Illuminating physics with gas-filled lamps: Exponent-rules

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Abstract

For gas-filled incandescent lamps we exploit the fact that their operation is significantly affected by coiling of the filament, convective-conductive loss in the Langmuir sheath and backscattering of tungsten atoms by the gas. These features are incorporated as correction factors in the temperature indices which parameterize the intrinsic properties of tungsten such as emissivity, resistivity, evaporation rate, etc together with the bulb observables e.g., life, lumen, power, etc. A chi squares procedure is adopted to the said index corrections leading to excellent reproduction of 14 exponent-rules. The resulting temperature parameterizations can be profitably employed in any application related to gas-filled lamps.

Keywords: Tungsten filament lamps, gas-filled, Langmuir sheath, Richardson evaporation, temperature parameterizations, index corrections, exponent-rules.

Resumen

Para lámparas incandescentes llenas de gas aprovechamos el hecho de que su funcionamiento es afectado significativamente por el bobinado del filamento, la pérdida convectiva-conductiva en la vaina de Langmuir y por la retrodispersión de los átomos de tungsteno por el gas. Estas características son incorporadas como factores de corrección en los índices de temperatura que parametrizan las propiedades intrínsecas del tungsteno tales como emisividad, resistencia, velocidad de evaporación, etc., junto con las observables de la bombilla, por ejemplo, la vida, intensidad, electricidad, etc. Se adopta un procedimiento de chi cuadrada para dichas correcciones índices llevando a una reproducción excelente de 14 reglas de exponentes. Las parametrizaciones de temperatura resultantes pueden ser rentables en cualquier aplicación relacionada con lámparas llenas de gas.

Palabras clave: Lámparas de filamento de Tungsteno, llenadas de gas, vaina de Langmuir, evaporación de Richardson, parametrizaciones de temperatura, correcciones de índice, reglas de exponentes**.**

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I. INTRODUCTION

Exponent-rules [1] for filament lamps are a beautiful set of experimentally determined empirical relations among various observables viz. life, lumen, lumen per watt, voltage, current, power and resistance. They serve to specify how the concerned observable changes when the bulb is operated at other than the rated voltage. For example, one of the rules telling about the way in which the life alters is

$$\frac{life}{LIFE} = \left(\frac{volts}{VOLTS}\right)^{d}$$

where the upper case letters refer to the rated voltage, lower case letters to general values and d=13.5 or 13.1 depending on whether the lamp is of vacuum or gas-filled type. There are fourteen relations of this kind well tabulated in the General Electric catalogue (Table I).

For vacuum lamps we have earlier derived [2] the functional form of such relations together with the algebraic/numerical value of the corresponding exponents with good accuracy using fundamental principles e.g., temperature dependence of tungsten properties, Joule heating of the filament, Planck's radiation law, spectral response of the eye, and Richardson evaporation of tungsten atoms. Our strategies adopted in ref. 2 was to select a pair of observables written as functions of temperature T and eliminate T between them to arrive at the desired 14 exponent rules. In a way that work amounted to an extension of van Horn [3] approach which had employed somewhat poor data on tungsten properties along with a rather tedious algebraic treatment of a limited number of lamp exponents.

The aim of the present work is to extend the exponent-rules with the following objectives in mind: (i) the analysis becomes applicable to the gas-filled lamps, (ii) the correction in the emissivity due to coiling effect is incorporated, (iii) the effect of the presence of the gas on some basic parameterization is accounted for by introducing correction factors and (iv) the accuracy of the resulting 14 exponents becomes 'excellent' vis-a-vis published catalogue. Section 2 below gives our formulation, Section 3 reports the numerical work and Section 4 summarizes the conclusions.

II. FORMULATION

A. Notations & Parameterizations

We shall consider a gas-filled incandescent filament lamp in a state of full brilliance operating at the steady state temperature T in the range 2100<T<3400 K. Many intrinsic properties of tungsten are quite sensitive functions of the temperature and it will be convenient to parameterize them in the form $\alpha T^{\beta+\delta\beta}$. The multiplicative coefficient α will not be needed in the sequel since only ratios of observables will be relevant when we shall discuss the exponent-rules although its value may be somewhat different from that of vacuum bulbs. The main index β corresponds to a straight wire burning in vacuum and its numerical value will be taken from ref. 2. The correction term $\delta\beta$ in the index accounts for the presence of the gas as well as coiling and its determination will be done through a procedure described in Section 3. Let us narrate concrete candidates below.

The emissivity ε reads

$$\varepsilon = \alpha_1 T^{\beta_1+\delta\beta_1};\beta_1 = 0.509\,. \quad (1)$$

It is expected that the correction term $\delta\beta_1$ should be significant because coiling effectively decreases the surface area and also gives rise to the so called shadow factor [4, 5, 6] which traps part of the emitted radiation causing an effective decrease of ε. Next, the resistivity ρ is accurately represented by

$$\rho = \alpha_2 T^{\beta_2} : \beta_2 = 1.203\,, \quad (2)$$

without the need of any correction $\delta\beta_2$. Since the coefficient of linear expansion of tungsten [7] is negligibly small, the change in filament's length and radius can be ignored, so that the resistance R also varies according to

$$R = \alpha_3 T^{\beta_3};\beta_3 = \beta_2\,. \quad (3)$$

At high temperatures, thermionic emission of tungsten atoms happens and the corresponding Richardson evaporation rate J per unit area is approximated by

$$J = \alpha_4 T^{\beta_4+\delta\beta_4};\beta_4 = 34.191\,. \quad (4)$$

A nonzero correction term $\delta\beta_4$ is a must owing to two reasons. First, due to coiling the net evaporation rate perceives a shadow factor. Second, because of the presence of the gas, some tungsten atoms originally ejected from the filament return back. Finally, light emission at incandescence occurs in accordance with Planck's radiation formula for a grey body. However, when this radiation is detected by the human eye the visual response is maximum at the wavelength $\lambda_m = 555$ nm. It is known that the corresponding Planck's energy density will depend on the factor

$$\left[\exp\left(hc/\lambda_m kT\right)-1\right]^{-1}_{gas} \approx \exp\left(-hc/\lambda_m kT\right)_{gas} = \alpha_5 T^{\beta_5+\delta\beta_5};\; \beta_5 = 9.600\,. \quad (5)$$

Here the suffix 'gas' emphasizes that the written Planck's function pertains to gas-filled lamps where the effect of coiling and the formation of Langmuir sheath [8, 9, 10] (i.e., a thin layer of gas of high viscosity around the filament) must be considered. This is achieved via the unknown correction index $\delta\beta_5$.

B. Bulb Observables

For gas-filled lamps although sizeable portion of input electrical power still goes into the radiative channel yet losses due to conduction through the lead/support wires and convection in the gas need to be considered. Since the conducted heat losses [6] are such a small part [8] of the overall power balance equation conduction in the lead/support wires can be neglected. Convection loss in the gas being substantial [8, 9, 10, 11, 12, 13] however, cannot be neglected and the best known theory to take this into account is conduction in the Langmuir sheath followed by convection [11]. Then the input electrical power can be written as

$$P \equiv V^2/R = \sigma\varepsilon A_0 T^4 + P_{Conv} = \alpha_6 T^{\beta_6} + \alpha_{Conv} T^{1.75}\,, \quad (6a)$$

where the radiative and convective components have been separately parameterized as functions of temperature. Now, Covington [14] has shown that these two components can be combined into a single effective term, in our notation viz

$$P \equiv V^2/R = \alpha_6' T^{\beta_6-0.28+\delta\beta_6};\beta_6 = \beta_1 + 4 = 4.509;\delta\beta_6 = \delta\beta_1\,. \quad (6b)$$

Here -0.28 represents a deterministic decrement in the relevant index due to convection loss and $\delta\beta_6 = \delta\beta_1$ arises because of the effect of coiling on the emissivity as mentioned in (1). Note that from the definition of $\delta\beta_6$ onwards all subsequent $\delta\beta$'s can be expressed in terms of $\delta\beta_1, \delta\beta_4, \delta\beta_5$. The other electrical observables of interest are voltage V and current I which vary according to

$$V = \alpha_7 T^{\beta_7+\delta\beta_7};\beta_7 = \left(\beta_2+\beta_6-0.28\right)/2 = \left(\beta_1+\beta_2+4-0.28\right)/2 = 2.716;\delta\beta_7 = 0.5\delta\beta_1\,, \quad (7)$$

$$I = \alpha_8 T^{\beta_8+\delta\beta_8}; \beta_8 = (\beta_6 - 0.28 - \beta_2)/2 = (\beta_1 - \beta_2 + 4 - 0.28)/2 = 1.513; \delta\beta_8 = 0.5\delta\beta_1 , \quad (8)$$

where use has been made of the parameterizations (1,2,3,6). Next, the question of the life τ of the bulb will be taken up for a filament which, at room temperature T_0 had length L_0, radius r_0 and density d_0. Remembering that an evaporation rate J is happening per unit area over the surface area A_0, the time taken for mass M to evaporate would be

$$\tau = M/JA_0 = r_0 d_0/2J = \alpha_9 / T^{\beta_9+\delta\beta_9}; \beta_9 = \beta_4 = 34.191; \delta\beta_9 = \delta\beta_4 \quad (9)$$

with the help of (4). Next, the calculation of the total visible light output Q lumens becomes relevant. Employing the standard Planck's distribution one can write

$$Q = \int_{\lambda_0}^{\lambda_f} \frac{683 S(\lambda)\varepsilon(\lambda,T)A_0(2\pi hc^2)d\lambda}{\lambda^5[\exp(hc/\lambda kT)-1]_{gas}} , \quad (10a)$$

where $S(\lambda)$ is the spectral luminous efficacy of the eye, $\lambda_0 = 380$ to $\lambda_f = 760$ nm gives the visible wavelength region, and $\varepsilon(\lambda,T)$ is the emissivity function depending simultaneously on the wavelength and temperature. Remembering that $S(\lambda)$ is sharply peaked at $\lambda_m = 555$ nm the slowly varying functions can be taken outside the integral getting

$$Q \approx \frac{683 A_0 (2\pi hc^2)\varepsilon(\lambda_m, T)}{\lambda_m^5[\exp(hc/\lambda_m kT)]_{gas}} \int_{\lambda_0}^{\lambda_f} S(\lambda)d\lambda. \quad (10b)$$

As the integral over $S(\lambda)$ is a constant parameterization of Q can be written as

$$Q = \alpha_{10} T^{\beta_{10}+\delta\beta_{10}}; \beta_{10} = \beta_1 + \beta_5 = 10.109; \delta\beta_{10} = \delta\beta_1 + \delta\beta_5 , \quad (10c)$$

in view of (1) and (5). Finally, the efficacy e (lumen per watt) of the bulb reads

$$e = Q/P = \alpha_{11} T^{\beta_{11}+\delta\beta_{11}}; \beta_{11} = \beta_{10} - \beta_6 + 0.28 = \beta_1 + \beta_5 - \beta_6 + 0.28 = 5.88; \delta\beta_{11} = \delta\beta_5 . \quad (11)$$

C. Derivation of Exponent-Rules

The rule which links the life to lumens will be considered first for illustration. Eliminating T between (9) and (10c) gives

$$\tau \propto \frac{1}{Q^a}; a = \frac{\beta_9 + \delta\beta_9}{\beta_{10} + \delta\beta_{10}} = \frac{34.191 + \delta\beta_4}{10.109 + \delta\beta_1 + \delta\beta_5} . \quad (12)$$

Considering this proportionality at the rated value designated by capital letters, and at general value distinguished by small letters and taking the ratio yields

$$\frac{life}{LIFE} = \left(\frac{LUMENS}{lumens}\right)^a , \quad (13)$$

as desired. Similarly all the remaining exponents-rules can be derived via suitable eliminations of temperature between observable-pairs as specified in Table II. It may be remarked that out of the14 exponents tabulated only four viz. $(\beta_1 + \delta\beta_1, \beta_2, \beta_4 + \delta\beta_4, \beta_5 + \delta\beta_5)$ are independent. The numerical work on the exponents will be taken up now.

III. NUMERICAL WORK

As is evident from 5th column of Table II there are three fundamental unknown correction terms $\delta\beta_1, \delta\beta_4$ and $\delta\beta_5$ since the remaining $\delta\beta$'s can be expressed in terms of these via Eqs. (6-11). To determine these fundamental $\delta\beta$'s a least square fit procedure was adopted by setting up

$$\chi^2 = \sum_{m=1}^{14} \left[(\text{Theoretical Exponent})_m - (\text{Experimental Exponent})_m\right]^2 , \quad (14)$$

where, for example, the starting term for m =1 has

$$(\text{Theoretical Exponent})_1 = a = (34.191 + \delta\beta_4)/(10.109 + \delta\beta_1 + \delta\beta_5) , \quad (15a)$$

$$(\text{Experimental Exponent})_1 = a = 3.86 . \quad (15b)$$

Employing the package *Nonlinear Least Squares Regression (Curve Fitter)* [15] we found

$$\delta\beta_1 = -0.178; \delta\beta_4 = 0.146; \delta\beta_5 = -1.047 , \quad (16)$$

whose substitution back in 5th column of Table II leads to the desired theoretical exponents displayed in Column 7. We now turn to discuss our results

IV. DISCUSSIONS

- It is clear from Table II that the agreement between the experimental and predicted lamp exponents is excellent giving credence to our whole approach**.**
- Our basic strategies adopted for gas-filled lamps was to incorporate the effect of coiling as well as the presence

of gas by introducing three correction indices $\delta\beta_1, \delta\beta_4$ and $\delta\beta_5$. These correspond to emissivity, evaporation rate and Planck's function, respectively. The conductive-convective power loss was taken care of by a correction index -0.28 in β_6 which was borrowed from the work of Covington [14].

- The determination of $\delta\beta_1, \delta\beta_4$ and $\delta\beta_5$ from first principles is rather impractical because many physical processes are competing [9, 10] simultaneously. For example, due to coiling the effective area of the filament decreases, part of the Stefan's radiation as well as evaporated atoms get trapped, affecting thereby the emissivity, Planck's function and evaporation rate. Similarly the presence of gas causes convection, conducts heat through Langmuir sheath, returns back a fraction of evaporated atoms [9] influencing thereby power-balance expression and evaporation rate [16]. There is no simple/neat mathematical theory dealing with these mechanisms implying that the magnitudes as well as signs of the said $\delta\beta$'s cannot be fixed *a priori*. That is why these were determined through a chi square procedure based on Eq. (14).
- The utility of Eqs. (1-11) is that having fixed the indices $\beta + \delta\beta$ from our analysis, one can employ these to parameterize tungsten properties along with bulb observables in any application.

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TABLE I. Chart of exponent-rules reproduced from General Electric Catalogue. The observables in CAPITAL (small) letters refer to operation at the rated (general) voltage.

$$\frac{life}{LIFE} = \left(\frac{LUMENS}{lumens}\right)^a = \left(\frac{LUMENS\ /WATT}{lumens\ /watt}\right)^b = \left(\frac{VOLTS}{volts}\right)^d = \left(\frac{AMPERES}{amperes}\right)^u ,$$

$$\frac{lumens}{LUMENS} = \left(\frac{volts}{VOLTS}\right)^k = \left(\frac{lumens/watt}{LUMENS/WATT}\right)^h = \left(\frac{watts}{WATTS}\right)^s = \left(\frac{amperes}{AMPERES}\right)^y = \left(\frac{ohms}{OHMS}\right)^z ,$$

$$\frac{LUMENS/WATT}{lumens/watt} = \left(\frac{LUMENS}{lumens}\right)^f = \left(\frac{VOLTS}{volts}\right)^g = \left(\frac{AMPERES}{amperes}\right)^j ,$$

$$\frac{amperes}{AMPERES} = \left(\frac{volts}{VOLTS}\right)^t ;\ \frac{watts}{WATTS} = \left(\frac{volts}{VOLTS}\right)^n .$$

TABLE II. Summary of the exponents for gas-filled lamps. Column one gives serial numbers *m*=1 to 14. Column two gives their nomenclature. Column three depicts their theoretical expression. Column four exhibits the pair of equations in the text used to derive the exponents in question. Column five gives their theoretical expressions in terms of known indices for vacuum bulbs and the three unknown correction terms. Columns six and seven show the numerical values of the exponents reported experimentally and predicted theoretically, respectively.

S. No. *m*	Symbol of the lamp exponent	Expression	Derived from equations	Expression in terms of three $\delta\beta$'s	Experimental value	Predicted value
1	a	$(\beta_9+\delta\beta_9)/(\beta_{10}+\delta\beta_{10})$	(9) & (10c)	$(34.191+\delta\beta_4)/(10.109+\delta\beta_1+\delta\beta_5)$	3.86	3.86
2	b	$(\beta_9+\delta\beta_9)/(\beta_{11}+\delta\beta_{11})$	(9) & (11)	$(34.191+\delta\beta_4)/(5.88+\delta\beta_5)$	7.1	7.1
3	d	$(\beta_9+\delta\beta_9)/(\beta_7+\delta\beta_7)$	(9) & (7)	$(34.191+\delta\beta_4)/(2.716+0.5\delta\beta_1)$	13.1	13.1
4	u	$(\beta_9+\delta\beta_9)/(\beta_8+\delta\beta_8)$	(9) & (8)	$(34.191+\delta\beta_4)/(1.513+0.5\delta\beta_1)$	24.1	24.1
5	k	$(\beta_{10}+\delta\beta_{10})/(\beta_7+\delta\beta_7)$	(10c) & (7)	$(10.109+\delta\beta_1+\delta\beta_5)/(2.716+0.5\delta\beta_1)$	3.38	3.38
6	h	$(\beta_{10}+\delta\beta_{10})/(\beta_{11}+\delta\beta_{11})$	(10c) & (11)	$(10.109+\delta\beta_1+\delta\beta_5)/(5.88+\delta\beta_5)$	1.84	1.84
7	s	$(\beta_{10}+\delta\beta_{10})/(\beta_6+\delta\beta_6)$	(10c) & (6b)	$(10.109+\delta\beta_1+\delta\beta_5)/(4.229+\delta\beta_1)$	2.19	2.19
8	y	$(\beta_{10}+\delta\beta_{10})/(\beta_8+\delta\beta_8)$	(10c) & (8)	$(10.109+\delta\beta_1+\delta\beta_5)/(1.513+0.5\delta\beta_1)$	6.25	6.24
9	z	$(\beta_{10}+\delta\beta_{10})/\beta_2$	(10c) & (2)	$(10.109+\delta\beta_1+\delta\beta_5)/1.203$	7.36	7.38
10	f	$(\beta_{11}+\delta\beta_{11})/(\beta_{10}+\delta\beta_{10})$	(11) & (10c)	$(5.88+\delta\beta_5)/(10.109+\delta\beta_1+\delta\beta_5)$	0.544	0.544
11	g	$(\beta_{11}+\delta\beta_{11})/(\beta_7+\delta\beta_7)$	(11) & (7)	$(5.88+\delta\beta_5)/(2.716+0.5\delta\beta_1)$	1.84	1.84
12	j	$(\beta_{11}+\delta\beta_{11})/(\beta_8+\delta\beta_8)$	(11) & (8)	$(5.88+\delta\beta_5)/(1.513+0.5\delta\beta_1)$	3.40	3.39
13	t	$(\beta_8+\delta\beta_8)/(\beta_7+\delta\beta_7)$	(8) & (7)	$(1.513+0.5\delta\beta_1)/(2.716+0.5\delta\beta_1)$	0.541	0.542
14	n	$(\beta_6+\delta\beta_6)/(\beta_7+\delta\beta_7)$	(6) & (7)	$(4.229+\delta\beta_1)/(2.716+0.5\delta\beta_1)$	1.54	1.54