Teaching thermal physics by *touching*

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Abstract
The study of the physical mechanisms involved in heat transfer phenomena is of great pedagogical importance due to their incidence in many aspects of our life. In this work we describe how a daily experience such as the touch of hot and cool objects with the hands can be used to learn concepts related to stationary and transient heat propagation in thermal physics courses.

Keywords: Conduction heat transfer, thermal conductivity, thermal effusivity.

Resumen
El estudio de los mecanismos físicos involucrados en fenómenos de transferencia de calor tiene gran importancia pedagógica debido a su incidencia en muchos aspectos de nuestra vida. En este trabajo describimos cómo una experiencia diaria tal como la manipulación de objetos calientes y fríos con las manos puede ser usada para aprender conceptos relacionados con la propagación de calor en regímenes estacionario y transitorio en cursos de física térmica.

Palabras claves: Transferencia de conducción del calor, conductividad térmica, efusividad térmica.

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To check the temperature of an object we frequently touch it with our hands and in a very short time interval we can tell if it is either warmer or cooler than our skin. But our perception of the temperature is often affected by several variables, such as the kind of material we touch, its absolute temperature and the time period of the "experiment", among others. For example, at room temperature wooden objects feels warmer to the rapidly touch with our hands than those made of a metal, but when a sufficient time has elapsed both seem to be at the same temperature.

This topic is of considerable pedagogical importance since it requires a good understanding of the physical mechanisms involved in heat transfer and can be exploited in introductory and advanced physics courses in different ways. Recently [1, 2] we have shown how the theory of heat conduction leads to the result that the relevant thermophysical parameter for the heat flux in the presence of transient heating is the thermal effusivity, ε, a rather unknown parameter for many people, instead of the much very well known thermal conductivity, k. We have shown how the non-stationary heat transfer theory show that if two semi-infinite regions at uniform temperatures T_1 and T_2 are placed suddenly in perfect thermal contact, their interface temperature, also called contact, feeling or sensation temperature, T_c, is given by

$$T_c = \frac{\varepsilon_1 T_1 + \varepsilon_2 T_2}{\varepsilon_1 + \varepsilon_2}. \qquad (1)$$

The thermal effusivity is defined as

$$\varepsilon = \sqrt{k\rho c} = k/\sqrt{\alpha} = \rho c\sqrt{\alpha} \qquad (2)$$

where α is the thermal diffusivity, k the thermal conductivity, ρ the density and c the specific heat. An extended explanation of the physical relevance of these parameters can be found elsewhere [2, 3].

If we identify region 1 with our hand at T_1=37^{0}C and the other with a touched object at a different temperature T_2, using Eq. (1) and tabulated values of the thermal effusivities we can determine the contact temperature that our hand will reach upon contact. In the above mentioned works we have made calculations for the contact temperature between human skin at 37^{0}C and different bodies at 20^{0}C as a function of their thermal effusivities, showing that when touching a highly thermal conductivity object such as a metal (e.g. Cu), as $\varepsilon_{\text{metal}} >> \varepsilon_{\text{skin}}$, the temperature of the skin drops suddenly to 20 ^{0}C (as one can see from Eq. (1)), and one sense the object as being "cold". On the other hand, when touching a body with a lower thermal conductivity, e.g. a wood's object ($\varepsilon_{\text{wood}} < \varepsilon_{\text{skin}}$) the skin temperature remains closest to 37 ^{0}C, and one sense the object as being "warm". This is the reason why a metal object feels colder than a wooden one to the touch, although they are both at the same, ambient equilibrium temperature. This is also the cause why human foot skin feels different the temperature of floors of different materials which are at the same room temperature and the explanation of why, when a person enters the cold water in a swimming pool, the temperature immediately felt by the swimmer is near its initial, higher, body temperature [4].

Now, consider the extreme situation of a man walking barefoot across the very hot sand of a beach, i.e., suppose that the touched object is very hot (Fig.1). Why does not the man's skin get burnt? Or this other case: How is it possible to put a very cold object, such as liquid Nitrogen, into a hand without damaging the skin?

Inspired in these questions and in a paper by Byalko [5], we will show in what follows how the laws of thermal conduction can also explain such paradoxical phenomena.

FIGURE 1. An apparently impossible act: a man touching a burning iron.

First, it is worth to notice that using Eq. (1) we can easy show that the contact temperature at the body surface in contact with the object tends to be, in both cases, closer than that of the skin, as can be seen in Fig. 2, where the calculated contact temperature between human skin at 37^0C and bodies of different materials at 1000^0C (circles) and 0^0C (squares) as a function of their thermal effusivities is represented (see Table 1 for values of thermal properties). This is one of the reasons why our skin is not burning when we make a suddenly (transient) contact to a hotter object or freezing when touching a very cold one (despite we fill that the object is hotter or colder, indeed).

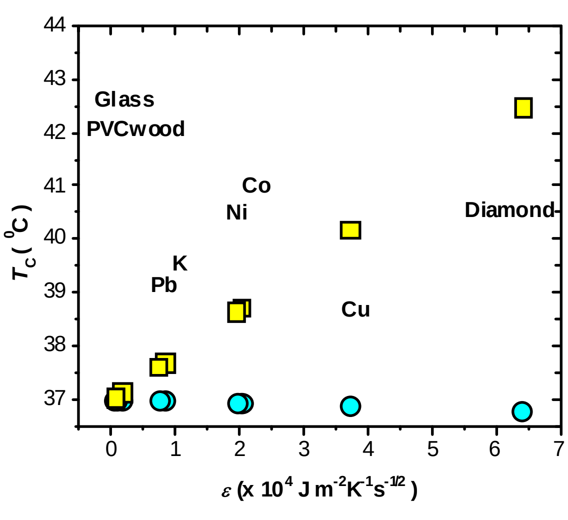

FIGURE 2. Contact temperatures as a function of thermal effusivity when touching with the hand at 37 ^{0}C objects of different materials at 0 ^{0}C (blue circles) and 1000^0C (yellow squares).

However, the following question arises at this point: For long contact times, for which there is no more a transient situation, the daily experience show us that these experiments should be dangerous for the skin. How long can be the contact time so that the above described experiments can be performed with safety?

TABLE I. Thermal properties of different materials at room temperature taken from references [3] and [5].

Material	K W m^{-1} K^{-1}	α 10^6 m^2 s^{-1}	ε Jm^{-2}K^{-1}s$^{-1/2}$	ρc 10^{-6}Jm^{-3}K^{-1}
Diamond	2300	1290	64040	1.78
Cu	400	116	37140	3.45
K	102	158	8150	0.65
Co	100	24.6	20150	4.05
Ni	91	23	19400	3.95
Pb	35	23	7300	1.52
Glass	1.11	0.56	1480	1.98
PVC	0.20	0.15	515	1.33
Hard Wood	0.16	1.77	380	0.09
Water	0.631	0.15	1589	0.004
Air	0.026	0.21	1.6	0.0001
Human Skin	0.37	0.109	1120	3.39

The answer has to do with the well known fact that a very thin layer of gas is produced when the skin touch very hot or cold objects [5]. In the above examples this gas is evaporated nitrogen in the case of liquid Nitrogen (note that we have taken the same illustrative example as Byalko) and vapour (mainly of water, since biological tissue contains more than 90% of this substance) exhaled when the outer layers of the skin are heated. Let us assume that this layer has a thickness L and suppose that the outer layer of skin can tolerate temperatures between 0^0C and 100^0C without frostbitten or roasting [5]. Inside the gas layer the temperature is distributed from that corresponding to the touched cold or hot object (T_1) and the contact temperature T_c. The temperature distribution can be calculated using Fourier's law

$$q = -k\nabla T\ , \tag{3}$$

where q is the heat flux density (Heat energy per unit time and unit area). For one-dimensional steady state conduction in extended samples of homogeneous and isotropic material, such as our thin gas layer of thermal conductivity k_g, Fourier's law can be integrated in each direction to its potential form. In rectangular coordinates it reads [2, 5]:

$$q = k_{gas}\frac{T_c - T_1}{L}\ . \tag{4}$$

At the same time heat will be removed from the touched surface of a semi-infinite region. A straightforward calculation [2], lead for the heat flux between the surface at T_C and the inside of the skin, say at T_2, to the well known result [6]:

$$q = \frac{\varepsilon\left(T_2 - T_c\right)}{\sqrt{\pi\, t}}. \tag{5}$$

Note that this heat flow, obtained for transient heating, is not proportional to the thermal conductivity of the material, as under steady state conditions (see Eq. (3)), but to its thermal effusivity (Care must be taken with the sign of the right hand side of the above equations, for example, in Eq. (4) it must be negative for $T_c > T_1$, since the heat flow is always directed from the region at a higher temperature to the coldest one). It must be the same than the heat flux given by Eq. (4):

$$k_{gas}\frac{T_c - T_1}{L} = \frac{\varepsilon\left(T_2 - T_c\right)}{\sqrt{\pi\, t}}. \qquad (6)$$

Then, the time τ required for the skin to reach the limiting values of the contact temperature given above (0 and 100^0C) can be then obtained from Eq. (12) as

$$\tau = \frac{\varepsilon^2 L^2}{\pi k^2_{gas}}\left(\frac{T_2 - T_c}{T_c - T_1}\right)^2. \qquad (7)$$

It is represented in Fig. (3) for different thicknesses of the gas layer. Note that we do not know well its exact value, although Byalko [5] has used a value of 0.1 mm for this parameter. In our estimation we have used the value of the air thermal conductivity given in Table I, because all gases have thermal conductivities of the same order of magnitude. Air, for example, is mainly composed by Nitrogen. We have taken for the skin temperature the value T_2=37^0C and the temperature of liquid Nitrogen T_1=-196^0C (the corresponding limiting contact temperature will be T_c=0^0C) for the case of the colder object (solid curve). In the case of the hot object (dotted curve) we have taken T_1=600^0C (T_c=100^0C).

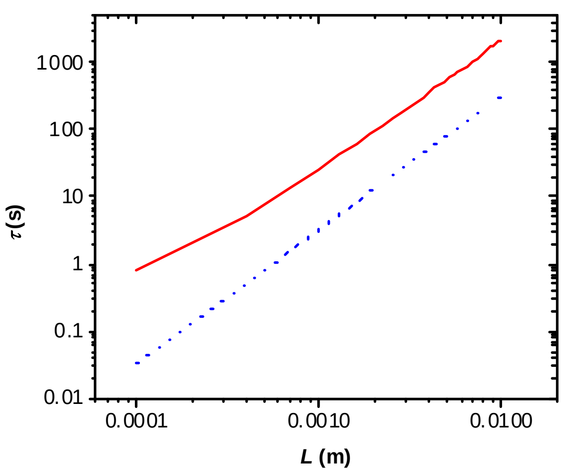

FIGURE 3. The time τ required for the skin to reach values of the contact temperature of 0^0C and 100^0C without frostbitten or burning up respectively (see text), as a function of the hypothetical thickness of the gas layer evaporated at the surface. The solid (red) and dotted (blue) curves correspond to the case of touching a cold (-196^0C) and a hot (600^0C) object, respectively.

From the figure we can observe that for gas layer thicknesses smaller than 10^{-3} m (a reasonable value) the time required to heat the skin to 100^0C by contact with an object at 600^0C is lower than 3s, enough to hop across the hot sand or to touch a hot iron. On the other hand liquid Nitrogen can be handled safely for a longer period of time which, in the figure, is about 25 s for the same layer thickness. In order to make these times longer, a gas layer of greater thickness must be achieved, as one can see from Fig. 3. I remember my mother dampening her fingers to check how hot the iron was so that clothes would not get burnt. In an empirical way she created an artificial gas layer to protect her hands. This is the situation schematically shown in Fig. 1.

The here presented results, of course, are in good agreement with our daily experience, when we use our hands to touch cold and hot objects, but the analysis here presented can be used to illustrate, in a simple way, concepts related to heat transfer under stationary and non-stationary conditions. Hopefully this paper will aid in broaden this theme to a wider audience.

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