A research on undergraduate students' conceptualizations of physics notions related to non-sliding rotational motion

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Abstract

This paper presents research findings of a study on specific conceptions held by college students in an introductory physics course when they explain a non-sliding rotational motion from the kinematical standpoint, as a uniformly accelerated rectilinear motion, and from the rotational dynamics framework, as well as the role of rotational inertia in this situation. Students' written answers to a paper and pencil problem are analysed in the light of Vergnaud's conceptual fields theory. The research was carried out under the qualitative paradigm in which data are grouped in categories which are not previously defined by the theoretical framework. The analysis of the results allowed the identification of some elements of the schemes students would use to handle the task. The findings show the potentiality of such a framework to interpret the construction processes of students' representations, as well as to design instructional strategies to facilitate critical meaningful learning.

Keywords: students' representations, operational invariants, reasonings, non-sliding rotational motion.

Resumen

En el trabajo se presentan los resultados de una investigación sobre concepciones y competencias específicas en estudiantes universitarios de primer curso acerca de los distintos modos de explicar el "mecanismo" de un movimiento de rodadura sin deslizamiento desde la dinámica rotacional y desde un punto de vista cinemático como movimiento rectilíneo uniformemente acelerado y el rol de la inercia rotacional en el mismo. Las respuestas principalmente a un trabajo escrito sobre rotaciones se analizan a la luz de la teoría de los campos conceptuales de Vergnaud. Es una investigación de tipo cualitativa, donde los datos se agrupan en categorías que no son provistas a priori por el marco teórico. El análisis de resultados permite identificar algunos elementos de los esquemas que usarían los estudiantes para resolver la tarea. Las conclusiones muestran la potencialidad de este marco teórico para interpretar los procesos de construcción de las representaciones de los alumnos, y para la elaboración de propuestas instruccionales tendientes a un aprendizaje significativo crítico.

Palabras claves: representaciones internas, invariantes operatorios, razonamientos, rodadura.

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I. INTRODUCTION

This study is part of a broader project which aims at searching for tokens that indicate the presence of operational invariants during the physics problem solving process and their relation to mental representations.

As the research in problem solving advances, the bonds with the learning of concepts and with the implicit relations to meaning in its broadest sense start to line up. That is to say, which are the general and particular conditions one has to rely on in order for learning to exist, considering from the context and the environment up to the minimum requirements which the statement of the problem must have so that certain levels of comprehension are accessible. In a detailed analysis of a problem situation, one can infer the presence of some implicit knowledge, traditionally difficult to be detected, whose quality and organization influence notably in the procedures people undertake trying to solve such problem situation.

A critical review of the processes and results in problem solving research and the configuration of Vergnaud´s theory of conceptual fields as an alternative theoretical framework for research in problem solving in sciences [1, 2] as well as a plausible referent for integrating the mental models of Johnson-Laird to the action schemes of Vergnaud [3, 4] have provided some light to interpret research findings of this study.

Precisely, what makes possible the characterization of some operational invariants – theorems and concepts in action – is their use in problem solving. The need emerges, then, for identification and, therefore, for investigation and documentation of them. Specifically, the problem solving in non-sliding rotational motion requires the use of many concepts and their relationship, the comprehension of which presents different levels of difficulty, especially in solving problem-situations that interrelate them.

II. THEORETICAL FRAMEWORK

Vergnaud's theory of conceptual fields[1] is a psychological theory of concepts [5], a cognitive theory of the process of conceptualization of reality. It is a pragmatic theory inasmuch as it presupposes that knowledge acquisition is shaped by situations, problems and actions of the subject. It is, therefore, through the situations that a concept acquires meaning to a student. It is, furthermore, a theory of the cognitive complexity, which contemplates the development of progressively dominated situations, of the concepts and theorems needed to successfully operate in these situations and of the words and symbols that can effectively represent these concepts and operations to the individual, according to his/her level of cognition.

Gérard Vergnaud, Piaget´s disciple, in his theory, enlarges and redirections Piaget's focus on the general logic operations, on the general structures of thought, to the study of the cognitive functioning of the "subject-in-action". Besides that, differently from Piaget, he assumes as frame of reference the content of knowledge itself and the conceptual analysis of the domain of such knowledge [6, 7]. To Vergnaud, Piaget did not realize how the cognitive development depends on situations and on specific conceptualizations necessary to deal with them [6].

Vergnaud considers that a concept is a triplet of sets [8, 5, 7]: **C=(S, I, L)** where

S: set of situations which give sense to a concept (*the referent*);

I: set of operational invariants associated to the concept (*the meaning*);

L: set of linguistic and non linguistic representations which allow for the symbolic representation of a concept, its attributes, the situation to which it applies and the procedures which it nourishes (*the significant*).

Vergnaud assigns to the term *situation* a limited, though sometimes ample and varied, meaning, the one of task or problem to be solved. To him, the situations are the ones that give meaning to a concept and the meaning is not in the situation itself. A concept becomes meaningful to a subject through a variety of situations and of different aspects of the same concept which involve such situations.

Among the individuals, what is developed are ways of organizing the activity. To develop such notion, Vergnaud used and redefined Piaget's concept of scheme[2]. He calls scheme an *invariant organization of behavior to a given class of situations* [5, 6, 9, 10]. It is not the behavior which is invariant, but its organization. Therefore, a scheme is a universal which is efficient to a range of situations that may generate different sequences of actions, of recollection of information and of control, depending on the characteristics of each particular situation [10].

According to Vergnaud [5, 6, 11], the components of the schemes are:

-*Anticipations of the objective* to be achieved, of the effects to be expected and of the occasional intermediate stages;

-*Rules of action* such as "if… then" which allow the generation of the subject's sequence of action; that is, rules of information search and of control of the results of the actions;

-*Operational invariants* (…) which guide the recognition of the elements belonging to the situation and the information taking on the situation to be dealt with. These are the knowledge contained in the schemes;

-*Possibility of inferences* (or reasoning) which allow "to calculate" – here and now – the rules and anticipations from the information and operational invariants which the subject has available.

To Franchi [7] the absence of an appropriate conceptualization is at the origin of the systematic mistakes made by students. However, the *operational invariants* are the ones that articulate practice to theory, that is, the ones to make the articulation essential, once the perception, the search and the selection of the information would be based completely on the *concepts-in-action* system available to the student (objects, attributes, relations, conditions, circumstances) and on the *theorems-in-action* subjacent to his/her behavior.

The *operational invariants* refer to objects, properties and relations which are kept through a series of situations. They determine what belongs, or not, to a specific concept. This knowledge, obviously, does not appear as in its disciplinary formulation – physics, mathematics, etc.- but is used in the action and in the resolution of tasks, situations, problems. Vergnaud denotes them to show their similarity to the corresponding categories of thought as defined in the light of logic, stressing here its implicit character: "*The operational invariant implies the construction of stable objects of thought which allow the construction the subject's rules of actions*" [12].

A theorem-in-action is a proposition considered true upon reality; a concept-in-action is a category of thought considered pertinent [9, 10].

On the other side, to Vergnaud, "*problem is everything that, in one way or another, implies, from the student, the construction of an answer or an action which produces a certain effect*" [8]. The most important criteria – as for this

[1] A comprehensive description of Vergnaud's theory of conceptual fields and its implications to the research and teaching of science can be found in [3] and in [2].

[2] This is not a simple concept, for the same word has been used with many different meanings in cognitive psychology. To this aspect one may consider its differentiation in [1].

expert – is the activity and action in situation, or what psychologists denote as "problem solving" with a much broader meaning than it has for physicists and mathematicians. The notion of problem *contains*, therefore, the idea of *novelty*, of something never done, of something still not understood (of a challenge). This does not mean, however, that the cognitive system through which the subject approaches the new problem is also new, quite the contrary, it is usually an old system, solidly acquired [8]. This conception of problem opens way, on one side, to an inclusive teaching of physics, and, on the other, to a physics that searches for more meaning [2].

The research on science education, traditionally, has identified problem solving and concept formation as disassociate and differentiated; seeing problem solving, many times, as a new combination of actions and rules which rely on the knowledge already formed, and the elaboration of concepts, as the emerging of new categories, of new ways of conceptualizing the world, of new objects and of new properties of these objects.

According to Vergnaud, and to our understanding, considering problem solving and concept formation this way is a mistake, for it underestimates two aspects: the symbolic representation and the concepts present in the resolution of problems on one side; and the problem solving which appear, in concept formation on the other. These two elements form the same thing: *the conceptualization*.

The study of conceptual fields, undertaken in mathematics by Vergnaud, may be easily extended to other areas. In physics there are many conceptual fields which can not be immediately taught, neither as system of concepts nor as isolated concepts. An evolutionary perspective of learning in these fields is necessary. We believe that the topic non-sliding rotational motion may offer important advances in this direction.

In synthesis, the key concepts of the conceptual fields' theory are, besides the concept of *conceptual field* itself, the concepts of *scheme, situation, operational invariant (theorem-in-action and concept-in-action)* and the conception of *concept*.

As stated before, this theory stresses that the acquisition of knowledge is shaped by the situations and problems previously dominated and that this knowledge has, consequently, many contextual characteristics.

III. RESEARCH METHODOLOGY

This is a qualitative, exploratory, kind of research, where the data is grouped according to categories which are not foreseen by the theoretical framework. The categories emerge from the analysis of the data [13] grouping the ones that have similar characteristics. This implies an immersion in them which allows to get to know their similarities and differences in such a way as to be able to find a quality that describes them the most accurate possible. We have analyzed more the processes than the results, although it is also our interest to know if the students are able to reach a correct result.

In classic cognitive psychology, language is conceived, above all, as a vehicle for expressing thought, the representations that an individual has built in his/her relation to the physical world. It is shown here that the language – mathematical, graphical, etc. – of the solver is used as a vehicle of meanings (as done in other studies) and of inferences made when elaborating a solution.

One of the teachers' tasks is seeing that the errors come to light so that they can analyze them and, thus, to detect which are the obstacles to overcome [14]. This means that the understanding of the problems of teaching and learning rely at the same time in the analysis of the predicative[3] forms and of the operative forms of knowledge.

With such studies we intend to value the use of problem-situations. Here, we analyze one, which has the objective of characterizing the motion of bodies along a slope, by means of two approaches: from the kinematics point of view, as a uniformly accelerated rectilinear motion, and from the rotational dynamics standpoint.

Records were taken of the interaction with students in consulting appointments as well as in moments of group or individual problem solving, as, for example, during and after course evaluations. This set of techniques: informal interview in appointments, field observation and the productions of the students, allowed us to obtain a good amount of material, which supplied the data in context. The research was carried out in real classroom situation, during the second semester of 2004, in the subject Physics I of the School of Engineering and in Physics I of the *Licenciatura* course in Geology of the Exact, Physics and Natural Sciences College of the National University of San Juan, Argentina. The credit course time – in the first case – was 10 (ten) hours a week, in a quadrimestral schedule and with a previous course on calculus, algebra and mathematical analysis; while In the *Licenciatura* course in Geological Sciences, Physics I is a subject with six hours a week, annual schedule and parallel course in Mathematics I. We analyzed the written resolutions of 41 students of the first year of Engineering (during the third evaluation undertaken by the students) and of 16 students of the first year of Geology (during their fourth partial evaluation), respectively; to the following problem situation:

A hollow sphere and a cylinder, both having the same mass and radius, set out from rest and roll along the same slope. Which one of them reaches the bottom first? Why?

The solution can be expressed by making considerations on energy, or even, on force and torque. The resolution that sets out from considerations on energy, valuing its conservation, bases the election of the first body to reach the bottom according to the higher velocity of the mass center associated to it:

$$WF_{NC} = \Delta E_M = 0 \quad \Rightarrow \quad \Delta E_{PG} + \Delta E_C = 0$$

$$\Delta E_{PG} = - \Delta E_C$$

[3] That is, be able to explicit the objects, concepts and their properties [9]. For instance, the same concept changes the conceptual level when is appears in a statement as a noun (in this case, it is object of thinking and theme of assertion), or as an adjective, a verb or as a relation (in this case, it is a predicate) [15]. Expliciting leads to learning to use systems of external representations and its use modifies the structure, according to a Vygotskian perspective.

$$\Delta E_{PG} = -W_P = mgh$$
$$mgh = ½\, I_{EIR}\, \omega^2 = cte \quad ; \quad v_{CM} = wr$$
$$t \;\; = f\,(I, \omega)$$

Whereas the resolution that sets out from force and torque justifies its election from the identification of the higher acceleration taken during the full time. Their answers indicate that the acceleration will relate to the different magnitudes through the fundamental equation of the rotational dynamics explaining that the only force that produces torque is the weight which acts upon the mass center of the body, for the reactions (***N*** and ***f**$_{re}$*) apply upon the same rotational axis (contact generatrix). Using the properties of the instantaneous rotational axis a solution is:

$$\sum \tau_{EIR} = m\, g\, sen\theta\, r = I_{EIR}\alpha = I_{EIR}\frac{a}{r} = const.$$
$$\rightarrow \quad t \;\; = f\,(I, a_{CM})$$

IV. ANALYSIS AND DISCUSSION OF THE RESULTS

In order to be able to solve this problem situation, a student needs to understand that both bodies set out together from rest, from the same height of the slope and roll without sliding. In this case, the rotational axis is predetermined, but it isn't fixed in space: in each instant there is a different rotational axis, given by the contact generatrix; or better, by an axis that passes through the mass center. The forces that act upon each body are their weight ***P***, the vinculum reaction ***N*** and the static friction force **f$_{re}$**, which prevents the point of contact from sliding.

To establish the equation of motion we can consider the non-sliding rotational motion as the rotational motion around an axis that passes through the mass center, to which the motion of translation of the axis of velocity ***Vcm*** is superposed. That is, every point of the body will have a velocity $\boldsymbol{v} = \boldsymbol{v_{cm}} + \boldsymbol{w} \times \boldsymbol{r}$. The vector ***w*** isn't arbitrary: it is fixed by the condition through which the points of the generatrix of contact with the slope have null velocity (rolling without sliding) $\boldsymbol{vp} = \boldsymbol{v_{cm}} + \boldsymbol{w} \times \boldsymbol{r_p}$. Since $\boldsymbol{r_p} \perp \boldsymbol{w}$, the modulus of ***w*** ends up as $w = vcm/rp$.

This motion can also be considered as pure instantaneous rotation around the contact generatrix, with the same angular velocity ***w***. Notice that in this case, in a posterior *dt* instant, the rotational axis is *other* (determined by the new contact generatrix), called, therefore, instantaneous axis. The velocity of any point is now $\boldsymbol{v} = \boldsymbol{w} \times \boldsymbol{r'}$, where ***r'*** is the position regarding a point of the rotational axis. To establish the equation of motion it will be enough to use again the fundamental equation of the rotations and its projection upon the contact generatrix is enough.

This problem links founding concepts of mechanics and their relations. The use of the error as mark or trace of a genuine intellectual activity, together with its analysis allowed us to observe that , besides the elements described in the literature, [16, 17, 18, 19, 20, 21, 22], some students showed certain problems of conceptualization of great value for basic research and for its implications for a new didactics of physics.

Categories have been designed in terms of explicit concepts and operational invariants, that is, in terms of which knowledges-in-action are being used. The valuing of these aspects allowed for the differentiation, in first instance, of five groups of difficulty, which were conceived according to limitations of the physical motion that students perceived and their interpretation from the assumed theoretical framework.

a. Students who considered that the body to reach the bottom first is related to that of greater value of *inertia*.

b. Students who identify the dependence of this arrival exclusively with a greater *kinetic energy*.

c. Students who operate inferring that at the end the speed "v" to the bottom depends only on a numerical factor obtained from considerations on energy.

d. Students who deduct from the fundamental equation of rotations that the acceleration depends only on the numerical coefficient.

e. Students who assume properties and principles which, applied to the body, allow for the determination of a winner.

As for the **first group (a)**, the kind of difficulties found appear more distributed and associated to quite elementary schemes. Reference to the magnitude "*rotational inertia*", though with a meaning reduced to mass as translational inertia and centered in a very deep rooted perceptual aspect: the visual image of a massive body (in a sense of greater volume) at fall. The students seem to present a *partial form* of explanation centered in the "*moment of inertia*" as the only variable.

Rotational inertia or moment of inertia is a simple[4] predicate. The students in this group deal with this notion in a very elementary way. The "execution" of this movement would be a strong invariant in the construction of the concept of non-sliding rotational motion, which, together with new concepts acquired during instruction (particle, inertia), would supply elements to focus the attention on the descending movement of a large size body[5].

Despite that, the absence of concepts such as rigid body, rotational inertia, "shape" of the body, rotational axis, principle of superposition, instantaneous rotation, among others, prevents from focusing the attention in critical aspects of the problem situation. In the following box we have synthetized the knowledge-in-action the students seem to sustain.

Concepts-in-action: mass, inertia, speed, particle, external force, acceleration.
Theorems-in-action: "the faster, the lesser time", "the body with more inertia arrives first", "the rotational inertia is bigger when the mass of the body moves away from the rotational axis".

[4] The body (sphere, cylinder, etc.) is the one having moment of inertia.
[5] Many people confound mass and volume and/or mass and weight. They think that an object with large mass must have a large volume and/or that something that has a large amount of matter is very heavy as well. Weight is a measure of the force exerted on a body due to gravity. Mass and weight are proportional but not equal.

The coherence of these mistakes allowed us to formulate the main theorem-in-action put at play: **The body with greater inertia[6] arrives first**. In their resolution, they used implicit rules of action, theorems and concepts-in-action which had a local range of validity.

These ideas are logical and consistent with everyday observations. The answers revealed the stability of such conceptions:

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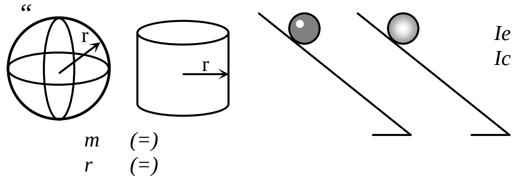

- *The sphere has a greater moment (of inertia), therefore it will arrive first at the bottom"*. Student 26.
- *"The one to arrive first is the sphere because of the greater I"*. Student 30.
- *"The sphere will arrive first because it has greater moment of inertia than the cylinder"*. Student 10.
- *"The one with the greater moment of inertia arrives first."* Student 39.
- *"The sphere arrives first because the cylinder takes longer to reach velocity due to lower moment of inertia"*. Student 18.
- *"The sphere has greater acceleration than the cylinder because the sphere has greater I than the cylinder. The sphere arrives first"*. Student Geo 04.

Besides the method of resolution employed, none of the students made a free body diagram. Some made just a scheme of the physical situation. Most just solved it using considerations on energy. In this case, they prefer to conceive the non-sliding rotational motion as a combination of the center of mass rotational motion and the rotation around itself. In exchange, if they employ torque and forces, fundamentally they solve it as pure instantaneous rotation.

With respect to the **second group (b)**, we distinguish those aspects which allude to the *kinetic energy* magnitude associated mainly to speed and/or velocity (and, in turn, to rotational inertia). That is, they show a *quite more global form* of explanation. As we know, kinetic rotational energy is a function of two physics magnitudes: rotational inertia and angular velocity, but the students seem to "synthesize it" as the only way to explain, without realizing that the variation of the kinetic energy is the same in both bodies along the considered section. This dependency of the information related to the conservation of the mechanical energy was not considered. The absence of such operational invariants allowed the selection of relevant information, making it difficult for a scientifically acceptable resolution to appear.

Kinetic energy (of rotation) is also a simple predicate. The students characterized in this group employ it in an elementary way and joined to a perceptual aspect: the visual image of a body of great rotational inertia and, therefore, of greater energy converted in kinetic energy of rotation. This execution would be another strong invariant in the construction of the concept of non-sliding rotational motion. In a body which rolls along a slope from rest until diminishing its height in "*h*", the variation of potential energy divides itself between the variation of the rotational and translational kinetic energy. The equality of total mass, "the greater the rotational inertia, the greater will be the fraction of energy converted into rotational kinetic energy". We synthetized the main knowledge-in-action the students seem to manifest:

Concepts-in-action: speed, lineal velocity, kinetic energy of rotation, body, incipient rotational inertia, incipient superposition principle, sliding.
Theorems-in-action: "The greater the rotational inertia, the greater will be the energy converted into kinetic energy of rotation". "The greater the kinetic energy of rotation, the faster". "The faster, the sooner the arrival".

The key theorem-in-action seems to be: **The greater rotational inertia, the greater energy converted into kinetic energy of rotation, therefore, sooner arrival**, remaining implicit "the greater kinetic energy of rotation, the faster" and "the faster, the sooner". The absence of concepts such as conservation of the mechanical energy, work of a force, static friction force, rotational inertia, rolling condition, would prevent from focusing the attention on the non-sliding rotational motion, resulting in difficulties to explain. That is, in explaining objects, concepts and properties:

- *"The one to arrive first is the sphere since it has greater kinetic energy at the arrival: Ec e > Ec c"*. Student 41.
- *"The cylinder arrives first. The cylinder has greater rotational inertia, therefore has greater kinetic energy"*. Student 36 (compares inertia).

Student 41 limited himself to comparing kinetic energy at the bottom of the slope, without relating neither work nor gravitational potential energy.

A difference that seemed important to us in relation to the first group is that we noticed the presence of an incipient notion of "body" different from the notion of "the particle model" (model which had been being worked with up to here). The frequency of appearance of the second category is quite smaller, though they continue sharing a scarce work with the operational forms of knowledge.

The need for a **third group (c)** is based in the reference to both magnitudes: rotational inertia and kinetic energy as cause of the type of motion.

This mental representation has resulted as more difficult to characterize, it has depended on the amount of available operational invariants and on which aspects of

[6] Galileu established that every body presented resistance to change its state of motion. The concept of inertia, proposed by him, discredited the aristotelic theory.

the situation are more significant. Rotational inertia is present in a more complete way, although still reduced.

The quality of the invariants was more elaborate. They are centered in the use of considerations on energy ($W_R = \Delta E_C$ or $W_{fr} = \Delta E_M = 0$) to the rolling motion. The operational forms are more developed. They could conclude, although always linked to the calculated kinetic variable **v**. As it seems, to this group of students, prevails an operational action more important which, at not interlacing with the predicate, limits the conceptualization of reality, reaching meaning through simple comparison of speeds. That is, to each of the rolling objects, they established the conservation of mechanical energy, the theorem of work and energy, and they concluded that the speed *v* at the bottom depends, therefore, on a numerical factor (not expliciting its dependence on more conceptual aspects such as the rotational inertia (I) which alludes to how the mass is distributed).

- "$Wfre = \Delta E_M = 0$; $-\Delta Ep = \Delta Ec$; .....
$v_{eh} = \sqrt{6/5\, gh}$; $vc = \sqrt{4/3\, gh}$

The cylinder arrives first since it reaches greater final velocity than the sphere". Student 04

- " $W_R = \Delta Ec$; .....
$v_c = \sqrt{4/3\, gh} > v_{eh} = \sqrt{6/5\, gh}$

The final velocity of the CM is greater to the cylinder than to the sphere. If both bodies went over the same distance, it implies that the cylinder accelerated more than the sphere, that's why it arrived before the sphere". Student 17

The kinetic approach as uniformly varied rectilinear motion was revealed by few students. The relation between the acceleration (or the velocity) and the arrival of a body to the bottom seems to be implicit. In this last answer, other schemes appear in relation to algorithms such as: $l = ½\, a_{CM}\, t^2$; $v_{CM} = a_{CM}\, t$; $l = ½\, v_{CM}\, t$.

A rigorous and systematic interpretation of the solutions allowed for the elaboration of a grouping based on the identification of several concepts and theorems-in-action in a moderately articulated way:

Concepts-in-action: rotational inertia (incipient), rigid body, speed, lineal velocity, angular velocity, kinetic energy, mechanical energy, potential energy or work, static friction force, rotational motion, translational motion, superposition principle, rolling condition, mechanical energy conservation.
Theorems-in-action: "The loss of potential energy is equal to the increase in kinetic energy". "The body with greater velocity arrives first".

In the **fourth group (d)**, another system of concepts emerges, such as: torque, mass distribution, instantaneous rotational axis, etc. These students presented a more developed notion of rotational inertia. In fact, although not mentioning the dependency of the rotational inertia (*I*), the idea that the rotational inertia of a body isn't necessarily a fixed quantity appears, despite not having worked with different rotational inertia around the axis *x*, *y*, and *z*[7]. The knowledge-in-action was summarized as follows:

Concepts-in-action: rotational inertia, torque, rigid body, vector, vectorial product, acceleration (angular and lineal), instantaneous axis of rotation, pure rotation, external forces.
Theorems-in-action: "The body with greater acceleration arrives first". "To change the state of rotational motion of a body, the application of a torque is necessary". "The greater the torque, the greater the acceleration".

The characterization of this category was difficult. It depended strongly on the quantity and quality of varied knowledge-in-action put at play and on the aspects of the situation which were more relevant to them. All of them inferred from the external representation systems used[8]. They were able to conclude, though always recognizing that the kinetic variable calculated ***a*** is greater to the cylinder than to the sphere depending on the numerical factor, not expliciting the dependency on the rotational inertia. That is, from the particular axis over the one which it rotates and from the way in which the mass is distributed around the axis of rotation:

- "$\sum\tau\ ext\ EIR = I\,\alpha$; (.....)
$aCM\ e = 3/5g\ sen\theta < aCM\ c = 2/3g\ sen\theta$
The cylinder arrives first because it has greater acceleration". Student 16

- "$\sum\tau = I\,\alpha$; (.....) (distinguishes different I to both bodies and compares acceleration)
So the cylinder arrives first because it has greater acceleration". Student 01

- "*As we can see, the acceleration of the mass center of the cylinder at the bottom of the slope is greater than that of the sphere, therefore, the cylinder will arrive first at the bottom".* Student 32

The consideration of the motion as pure rotation around the instantaneous axis of rotation (IAR) does not have to put at play concepts and relations such as: rolling condition, static friction force, work of a force, mechanical energy conservation. To our understanding, their only requirement as a method of solution would impoverish the gain of the so anxiously expected conceptualization. Despite a greater development of the operatory forms of knowledge in the last group, they continue entailing the rotational inertia weakly.

In a fifth group (e), the students establish the fundamental equation of the rotational dynamics and obtain an expression for the acceleration (lineal and angular) based on the involved physical magnitudes. They all make a free fall diagram. These students assume (complete or incompletely) relevant mechanical properties and principles which, applied to a body, allow to determine a winner.

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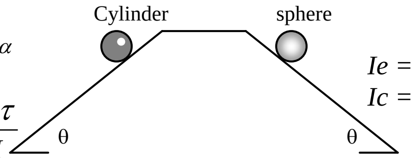
[7] This means that the rotational inertia is a magnitude still more complex than the simple scalar form we have been using in the course and which is part of a longer term psychogenetic process.
[8] None of the students of this group considered, even in an implicit way, the concepts of superposition principle and, therefore, of its equivalence to the explanation of rolling with as pure instantaneous rotation.

Some answers establish the fundamental equation of the rotational dynamics and obtain an expression for the acceleration (angular or lineal), declaring:

- "

$\Sigma \tau = I \alpha$

$\alpha = \frac{\Sigma \tau}{I}$



$Ie = ⅔\ mr^2$

$Ic = ½\ mr^2$

The cylinder falls faster[9] *because it has less moment of inertia, reaching, that way, greater angular acceleration"*. Student 28

- "*Having different moments of inertia, the angular acceleration will be affected. The one of the cylinder will be greater due to its lower rotational inertia (its particles are more distant from the rotational axis). If the acceleration of the cylinder is higher, it will take less time for it to run along the slope"*. Student Geo 04

In the light of the records and of the theoretical framework, the following knowledge-in-action may be identified:

Concepts-in-action: rotational inertia, torque, rigid body, vector, vectorial product, acceleration (angular and linear), shape of a body, $Vp= 0$, $Vcm=w\ r$, instantaneous axis of rotation, pure instantaneous rotation, external forces.

Theorems-in-action: "To the same applied torque, the body with less rotational inertia reaches higher acceleration and arrives first". "To change the state of rotational motion of a body, the application of a torque is needed".

We notice, thus, how the thinking operations are analyzed in close relation to the content worked. These operations are the main axis of conceptualization. The operational form of the distinct students evolved, as we have seen – although, sometimes, not articulated with the predicate form. We agree with Vergnaud [6] in that explanation and symbolization are important ways through which we gain or reach cognitive complexity.

VI. CONCLUSIONS

The resolution of new and partially new problem-situations requires meanings. To learn is to acquire useful information as conceptual tool to facilitate the resolution of such problems. In this study, above all, the concept of theorem-in-action was fundamental for the understanding of how the resolution of problems has its base in a conceptual or nearly conceptual representation of reality and how it habilitates the analysis of intuition in physics terms.

The analysis of the students' answers from the theoretical framework allowed the interpretation of some difficulties in learning about non-sliding rotational motion related to the structure of reasoning which would be used to solve the task; and, in particular, of the difficulties associated to the understanding of the role of rotational inertia and of acceleration of the mass center (or yet, of the velocity of the mass center when they opt for considerations on energy) in determining what bodies of different forms arrive first at the bottom of the same slope.

We are facing knowledge in construction. Neither rotational inertia nor kinetic energy are alternative conceptions. Nevertheless, we are dealing with methodologies of work based on alternative conceptual systems with different *knowledge-in-action* at its base. The properties and relations of the concepts (rotational inertia, kinetic energy, torque, etc.) come to play from the situations in which the students are involved and will, probably, be involved in. Their learning relies in common sense and is built from it. As meaningful learning occurs, the individual's mind organizes itself. Generally, we have a simple sequential mental representation in close relation to the intuition and to the step by step.

The most interesting aspect of the conceptual richness of the analysis and of the students' reasoning was the identification and signification of the dependency and independence of the information.

We could notice that in some "organization forms" the students offered their conclusions relating numbers and not physics magnitudes. They were not reasoning, despite the important unfolding of operational forms, which are not in themselves enough for reaching conceptualization once it is necessary that they evolve joined to the predicate forms. It's fundamental to learn and to teach to reason based on properties and principles.

We verified through this study that the students used different types of significant to specify, precise, represent and communicate invariants. According to Vergnaud, the operational invariants "support" the representation in the level of the signifier, while the language and other symbols "support" it in the level of the significant. At last, the meaning of the concepts is in the operational invariants.

All of this has important implications to the classroom, to the formal or non-formal education. Often we, teachers and researchers, and even education authorities, lose track of the long way it takes to the construction of knowledge, and of the most basic intervention necessary to be done in a systematic and intentional form, with specific strategies derived from the associated difficulties.

The resolution of problems has, many times, reduced its function to a simple instrument of information transposition: a straightly technocratic perspective, which has left aside all the weight that the scientific culture and history have exerted upon the resolutions. Nevertheless, when we study the history of the resolution and of the solved problem, we are impressed by the importance these factors have had.

This kind of study aims at acting, on one side, as a starting point of reflections and strategies which specially guide the attention to the formal teaching of the content, increasing the theoretical support for the mediation in physics education. On the other side, from a greater

[9] "Falling" faster seem to be like "arriving" first, in the sense of dropping faster. Although our objective is not to catalog alternative conceptions, the relation between intuitive knowledge and construction of scientific knowledge is narrow. The latter may find support in the first.

deepening which would allow the identification of some medullar difficulties that, in its turn, with the integration of other data, could be object of hypothesis in additional studies with an important degree of specificity.

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