscope under controlled illumination conditions. The precession and nutation of the gyroscope are registered as light patterns in space that allows us to exhibit the characteristic features of this important physical phenomenon. The aim of this work is to provide with new elements that contribute to facilitate the teaching of this important physical phenomenon for all-level physics students.

## II. EXPERIMENTAL DESIGN

### A. Experimental setup

The experimental setup is shown in Fig. 1. It consists on a commercial gyroscope PASCO ME-8960, a digital camera mounted on a tripod, two strobes, a light emitting diode (LED) powered by two small batteries AG13, and a backdrop.

**FIGURE 1.** Experimental setup.

Fig. 2 shows the details of the gyroscope used in the present study. It includes a base supporting the Rotation axis, which at point P joints with a Gyroscope axis. At upper side there is a massive rotating disk and the opposite side there is a counterweight. At the tip of Gyroscope axis there is a LED. The inclination of the Gyroscope axis allows the gravitational force to exert a torque on it, provided that the center of mass (CM) is shifted from the supporting point *P*. The position $z_{cm}$ of the CM is controlled by manipulating the position of the counterweight.

Three angles ($\theta$, $\emptyset$, $\psi$) are used to characterize the dynamics of the system, are shown in Fig. 3. The inclination of the Gyroscope axis with respect to the vertical line ($Z_0$ axis) is measured by angle $\theta$, while the rotation of the Gyroscope axis around the vertical axis $Z_0$ is described by $\emptyset$. The angle $\psi$ measures the rotation of the disc M around the Z axis. These special angles are known as *Eulerian Angles* in theoretical mechanics textbooks [2, 3], and very important in the mathematical formal description of the rigid body rotation.

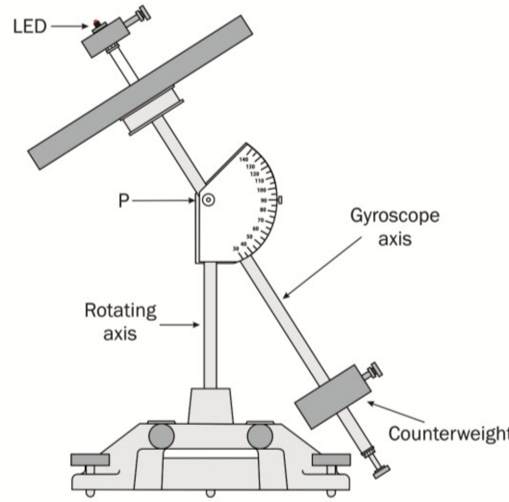


**FIGURE 2.** Gyroscope scheme.

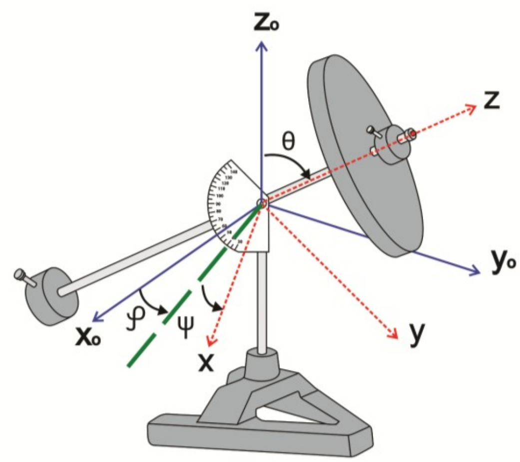


**FIGURE 3.** Gyroscope position, respect to laboratory reference system ($X_0Y_0Z_0$) and body-fixed system (XYZ). The line of nodes (intersection of the planes $z_0$=0 y z=0) is also shown (green dashed curve).

### B. Photographic technique

The LED and batteries are attached at the head of the gyroscope as indicated in Fig. 2. We used small batteries in order to have a negligible mass at the tip of the axis. The chosen position of the camera is in front of the object of photograph, the gyroscope. The plane of the photograph must be perpendicular to the line between the gyroscope and the camera, and such a plane is bounded by the maximum height that gyroscope can reach and the minimum is its base. The lateral width of the plane of the photograph is chosen such that the device fits inside in a complete revolution. Since the light painting technique requires a completely dark room and a long-exposure time, the shutter of the camera is open as long as necessary, *i.e.,* working the camera in manual form.

In order to remove any unwanted glare and provide a homogeneous appearance, a black velvet hanging vertically is used as the background, which has a slight curvature before touching the floor for helping to miss the line formed by the wall and the floor. The color and texture of the fabric prevents the reflecting light from spreading in preferential directions.

In the model of gyroscope used here, the spinning is operated manually by pulling a thread wounded around a small pulley. Once the disc is spinning, the Gyroscope axis starts with its soft movements, so that the LED starts painting in the air the trajectory followed by the upper tip of the axis, which is captured by the long exposure camera, resulting in beautiful precession and nutation patterns.

When gyroscope is placed in the observation plane there are positions where the LED cannot be observed from the camera because it is eclipsed by rotating disc. We set the initial conditions such that the LED is hidden at the initial time so that the camera is unable to record it when we are still holding the axis. When the gyroscope starts spinning, we can also turn off the light and wait for the appearance of the LED light; at this time the photographic capture begins. The shutter is closed when it is eclipsed again.

At an intermediate point it is possible to register completely the position of the gyroscope by taking a snapshot. As the flash operates manually, the photographer must decide when register its position. To obtain an image where the object is located at a desired moment, it is necessary to illuminate it at that instant. In order to do that, two flashes are used and the camera is operated manually to take the snapshot. The flashes are placed in positions slightly outside the plane of focus (about 20 or 30 degrees from it. See Fig. 1). In order to synchronize the strobes, one of them is put in slave mode so that the master flash drives the slave and both fire simultaneously. The master flash is operated manually at the chosen time.

## III. RESULTS AND DISCUSSION

For all cases considered in this study, the initial angular position of the Gyroscope axis with respect to the vertical axis is set to $\theta(0) = 70°$, and the initial angular speed of the rotating disc M is chosen in the positive direction ($\dot{\psi}(0) > 0$).

### A. Upward cusps

Let us consider the gyroscope in a situation in which the CM is located between the rotating disc and the supporting point P, of the Gyroscope axis (*i.e.*, $z_{cm} > 0$). Once the disc is spinning with high enough angular speed, the Gyroscope axis is released from rest ($\dot{\phi}(0) = 0$) allowing it to move according to the underlying conservation physical laws in the system. The resulting motion is shown in Fig. 4, which is a long exposure time photograph that illustrates the cusp-like precession and nutation pattern, painted by light in the air by the LED installed in the upper pole of the gyroscope. This beautiful curve that has the shape of a crown with upward pointing cusps, is the trajectory followed by the LED placed at the end of the Gyroscope axis.

A simple and intuitive way of interpreting this motion is the following. Once the Gyroscope axis is released, the fast spinning disc tends to fall due to the gravitational torque. However, the disc cannot continue falling due to the requirements of conservation of angular momentum and total energy. It instead departs from the expected falling course accomplishing a short U-shaped trajectory, so that the Gyroscope axis returns to the original angle $\theta = 70°$, although with different $\emptyset$. At this time, the Gyroscope axis stops momentarily and the LED is ready to paint another "U" in the air. This cycle is repeated over and over again, resulting in the crown-shaped trajectory that we can appreciate in the photograph of Fig. 4.

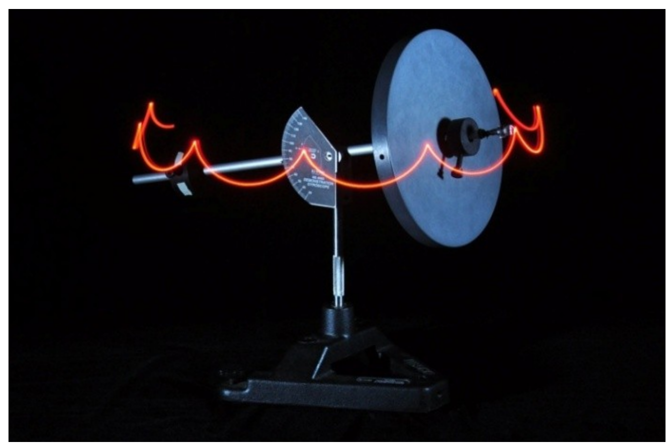

**FIGURE 4.** Precession and nutation pattern for $z_{cm} > 0$ when the axis is released from rest ( $\dot{\theta}(0) = \dot{\phi}(0) = 0$). Exposition time is 9.7s.

### B. Upward loops

Let us consider now a slight variation of the above case. For the same initial conditions as above, we now give a tiny impulse to the Gyroscope axis in the counterclockwise direction (as seen from the top of the gyroscope), that is with $\dot{\phi}(0) > 0$, so that instead of moving from rest, its motion starts also with a horizontal component. The resulting motion is illustrated in Fig. 5. As predicted by mathematical solutions [2, 3]. The angular velocity $\dot{\phi}(t)$ takes both positive and negative values for $t > 0$, which is consistent with the formation of loops in the trajectory of Fig. 5.

**FIGURE 5.** Precession and nutation pattern for $z_{cm} < 0$ when the axis is released with horizontal backward impulse ( $\dot{\theta}(0) = 0$, $\dot{\phi}(0) > 0$. Exposition time is 17.15s.

**C. Downward cusps**

In the previous examples the position of the CM was placed above the point P. We now shift the CM to a new position on the other side of P ($z_{cm} < 0$). With the same initial conditions used in the case shown in Fig. 4 and this CM shift, we now release the Gyroscope axis from rest, and the subsequent motion is presented in Fig. 6. Note that the resulting precession and nutation pattern is crown-shaped like in Fig. 4 but with cusps pointing downward. This kind of movement can be readily explained in a similar way as we did in Fig. 4 in terms of conservation laws, but the inverted gravitational torque implies the inverted "U"s forming the pattern of Fig. 6.

**FIGURE 6.** Precession and nutation pattern for $z_{cm} < 0$ when the axis is released from rest ( $\dot{\theta}(0) = \dot{\phi}(0) = 0$). Exposition time is 14.2s.

**D. Downward loops**

As a final example, let us consider the same initial situation as the previous case but we now apply a tiny horizontal impulse to the Gyroscope axis in the same direction as in the case of Fig. 5 ($\dot{\phi}(0) > 0$). As expected due to the inversion of the gravitational torque, the resulting motion is similar to the case of Fig. 5, but with downward loops. The precession and nutation pattern for this case is shown in Fig. 7.

**FIGURE 7.** Precession and nutation pattern for $z_{cm} < 0$ when the axis is released with horizontal backward impulse ( $\dot{\theta}(0) = 0$, $\dot{\phi}(0) > 0$). Exposition time is 14.5s.

## IV. CONCLUSIONS

We have introduced a didactic experiment for the visualization of precession and nutation patterns generated by a rotating rigid body. We described in detail how to apply the light painting photographic techniques to the gyroscopic motion as a pedagogical tool, providing with new elements that contribute to facilitate the teaching of this important physical phenomenon for all-level physics students. Moreover, the kind of analysis contributes to circumvent the practical difficulty of directly observe this phenomenon in the laboratory, and we consider that these ideas can be incorporated in the laboratory practices in physics schools.

## ACKNOWLEDGEMENTS

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