On the Relativistic Concept of the Dirac's electron Spin

N. Hamdan[1], A. Chamaa[1] and J. López-Bonilla[2]
[1]*Department of Physics, University of Aleppo, Aleppo-Syria.*
[2]*SEPI-ESIME-Zacatenco, Instituto Politécnico Nacional, Edif. Z-4, 3er. Piso, Col. Lindavista CP 07 738 México DF*

E-mail: nhamdan59@hotmail.com

(Received 3 November 2007; accepted 10 January 2008)

Abstract

Despite the success of the Dirac equation, the incorporation of the special theory of relativity into quantum mechanics predicts some paradoxes, like that the spin prediction from Dirac equation can be only identified with non- relativistic approximations (Pauli and Foldy-Wouthysen), as well as the spin predication is a relativistic quantum phenomena because the spin prediction is a necessary requirement of the relativistic quantum mechanics only. In this paper we show that the derivation of the spin and its magnetic moment can be done with a pure classical treatment. Since we start from the classical physical laws and the classical relativity principle to get the linear Schrödinger equation as a result the derivation of the spin and its magnetic moment can be done with a pure classical treatment. This approach result a Schrödinger equation, in which we show that the spin of the electron is a non relativistic quantum phenomenon too.

Keywords: Dirac equation, electron spin, non relativistic Quantum Mechanics.

Resumen

A pesar del éxito de la ecuación de Dirac, la incorporación de la teoría especial de relatividad en la mecánica cuántica predice algunas paradojas, como la predicción del spin de la ecuación Dirac que puede ser identificada solamente con aproximaciones no-relativistas (Pauli y Foldy-Wouthysen), así como el pronóstico del spin que es un fenómeno cuántico relativista porque la predicción del spin solo es un requerimiento necesario de la mecánica cuántica relativista. En este artículo mostramos que la derivación del spin y su momento magnético puede ser realizada con un tratamiento puramente clásico. Ya que comenzamos de las leyes físicas clásicas y del principio de relatividad clásico para obtener la ecuación de Schrödinger lineal como un resultado, se puede realizar la derivación del spin y su momento magnético con un tratamiento puramente clásico. De esta aproximación resulta una ecuación de Schrödinger, en la cual mostramos que el spin del electrón ya no es un fenómeno relativista cuántico.

Palabras clave: Ecuación de Dirac, spin del electrón, mecánica cuántica no relativista.

PACS: 03.30.+p, 03.50.-z, 03.65.-w, 03.65.Pm. **ISSN 1870-9095**

I. INTRODUCTION

In 1926 while Schrödinger was publishing his non-relativistic single particle wave equation [1], Dirac [2] was searching for a relativistic invariant form of the one-particle Schrödinger equation for electrons starting from the relativistic equation, which was known as Klein–Gordon equation (KGE). However, at that time several objections emerged against the KGE as a single particle equation because its solutions allowed negative probability densities, besides there was the possibility of negative energies and their solutions did not have clear spin dependence. In 1928 Dirac published an equation [2, 3] which was presented as a definite solution to the above mentioned problems where he has shown that the spin belongs to the relativistic wave equation. The integration of the special relativity theory with quantum mechanics has yielded many paradoxes that remained unsolved that it was impossible to directly write a non-relativistic equation for spin-1/2 particles and that it could therefore only be derived as a non-relativistic limit of the relativistic Dirac equation. So it was known in standard quantum mechanics that the spin of electron has only relativistic nature. However, in 1984, this supposition was questioned by W. Greiner [4] when he derivates the spin from the non-relativistic quantum mechanics, *i.e.,* he derivates the spin from the Schrödinger equation.

In this paper we obtain additional advantage concerning the same result of Greiner, where we revealed that the spin of the electron and its magnetic moment can be derived from the modified Schrödinger equation without using any kind of approximations (non-relativistic limit of Dirac equation), and that the derivation of the spin and its magnetic moment can be done with a pure classical treatment.

II. THE RELATIVISTIC DIRAC EQUATION AND THE PAULI EQUATION AS A NON-RELATIVISTIC LIMIT OF THE DIRAC EQUATION

The early twentieth century saw two major revolutions in the way physicists understand the world. The first one was quantum mechanics itself and the other was the theory of relativity. Important results also emerged when these two theories, *i.e.*, quantum mechanics and the theory of relativity were brought together and one of these results is the Dirac equation which leads to the spin of an electron that was known as a relativistic effect.

When calculating kinetic energy relativistically using Lorentz transformation instead of Newtonian mechanics, Einstein discovered that the amount of energy is directly proportional to the mass of body:

$$E = mc^2, \tag{1}$$

where E is the total energy and m the relativistic mass.

The energy and momentum of a particle momentum are then related by the principal equation governing the dynamics of a free particle:

$$E^2 = c^2 \boldsymbol{p}^2 + m_0 c^4, \tag{2}$$

where c is the speed of light, m_o is the rest mass of the particle and $\boldsymbol{p}$ is the momentum.

Following Dirac, we take into account the time dependent of Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t}\psi = \hat{H}\psi , \tag{3}$$

using (2) and (3) Dirac assumed that

$$i\hbar \frac{\partial}{\partial t}\psi = \sqrt{c^2 \hat{\boldsymbol{p}}^2 + m_0^2 c^4}\, \psi .$$

One of the conditions imposed by Dirac in writing down a relativistic equation for the electron was that the ‘‘square’’ of that equation will give the Klein-Gordon equation. Imposing the additional condition of linearity of □ in the components of $\hat{\boldsymbol{p}}$ led Dirac to following relation

$$i\hbar \frac{\partial}{\partial t}\psi = \hat{H}_D \psi , \tag{4}$$

where

$$\hat{H}_D = c(\boldsymbol{\alpha}\hat{\boldsymbol{p}}) + m_0 c^2 \beta , \tag{5}$$

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix} , \quad k = 1,2,3, \quad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} , \tag{6}$$

and I is the two-by-two identity matrix.

In standard quantum mechanics, it is not possible to directly extend the Schrödinger equation to spinors, so the Pauli equation must be derived from the Dirac equation by taking its non-relativistic limit. This is in particular the case for the Pauli equation which predicts the existence of an intrinsic magnetic moment for the electron and gives its correct value only when it is obtained as the non-relativistic limit of the Dirac equation.

In the Dirac equation for the relativistic charged particle moving in a constant magnetic field

$$i\hbar \frac{\partial}{\partial t}\psi = \left[c\boldsymbol{\alpha}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\boldsymbol{A} \right) + \beta m_0 c^2 \right]\psi , \tag{7}$$

we can follow Pauli's approach by eliminating small components to derive the Pauli equation. We consider a two-component representation, where the four-component spinor □ is decomposed into two spinors □$_b$ and □$_s$,each one with two components

$$\psi = \begin{pmatrix} \psi_b \\ \psi_s \end{pmatrix}, \quad \psi_b = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \psi_s = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}. \tag{8}$$

In the non-relativistic limit, the rest energy, $m_o c^2$ becomes dominant; therefore, the two component solution is approximately

$$\psi_{b,s} = e^{\frac{-i m_0 c^2 t}{\hbar}} \psi^0_{b,s} . \tag{9}$$

Substituting Eq. (9) into Eq. (7), and using Eq. (6), it gives

$$i\hbar \frac{\partial}{\partial t}\psi_b^0 = c\boldsymbol{\alpha}\left(i\hbar\nabla - \frac{e}{c}\boldsymbol{A} \right)\psi_s^0 , \tag{10a}$$

$$i\hbar \frac{\partial}{\partial t}\psi_s^0 = c\boldsymbol{\alpha}\left(i\hbar\nabla - \frac{e}{c}\boldsymbol{A} \right)\psi_b^0 - 2m_0 c^2 \psi_s^0 , \tag{10b}$$

$$\left| i\hbar \frac{\partial}{\partial t}\psi_s^0 \right| \ll \left| m_0 c^2 \psi_s^0 \right| , \tag{11}$$

and with this last approximation, Eq. (10b) becomes to

$$0 = c\boldsymbol{\alpha}\left(i\hbar\nabla - \frac{e}{c}\boldsymbol{A} \right)\psi_b^0 - 2m_0 c^2 \psi_s^0 ,$$

which gives

$$\psi_s^0 = \frac{\boldsymbol{\alpha}\left(i\hbar\nabla - \frac{e}{c}\boldsymbol{A} \right)}{2m_0 c}\psi_b^0 . \tag{12}$$

The lower component ψ_s^0 is generally referred to as the 'small' component of the wavefunction □, relative to the 'large' component ψ_p^0.

Substituting the expression ψ_s^0 given by Eq. (12), into Eq. (10a), we obtain

$$i\hbar \frac{\partial}{\partial t}\psi_b^0 = \frac{\boldsymbol{\alpha}\left(i\hbar\nabla - \frac{e}{c}\boldsymbol{A} \right)\boldsymbol{\alpha}\left(i\hbar\nabla - \frac{e}{c}\boldsymbol{A} \right)}{2m_0}\psi_b^0 .$$

Finally, by using the well-known identities

$$(\boldsymbol{\sigma a})(\boldsymbol{\sigma b}) = \boldsymbol{ab} + i\boldsymbol{\sigma}(\boldsymbol{a} \times \boldsymbol{b}) ,$$

we deduce that, being $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ the magnetic field,

$$i\hbar\frac{\partial}{\partial t}\psi_b^0 = \left[\frac{\left(i\hbar\nabla - \frac{e}{c}\mathbf{A}\right)^2}{2m_0} - \frac{e\hbar}{2m_0 c}\boldsymbol{\sigma B}\right]\psi_b^0 . \qquad (13)$$

The Pauli equation for the theory of spin was derived as a non-relativistic limit of the relativistic equation and it was well known in standard quantum mechanics as a direct proof of the fundamentally relativistic nature of the spin. As we shall verify, the modified Schrödinger equation will lead directly to the Pauli equation and therefore to the spin of the electron without using the non- relativistic limit, with the same results as in Dirac's theory of the relativistic electron.

III. DERIVATION OF LINEAR SCHRÖDINGER EQUATION

It is well known that the nature of spin defies non relativistic QM. Therefore a statement was known that the spin must have to do with special relativity although its connection is not entirely understood. In contrast to this statement, W. Greiner [4] has followed the Dirac's approach where he started from the same premise: the Schrödinger operator $\hat{K} = \hat{E} - \frac{\hat{\boldsymbol{p}}^2}{2m_0}$ must be linear in momentum. Then Greiner writes the free Schrödinger equation $\hat{K}\Psi = 0$ as

$$\hat{\Theta}\Psi = \left(\hat{A}\hat{E} + \hat{\boldsymbol{B}}\hat{\boldsymbol{p}} + \hat{C}\right)\Psi = 0 , \qquad (14)$$

where the operator $\hat{\Theta}$ would be linear in momentum, thus there must be an operator such that

$$\hat{\Theta}' = \hat{A}'\hat{E} + \hat{\boldsymbol{B}}'\hat{\boldsymbol{p}} + \hat{C}' , \qquad (15)$$

so the multiplication of Eqs. (14) and (15) result again the Schrödinger equation

$$\hat{\Theta}'\hat{\Theta}\Psi = 2m_0\hat{K}\Psi . \qquad (16)$$

According to Eq. (16) Greiner determines the matrix *A, B* and *C.* He obtained the linear Schrödinger equation

$$\left(\hat{A}\hat{E} + \hat{\boldsymbol{B}}\hat{\boldsymbol{p}} + \hat{C}\right)\Psi = 0 , \qquad (17)$$

where

$$\hat{A} = -i\begin{pmatrix} 0 & 0 \\ I & 0 \end{pmatrix}, \quad \hat{C} = i\begin{pmatrix} 0 & 2m_0 I \\ 0 & 0 \end{pmatrix} , \qquad (18)$$

and

$$\hat{B}_5 = -i\hat{M} , \quad \hat{B}_\alpha = \hat{M}\gamma_\alpha , \quad \alpha = 1 \text{ to } 4 , \qquad (19)$$

as well as

$$M = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} = M^{-1} .$$

Greiner gets also that the □□ have the usual representation

$$\gamma_j = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix} \ (j = 1,2,3) , \quad \gamma_4 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} . \qquad (20)$$

IV. DERIVATION OF MODIFIED SCHRÖDINGER EQUATION FROM THE CLASSICAL PHYSICAL LAWS AND ITS SOLUTIONS

In several recent papers [5, 6] we suggested another way to account for the Lorentz transformation and its kinematical effects in relativistic electrodynamics as well as in relativistic mechanics. And by following the same approach we derived Einstein's equation as well as the De Broglie relation from classical physical laws such as the Lorentz force law and Newton's second law [7, 8, 9]

$$E_t = mc^2 , \qquad (21a)$$

$$p = \frac{h}{\lambda} . \qquad (21b)$$

We showed also that Eq. (21a) could be written as

$$E_t = mv^2 + m_0 c^2\sqrt{1 - \frac{v^2}{c^2}} , \qquad (22)$$

and from the last relation we have by definition the kinetic energy

$$E_k = E_t - m_0 c^2 = mv^2 + m_0 c^2\left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right),$$

and for non-relativistic velocities, $v<<c$, Eq. (22) reduces to

$$E_t = E_k + m_0 c^2 = \frac{1}{2}m_0 v^2 + m_0 c^2 . \qquad (23)$$

In the paper [10], D. Ward, S. Volkmer started from the classical electromagnetic wave equation as well as the basics of Einstein's special theory of relativity. And by extending this wave equation for photons, generalize to non-zero rest mass particles they get the free Schrödinger equation. So following a similar approach to that used in [10], *i.e.,* by starting from the classical physical laws and the classical relativity principle we get also the linear Schrödinger equation, *i.e.*, Eq. (17) without using Einstein's special theory of relativity as in [10] did. Therefore, we can go beyond the mathematical similarities of the classical and quantum theories of the electron if we recognize that we get Eqs. (21) and (23) without using of Einstein's special theory of relativity this being the new input. Since our work carries D. Ward's work a step further by deriving Eqs. (21) and (23) without using the special relativity theory and we obtain also the same result of Greiner, *i.e.*, Eq. (17). As a result the derivation of the spin and its magnetic moment can be done with a pure classical treatment.

Our starting point is Eq. (17), but first we rewrite the matrices *A, B* and *C* as follows

$$A = M\begin{pmatrix} -iI & 0 \\ 0 & 0 \end{pmatrix}, \tag{24a}$$

$$C = M\begin{pmatrix} 0 & 0 \\ 0 & 2miI \end{pmatrix}, \tag{24b}$$

$$B_j = M\begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}. \tag{24c}$$

Setting Eqs. (24) in Eq. (17), and multiplying by M^{-1}, we get

$$\begin{pmatrix} iI & 0 \\ 0 & 0 \end{pmatrix}\hat{E}\Psi = \begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}\hat{\boldsymbol{p}}\,\Psi + \begin{pmatrix} 0 & 0 \\ 0 & 2m_0 iI \end{pmatrix}\Psi. \tag{25}$$

We shall continue to find the eigenfunctions of the new linear Schrödinger equation to prove that there is contradiction with Dirac's conceptions of relativistic spin of electron. That means we will show that the new linear Schrödinger equation is an equation for describing the spin of electron. Solutions to Eq. (25) are plane waves which can be written in the following form

$$\Psi(x,t) = N\begin{pmatrix} \varphi(x,t) \\ \chi(x,t) \end{pmatrix} = N\begin{pmatrix} \varphi(x) \\ \chi(x) \end{pmatrix} e^{\frac{-iEt}{\hbar}}, \tag{26}$$

where N is the normalization constant, and $\begin{pmatrix} \varphi(x) \\ \chi(x) \end{pmatrix}$ is a four component spinor. Substituting Eq. (26) in Eq. (25), and considering that

$$\begin{pmatrix} \varphi(x) \\ \chi(x) \end{pmatrix} = \begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} e^{\frac{ipx}{\hbar}},$$

where

$$\varphi_0 = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix},$$

and

$$\chi_0 = \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix},$$

are two-component spinors, we find that

$$E\begin{pmatrix} iI & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} = p_j\begin{pmatrix} 0 & \sigma_j \\ \sigma_j & 0 \end{pmatrix}\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} + 2m_0 i\begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} \varphi_0 \\ \chi_0 \end{pmatrix} \tag{27}$$

it implies the two equations

$$iE\varphi_0 - \sigma_j p_j \chi_0 = 0, \tag{28a}$$

$$\sigma_j p_j \varphi_0 + 2m_0 i\,\chi_0 = 0, \tag{28b}$$

which have solution if

$$\begin{vmatrix} iE & -\sigma_j p_j \\ \sigma_j p_j & 2m_0 i \end{vmatrix} = 0, \tag{29}$$

and by using the same known identities, that used above, then Eq. (29) goes into

$$E = \frac{\boldsymbol{p}^2}{2m_0}. \tag{30}$$

For a given E from Eqs. (28) follows that

$$\chi_0 = \frac{-\sigma_j\, p_j}{2m_0 i}\varphi_0, \tag{31}$$

and from Eqs.(28) and (31) we find

$$E\varphi_0 = \frac{(\sigma_j \cdot p_j)^2}{2m_0}\varphi_0. \tag{32}$$

Using Eq. (30) in Eq. (32), we find that φ_0 can take one of the representations

$$\varphi_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

or

$$\varphi_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and from the above calculations we deduce that

$$\Psi(x,t) = N\begin{pmatrix} \varphi(x,t) \\ \chi(x,t) \end{pmatrix} = N\begin{pmatrix} \varphi_0 \\ \dfrac{i\boldsymbol{\sigma}\hat{\boldsymbol{p}}}{2m_0}\varphi_0 \end{pmatrix} e^{\frac{i}{\hbar}(px - Et)}. \tag{33}$$

By calculating the normalization constant N for positive energy solution, we obtain

$$N = \sqrt{\frac{2m_0}{E + 2m_0}}.$$

If we consider the free electron motion along the z-direction, then the two states that represent free moving electron are

$$\psi_{\vec{p},+\frac{1}{2}} = \sqrt{\frac{2m_0}{E + 2m_0}}\begin{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \dfrac{i\sigma_z p}{2m}\begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} e^{i(pz - Et)}, \tag{34a}$$

$$\psi_{\vec{p},-\frac{1}{2}} = \sqrt{\frac{2m_0}{E + 2m_0}}\begin{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ \dfrac{i\sigma_z p}{2m}\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} e^{i(pz - Et)}. \tag{34b}$$

These are just the wave functions that can describe the spin up, and the spin down. For each value of p there is one positive eigenvalue, Eq. (30), and two eigenfunctions, Eqs. (34), in according with eigenvalue Eq. (25).

V. INTERACTION WITH THE MAGNETIC FIELD- THE PAULI EQUATION

The most important result of the relativistic Dirac equation was presenting a theoretical description of the electron spin and its magnetic moment, which means that the predictions of electron spin is the property of the Dirac equation only. That is not true for many reasons:

If one derives the spin of the electron and its magnetic moment from the non relativistic linear Schrödinger equation, Eq. (25). The predictions of electron spin did not occur directly from the Dirac equation, but using approximation like the non relativistic limit of Dirac equation to get the Pauli equation with the proper value of the spin of the electron and its magnetic moment from the non relativistic Pauli equation

As we know, the momentum $\boldsymbol{p}$ is replaced in the same way to include the effects of electromagnetic fields, and if we only consider the effect of a magnetic field $\boldsymbol{B}$ then the momentum is replaced as $\hat{\boldsymbol{p}} \rightarrow \hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A}$, and Eq. (25) becomes

$$\begin{pmatrix} iI & 0 \\ 0 & 0 \end{pmatrix} \hat{E}\Psi = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix} \left(\hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A} \right) \Psi + \begin{pmatrix} 0 & 0 \\ 0 & 2m_0 iI \end{pmatrix} \Psi, \quad (35)$$

which implies

$$\begin{pmatrix} i\hat{E}\varphi(x,t) \\ 0 \end{pmatrix} = \begin{pmatrix} \boldsymbol{\sigma}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A} \right)\chi(x,t) \\ \boldsymbol{\sigma}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A} \right)\varphi(x,t) \end{pmatrix} + \begin{pmatrix} 0 \\ 2im_0 I\,\chi(r,t) \end{pmatrix}, \quad (36)$$

and

$$i\hat{E}\varphi - \boldsymbol{\sigma}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A} \right)\chi = 0\,, \quad (37a)$$

$$\boldsymbol{\sigma}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A} \right)\varphi + 2m_0 i\,\chi = 0\,, \quad (37b)$$

therefore

$$\hat{E}\varphi(x,t) = \frac{\left\{ \boldsymbol{\sigma}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A} \right) \right\}^2}{2m_0}\varphi(x,t)\,. \quad (38)$$

Using the well-known identities, we get

$$\left\{ \boldsymbol{\sigma}\left(\hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A} \right) \right\}^2 = \left(\hat{\boldsymbol{p}} - \frac{e}{c}\mathbf{A} \right)^2 - \frac{e}{c}\hbar\boldsymbol{\sigma}\boldsymbol{B}\,,$$

we recover finally an equation for the two- component spinor-□

$$i\hbar\frac{\partial\varphi(x,t)}{\partial t} = \left[\frac{\left(\hat{p} - \frac{e}{c}\mathbf{A} \right)^2}{2m_0} - \frac{e\hbar}{2m_0 c}\boldsymbol{\sigma}\boldsymbol{B} \right]\varphi(x,t)\,, \quad (39)$$

which is the Pauli equation with a spin g-factor of 2, the same result as in Dirac's theory, is derived from the non relativistic linear Schrodinger equation, this means that the electron has a magnetic moment $-e$□$/2m_o$□ and the magnetic moment interacts with an external magnetic field, the corresponding contribution to the energy is -□$\boldsymbol{B}$. An important characteristic of Eq. (39) is that we did not use any kind of approximation to reach it, *i.e.*, we did not use the condition (11) to eliminate the lower component □ of the wavefunction □, here the component □ of □ eliminates itself without any approximation.

In Schrödinger theory the orbital angular momentum $\hat{L}$ commutes with the Hamiltonian $\hat{H} = \hat{\boldsymbol{p}}^2/2m$, this is not the case in Dirac theory, since the Dirac Hamiltonian is linear in momentum and the total momentum $\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}$ commutes with it. For this reason, it is conventional to choose an operator similar to $\boldsymbol{J}$ which commutes with the linear Schrödinger Hamiltonian □$_s$. From Eqs. (24) and (25) we can define the linear Schrödinger Hamiltonian □$_s$

$$\hat{H}_s = \boldsymbol{\alpha}\hat{\boldsymbol{p}} + D\,, \quad (40)$$

where

$$D = \begin{pmatrix} 0 & 0 \\ 0 & 2miI \end{pmatrix}, \qquad \boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}.$$

The orbital momentum $\hat{L}$ doesn't commutes with □$_s$

$$[H_s, L] = [\boldsymbol{\alpha}\hat{\boldsymbol{p}} + D, L] = -i\hbar(\boldsymbol{\alpha}\times\hat{\boldsymbol{p}})\,, \quad (41)$$

however, to find an operator which commutes with □$_s$, we remind the operator B in Eq. (24c):

$$\boldsymbol{B} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix},$$

and its commutator with □$_s$ is

$$[H_s, \boldsymbol{B}] = 2i(\boldsymbol{\alpha}\times\hat{\boldsymbol{p}})\,. \quad (42)$$

Now if we define Dirac spin operator as

$$\hat{\boldsymbol{S}} = \frac{\hbar}{2}\hat{\boldsymbol{B}}\,, \quad (43)$$

it satisfies

$$[H_s, S] = i\hbar(\boldsymbol{\alpha}\times\hat{\boldsymbol{p}})\,, \quad (44)$$

and from commutators (41) and (44) we see that:

$$[H_s, \boldsymbol{L} + \boldsymbol{S}] = [H_s, \boldsymbol{J}] = i\hbar(\boldsymbol{\alpha}\times\hat{\boldsymbol{p}}) - i\hbar(\boldsymbol{\alpha}\times\hat{\boldsymbol{p}}) = 0\,,$$

which means that the operator

$$\hat{\boldsymbol{J}} = \hat{\boldsymbol{L}} + \hat{\boldsymbol{S}}\,, \quad (45)$$

commutes with □$_s$. It shows also that the total angular momentum, Eq. (45), is given similarly as in Dirac theory.

VI. CONCLUSION

Despite successes of the Dirac equation, there remain a number of misunderstandings about this equation. The first misunderstanding about Dirac equation is the spin of electron which is well known as a relativistic effect. Although of this the spin prediction from Dirac equation can not be allowed directly without approximations methods *i.e.*; the non-relativistic limit of the relativistic Dirac equation to get the Pauli equation which predicts the existence of an intrinsic magnetic moment for the electron and gives its correct value only when it is obtained as the non-relativistic limit of the Dirac equation.

The second misunderstanding about Dirac equation is zitterbewegung. In the Dirac relativistic equation for the spin 1/2 particle, there is a velocity operator $\hat{\boldsymbol{v}} = c\hat{\alpha}_j$. It is believed that this operator is inadequate in two aspects: The first one is that its eigenvalues are $+c$ and $-c$ with c being the light speed in a vacuum. The other is that it is not proportional to the linear momentum. In the papers [12, 13] it had been shown that the velocity of the particle obtained from the modified Dirac equation always moves with velocity $\pm v$ as observed in the laboratory, and this result eliminate the problem of the zitterbewegung.

In contrast to it stands the wrong statement, which attribute spin to relativistic characteristics and that non-relativistic quantum mechanics is a theory of spinless particle. Recently, many Authors have argued that the Schrödinger equation of non-relativistic quantum mechanics describes not a spinless particle as universally assumed, but a particle in a spin eigenstate [11, 12, 13]. The spin of electron can be derived gradually from the non relativistic linear Schrodinger equation, and everything result automatically as it is shown in the present paper.

REFERENCES

[1] Schrödinger, E. *Quantisierung als Eigenwertproblem*, Ann. der Physik **384**, 361-376 (1926); **384**, 489-527 (1926); **385**, 437-490 (1926); **386**, 109-139 (1926).

[2] Dirac, P. A. M., *The Quantum Theory of Electron*, Proc. R. Soc. London **A 117**, 610–624 (1928).

[3] Moyer, D. F., *Origins of Dirac's Electron, 1925–1928*, Am. J. Phys. **49**, 944–949 (1981).

[4] Greiner, W., *Quantenmechanik I, (Band 4)* (Verlag Harmi Deutsch, Thun, 1984) pp. 303–312.

[5] Hamdan, N., *Abandoning the Ideas of Length Contraction and Time Dilation,* Galilean Electrodynamics **14,** 83-88 (2003).

[6] Hamdan, N., *Newton's Second Law is a Relativistic law without Einstein's Relativity*, Galilean Electrodynamics **16**, 71-74 (2005).

[7] Hamdan, N., *Derivation of the de Broglie's Relations from the Newton Second Law*, Galilean Electrodynamics **18**,108-111 (2007).

[8] Hamdan, N., *The Dynamical de Broglie Theory*, Annales Fondation Louis de Broglie **32**, 1-13 (2007).

[9] Hamdan, N., K., Hariri, A. and López-Bonilla, J., *Derivation of Einstein's Equation, $E=mc^2$, from the Classical Force Laws*, Apeiron **14,** 435-453 (2007).

[10] Ward, D. and Volkmer, S., *How to Derive the Schrödinger Equation*, arXiv:physics/0610121, (2006).

[11] Gurtler, R. and Hestenes, D. *Consistency in the formulation of the Dirac, Pauli and Schrödinger Theories*, J. Math. Phys. **16**, 573–584 (1975).

[12] Bakhoum, E. G., *Fundamental Disagreement of Wave Mechanics with Relativity*, Physics Essays **15**, (2002).

[13] Hamdan, N., Altorra, A. and Salman, H. A., *The Classical Zitterbewegung*, accepted for publication in Galilean Electrodynamics.