Chaotic motion of a bimetallic circular plate

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Abstract

Considering the effect of geometric nonlinearity and uniformly distributed time-varying temperature, the bifurcation behaviors and chaotic phenomena of a bimetallic thin circular plate are investigated. First of all, the nonlinear dynamic equation for bimetallic plate is established and further reduced to Duffing equation of harmonic parametric excitation, from which the pitchfork bifurcation problem is discussed. Secondly, the critical conditions for occurrence of homoclinic and subharmonic bifurcations as well as chaos are studied theoretically by means of Melnikov function method. Finally, the chaotic motions are searched and simulated numerically with the application of Computer Algebra Systems Maple, and the Poincaré map and phase portrait are used to evaluate if a chaotic motion appears. The results indicate that there exist some chaotic motions in a heated bimetallic plate.

Keywords: Bimetallic thin plate, Time-varying temperature, Melnikov function, Subharmonic bifurcation, Chaotic motion.

Resumen

Teniendo en cuenta el efecto de la no linealidad geométrica y la temperatura variando en el tiempo distribuida de manera uniforme, se investigan los comportamientos de bifurcación y el fenómeno caótico de una placa bimetálica delgada. En primer lugar, la ecuación dinámica no lineal para una placa bimetálica está establecida y se reduce a la ecuación de excitación paramétrica armónica de Duffing, a partir de la cual se discute el problema de bifurcación de horca. En segundo lugar, en teoría se estudian las condiciones críticas para la aparición de bifurcaciones homoclínicas y subarmónicas, así como el caos por medio del método de la función de Melnikov. Por último, se buscan movimientos caóticos que son simulados numéricamente con la aplicación de sistemas de álgebra computacional de Maple, y el mapa de Poincaré y la fase retrato se utilizan para evaluar si aparece un movimiento caótico. Los resultados indican que existen algunos movimientos caóticos en una placa bimetálica calentada.

Palabras clave: Placa bimetálica delgada, temperatura dependiente del tiempo, función de Melnikov, bifurcación subarmónica, movimiento caótico.

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I. INTRODUCTION

The study of vibration for a thin plate is a very old issue. With the increasing development of nonlinear dynamics and the practical use of such plates in the last few decades, extensive analytical investigations, analogue and numerical simulations, as well as experimental observations have been dedicated to reveal the nonlinear bifurcational phenomena and chaotic characteristics of plates at large amplitude in previous publications. Yang and Sethna [1] used the averaging method to study the local and global bifurcations in parametrically excited nearly square plates, the results indicated the existence of heteroclinic loops and the occurrence of Smale horse and chaotic motion. Yang and Sethna [2] studied non-linear flexural dynamic behaviors of a nearly squared plate when the excitation frequency is close to one of the anti-symmetric modes. Based on the studies in reference [1], Feng and Sethna [3] made use of a global perturbation method to study further the global bifurcations and chaotic dynamics of thin plates under parametric excitation, and obtained the conditions in which Silnikov-type homoclinic orbits and chaos can occur. Pai and Nayfeh [4] presented a general nonlinear theory for the studies on dynamics of elastic composite plates. Hadian and Nayfeh [5] used the method of multiple scales to analyze asymmetric responses of nonlinear clamped circular plates subjected to harmonic excitations. Shu *et al*. [6] employed a double-mode approach to predict the chaotic motion of a large deflection plate by using a method of Melnikov[7]. Yeh *et al*. [8] characterized the conditions that can possibly lead to chaotic motion and bifurcation behavior for a simply supported large deflection thermo-elastic circular plate with variable thickness by utilizing the criteria of fractal dimensions, maximum Lyapunov exponents and bifurcation diagrams. Zhang [9] analyzed the global bifurcation and chaotic dynamics of a parametrically excited rectangular thin plate and found the chaotic motion from numerical simulation.

In the past decades, the researchers have conducted a number of studies on the nonlinear oscillations, bifurcations and chaos of thin plates undergoing periodic lateral or in-plane loading. However, the dynamics of nonlinear plates subjected to thermal loads has not received the same attention from investigators. Nowacki [10] derived and simplified the basic equations of thermoelastically coupled linear vibration in plates and solved the problem of transverse vibrations when the temperature field is the time harmonic function. Chang and Wan [11], Shu and his co-workers [12] extended the work of Nowacki toward nonlinear case for a rectangular plate and circular one respectively. Argyris and Tenek [13] simulated the nonlinear dynamic oscillations of laminated composite plates and shells under the action of a periodic heat load using the finite element method. Han *et al*. [14] studied the chaotic motion of a clamped elliptic plate under lateral harmonic excitation and steady-state uncoupled temperature field, the critical condition for chaotic motion was given by Melnikov function method and the path to chaotic motion was discussed by sub-harmonic theory.

It becomes obvious from the view of past literature that the motion of a large deflection plate may sometimes lead to bifurcation and chaos. As specific combinations of parameters are varied, a plate displays a wealth of nonlinear phenomena. If a system falls into chaos, its behavior is difficult to predict and control. So identifying chaotic motion and avoiding its occurrence are of great importance.

The present discussed bimetallic plates are widely used in precision instruments and micromachines. Much attention has received for the thermal stability problem of this kind of plates and shells [15, 16]. However, ther are few archival publications related to their chaotic motion and bifurcation behavior to the best of authors' knowledge. Recently, the authors adopted the selection method of reference surface of coordinates suggested by Radkowski [17] to the nonlinear vibration problem of heated thin bimetallic plates and shells to obtained the compact control equations and further their periodic solutions from the perturbation-variational method [18, 19], but still with no concern of their chaotic motion.

The objective of this paper is to study the nonlinear dynamic behaviors and chaotic motion of a thin circular bimetallic plate which has suffered a finite axisymmetric deformation under time varying temperature. The governing equations are set up in forms similar to those of classical Von Kármán's single-layered plates theory, these equations are further changed into dynamic version by Galerkin's method. The qualitative behaviors of the unperturbed system are analysed for static temperature parameter. The critical conditions for subharmonic bifurcation and chaotic motion are established by Melnikov method. Numerical simulations are executed with the aid of Maple program, the results show that chaotic motion can occur in a heated bimetallic plate.

II. DYNAMIC BASIC EQUATIONS

Consider a bimetallic circular plate with its total thickness h small in comparison with radius a is composed of two thin homogeneous isotropic metallic plates bonded at the common surface, such that no slippage can occur. The clamped immovable edge condition for plate under time varying temperature $T(t)$ is considered and the material properties of the plate are assumed to be independent of temperature.

Let h_i, ρ_i, E_i and α_i be the thickness, mass density, Young's modulus and thermal expansion coefficient of each layer. Here, $i=1,2$ represent the upper and lower layer respectively. Assuming Poisson's ratio $\nu_i=\nu$ [16], then the distance of reference surface from the lower surface is obtained as [17]

$$h_0=\frac{E_1h_1^2+2E_1h_1h_2+E_2h_2^2}{2\left(E_1h_1+E_2h_2\right)}. \qquad (1)$$

Based on Von Kármán's theory, the dimensionless governing equations of axisymmetrically large amplitude vibration under a uniform spatial distribution temperature change $T(t)$ can be derived from Hamilton's principle as follows

$$\nabla^4W+\frac{\partial^2W}{\partial\bar{\tau}^2}+\mu\frac{\partial W}{\partial\bar{\tau}}=L\left(W\ ,\Phi\right), \qquad (2)$$

$$\nabla^4\Phi=-\frac{1}{2}L\left(W\ ,W\right), \qquad (3)$$

$$\text{at } R=0\,,\quad W<\infty,\ \frac{\partial W}{\partial R}=0,\ \frac{1}{R}\frac{\partial\Phi}{\partial R}<\infty, \qquad (4)$$

$$\text{at } R=1\,,\quad W=0,\ \frac{\partial W}{\partial R}=0,\ \frac{\partial^2\Phi}{\partial R^2}-\frac{\nu}{R}\frac{\partial\Phi}{\partial R}+\lambda=0. \qquad (5)$$

By the redetermination of reference surface of coordinate, the above governing equations for double-layered plates are simplified into a form similar to those of classical single-layered plates theory.

The dimensionless quantities are related to the corresponding physical ones through the following relations.

$$R=r/a\,,\quad W=\left[C\left(1-\nu^2\right)\right]^{\frac{1}{2}}D^{-\frac{1}{2}}w\,,\quad \Phi=\varphi/D\,,\quad \lambda=\left(1-\nu\right)\alpha_m a^2T\ /D\ ,$$

$$\bar{\tau}=D^{\frac{1}{2}}\left(\rho ha^4\right)^{-\frac{1}{2}}t\,,\quad \mu=a^2\left(\rho hD\right)^{-\frac{1}{2}}\delta.$$

in which, r is the radial coordinate, t the time variable, δ the damping coefficient, w the deflection of reference surface, φ the stress function, ∇^4 and J are two partial differential operators defined as

$$\nabla^4 f = \frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial}{\partial R}\frac{1}{R}\frac{\partial}{\partial R}R\frac{\partial f}{\partial R},$$

$$L(f,g) = \frac{1}{R}\left(\frac{\partial f}{\partial R}\frac{\partial^2 g}{\partial R^2} + \frac{\partial^2 f}{\partial R^2}\frac{\partial g}{\partial R}\right),$$

and

$$C = \frac{1}{1-\nu^2}\sum_{i=1}^{2} E_i h_i, \; D = \frac{1}{3(1-\nu^2)}\sum_{i=1}^{2} E_i\left[\left(h - \sum_{k=0}^{i-1} h_k\right)^3 - \left(h - \sum_{k=0}^{i} h_k\right)^3\right],$$

$$\alpha_m = \frac{1}{1-\nu}\sum_{i=1}^{2} E_i \alpha_i h_i, \; \rho = \sum_{i=1}^{2}\frac{h_i}{h}\rho_i,$$

signify the effective extensional rigidity, flexural rigidity, first order thermal expansion coefficient and mass density, respectively(see references [18, 19] for detailed derivation of dimensionless governing equations).

The following single mode expression for W, in the usual way, is assumed.

$$W(R,\bar{\tau}) = A(\bar{\tau})\left(1-R^2\right)^2, \tag{6}$$

which has already satisfied the boundary conditions of W in equations (4-5).

Taking the time varying temperature to be of the form.

$$\lambda = \lambda_0 + \lambda_t \cos\varpi\bar{\tau}. \tag{7}$$

Substituting equations (6-7) into compatibility equation (3) and noting the boundary condition of φ, the solution for stress function may be arrived as

$$\frac{\partial\Phi}{\partial R} = -\frac{\lambda}{1-\nu}R + \left[\frac{5-3\nu}{6(1-\nu)}R - R^3 + \frac{2}{3}R^5 - \frac{1}{6}R^7\right]A^2(\bar{\tau}). \tag{8}$$

Substitution of equations (6-8) into equation (2), and application of Galerkin's method yield a nonlinear differential equation for A as

$$\frac{\mathrm{d}^2 A}{\mathrm{d}\bar{\tau}^2} + \mu\frac{\mathrm{d}A}{\mathrm{d}\bar{\tau}} - \alpha A + \gamma A^3 = QA\cos\varpi\bar{\tau}, \tag{9}$$

where

$$\alpha = \frac{320}{3} - \frac{20\lambda_0}{3(1-\nu)}, \quad \gamma = \frac{10(23-9\nu)}{63(1-\nu)}, \quad Q = \frac{20\lambda_t}{3(1-\nu)}.$$

Let $\alpha = 0$, one obtains the critical dimensionless temperature at which the plate is in buckled state

$$\lambda_{cr} = 16(1-\nu).$$

For the convenience of analysis, the new transformations of variables and parameters are introduced as

$$A = x|\alpha|^{\frac{1}{2}}\gamma^{-\frac{1}{2}}, \quad \bar{\tau} = \tau|\alpha|^{-\frac{1}{2}}, \quad \varpi = \Omega|\alpha|^{\frac{1}{2}},$$
$$\mu = \varepsilon\eta|\alpha|^{\frac{1}{2}}, \quad Q = \varepsilon f|\alpha|.$$

With this new notation, equation (9) be rewritten to the following equivalent system of first order equations

$$\begin{cases} \dot{x} = y, \\ \dot{y} = \alpha|\alpha|^{-1}x - x^3 + \varepsilon\left(fx\cos\Omega\tau - \eta y\right), \end{cases} \tag{10}$$

where dots define differentiation with respect to τ, ε is a small parameter.

III. QUALITATIVE ANALYSIS FOR UNPERTURBED SYSTEM

Apart from the ε-term, equations (10) becomes a unperturbed system, with Hamiltonian

$$H(x,y) = \frac{1}{2}y^2 - \frac{1}{2}\alpha|\alpha|^{-1}x^2 + \frac{1}{4}x^4 = h. \tag{11}$$

Such an equation describes a pitchfork bifurcation, and for different values of h, it indicates different dynamic behavior. Here three cases for the changes of static temperature parameter λ_0 are discussed as follows.

1. For $\lambda_0 < \lambda_{cr}$ or $\alpha < 0$, only one fixed point $(0,0)$, which is a center, exists in unperturbed system. The typical orbit is a closed periodic one that indicates the nonlinear oscillation in the neighborhood of the stable equilibrium position.

Following references [20, 21], the closed orbit involving the fixed point is

$$\begin{cases} x_k(\tau) = \frac{\sqrt{2}k}{\sqrt{1\text{-}2k^2}}\mathrm{cn}\left(\frac{\tau}{\sqrt{1\text{-}2k^2}}, k\right), \\ y_k(\tau) = -\frac{\sqrt{2}k}{1\text{-}2k^2}\mathrm{sn}\left(\frac{\tau}{\sqrt{1\text{-}2k^2}}, k\right)\mathrm{dn}\left(\frac{\tau}{\sqrt{1\text{-}2k^2}}, k\right). \end{cases} \tag{12}$$

With its period

$$T_k = 4\sqrt{1-2k^2}K(k).$$

Here $K(k)$ is the complete elliptic integral of the first kind and $k\in(0,1/\sqrt{2})$, satisfies $h(k)=2(1-k^2)k^2/(1-2k^2)^2$, is its modulus. sn, cn and dn are the Jacobi elliptic functions. As $\lim_{k\to 1/\sqrt{2}} T_k=0$, $\lim_{k\to 0} T_k=2\pi$, $\mathrm{d}T_k/\mathrm{d}k<0$, so $T_k\in(0,2\pi)$ decreases monotonically with k, and the increase in energy of the periodic orbit yeilds the decrease in its period.

2. For $\lambda_0>\lambda_{cr}$ or $\alpha>0$, $(0,0)$ becomes a hyperbolic saddle, connected by two homoclinic orbits, each surrounding the two new centers $(\pm 1,0)$, Small oscillation around $(\pm 1,0)$ when $h<0$ and large oscillation when $h>0$ are observed. For $h=0$, one obtains the two homoclinic orbits (here and henceforth, only the orbits in right half of phase space are discussed).

$$\begin{cases} x_o(\tau)=\sqrt{2}\operatorname{sech}\tau, \\ y_o(\tau)=-\sqrt{2}\tanh\tau\operatorname{sech}\tau. \end{cases} \tag{13}$$

When $k\in(0,1)$ satisfies $h(k)=(k^2-1)/(2-k^2)^2$, one gets a one-parameter family of periodic orbits within each of homoclinic orbit.

$$\begin{cases} x_k(\tau)=\dfrac{\sqrt{2}}{\sqrt{2-k^2}}\operatorname{dn}\left(\dfrac{\tau}{\sqrt{2-k^2}},k\right), \\ y_k(\tau)=-\dfrac{\sqrt{2}k^2}{2-k^2}\operatorname{sn}\left(\dfrac{\tau}{\sqrt{2-k^2}},k\right)\operatorname{cn}\left(\dfrac{\tau}{\sqrt{2-k^2}},k\right), \end{cases} \tag{14}$$

the period of these orbits is $T_k=2\sqrt{2-k^2}K(k)$, and $\mathrm{d}T_k/\mathrm{d}k>0$, that is to say, T_k increases monotonically with k, when $k\to 1$, $K(k)\to\infty$, T_k will approach infinity as a limit.

When $k\in(1/\sqrt{2},1)$ satisfies $h(k)=k^2(1-k^2)/(2k^2-1)^2$, one gets another one-parameter family of periodic orbits outside the homoclinic orbit.

$$\begin{cases} x_k(\tau)=\dfrac{\sqrt{2}k}{\sqrt{2k^2-1}}\operatorname{cn}\left(\dfrac{\tau}{\sqrt{2k^2-1}},k\right), \\ y_k(\tau)=-\dfrac{\sqrt{2}k}{2k^2-1}\operatorname{sn}\left(\dfrac{\tau}{\sqrt{2k^2-1}},k\right)\operatorname{dn}\left(\dfrac{\tau}{\sqrt{2k^2-1}},k\right), \end{cases} \tag{15}$$

now the orbits period becomes $T_k=4\sqrt{2k^2-1}K(k)$, and still $\mathrm{d}T_k/\mathrm{d}k>0$.

3. For $\lambda_0=\lambda_{cr}$ or $\alpha=0$, only one nonhyperbolic fixed point $(0,0)$ exists in unperturbed system, so $\lambda_0=\lambda_{cr}$ or $\alpha=0$ is a pitchfork bifrcation point.

IV. MELNIKOV FUNCTION METHOD FOR PERTURBED SYSTEM

For a pair of given prime integers m and n, as in reference [21], the Melnikov function of subharmonic orbits satisfies the resonance condition $T_k=2m\pi/n\Omega$ in perturbed system (10) is expressed by

$$M^{\frac{m}{n}}(\tau_0)=\int_0^{nT_k}\left[-\eta y_k(\tau)+fx_k(\tau)\cos\Omega(\tau+\tau_0)\right]y_k(\tau)\,\mathrm{d}\tau, \\ =-\eta J_1(m,n)+fJ_2(m,n)\sin\Omega\tau_0, \tag{16}$$

where τ_0 is the reference time. When $M^{\frac{m}{n}}(\tau_0)$ has simple zero, the parameters η and f satisfies

$$\frac{f}{\eta}>\frac{J_1(m,n)}{J_2(m,n)}=R_m(\Omega), \tag{16}$$

which gives the necessary condition for occurring subharmonic bifurcation in system. Here $R_m(\Omega)$ defines the threshold value for subharmonic periodic solution of order m. In the following, the bifurcation thresholds for three type of periodic orbits described by equations (12), (14) and (15) are given and discussed based on the results obtained in references [20, 21].

For the Melnikov function of periodic orbits (12), J_1 and J_2 in equation (16) are computed and expresses by

$$J_1(m,n)=\frac{8n}{3\sqrt{1-2k^2}}\left[\frac{1-k^2}{1-2k^2}K(k)-E(k)\right], \tag{17}$$

$$J_2(m,n)=\begin{cases} 0, & n\neq 1 \text{ or } m \text{ is odd}, \\ 2\pi\Omega^2\operatorname{csch}\left[\Omega\sqrt{1-2k^2}K'(k)\right], & n=1 \text{ and } m \text{ is even}. \end{cases} \tag{18}$$

Here $E(k)$ is the complete elliptic integral of the second kind, $K'(k)=K(k')$ the associated complete elliptic ingrals of the first kind, where k' is termed the complementary modulus and is related to k by $k'=\sqrt{1-k^2}$.

From the above two equations and the related theorem of Melnikov method, one can concludes that when $n=1$ and $f/\eta>J_1(m,1)/J_2(m,1)=R_m^{(1)}(\Omega)$, the subharmonic periodic solution with even order exists in the system. It has already been verified by Li [21] that for a given external exciting frequency Ω, the number of subharmonic bifurcation of even order that the perturbed system experiences is less than $[\Omega]/2$, here $[\Omega]$ is integer part of Ω.

With an analogous analysis for periodic orbits (14), in the present case,

$$J_1(m,n)=\frac{4nk}{3\sqrt{2-k^2}}\left[\frac{k^2-k^4}{2-k^2}\tilde{F}\left(\arcsin k,\frac{1}{k}\right)+\tilde{E}\left(\arcsin k,\frac{1}{k}\right)\right] \quad (19)$$

$$J_2(m,n)=\begin{cases}0, & n\neq 1,\\ \pi\Omega^2\operatorname{csch}\left[\Omega\sqrt{2-k^2}K'(k)\right], & n=1,\end{cases} \quad (20)$$

in which, $\tilde{F}$ and $\tilde{E}$ are the elliptic integrals of the first and second kind respectively. Thus when n=1 and $f/\eta > J_1(m,1)/J_2(m,1)=R_m^{(2)}(\Omega)$, the subharmonic periodic solution of order m exists in the system. The same analysis for periodic orbits (15) concludes that when n=1 and

$$\frac{f}{\eta}>\frac{J_1(m,1)}{J_2(m,1)}=\frac{8K(k)}{3m\pi^2\Omega}\left[\frac{1-k^2}{2k^2-1}K(k)+E(k)\right]\sinh\left[\frac{m\pi K'(k)}{2K(k)}\right]=R_m^{(3)}(\Omega) \quad (21)$$

the subharmonic periodic solution with even order exists in the system.

In a similar manner, the Melnikov function for homoclinic orbits (13) is easily given and explicity computed by

$$M(\tau_0)=\int_{-\infty}^{+\infty}\left[fx_0(\tau)\cos\Omega(\tau+\tau_0)-\eta y_0(\tau)\right]y_0(\tau)\mathrm{d}\tau=-\eta I_1+fI_2\sin\Omega\tau_0 \quad (22)$$

where, $I_1=4/3$, $I_2=\pi\Omega^2\operatorname{csch}\frac{\pi\Omega}{2}$. So if and only if

$$\frac{f}{\eta}>\frac{I_1}{I_2}=\frac{4}{3\pi\Omega^2}\sinh\frac{\pi\Omega}{2}=R_\infty(\Omega), \quad (23)$$

the Melnikov function has a simple zero and consequently the stable and unstable manifolds of the saddle intersect, this implies that there exists a horseshoe in Poincaré map, and the system may converts periodic motion into chaotic motion. Here $R_\infty(\Omega)$ is defined as the threshold value for chaotic motion.

In the space of parameters $(\Omega,\ f/\eta)$ or $(\varpi,\ f/\eta)$, the function R_∞ is the limit curve separating the choatic zones from the non-chaotic zones. This will be discussed numerically in the next section.

So far, four thresholds, $R_m^{(1)}$, $R_m^{(2)}$, $R_m^{(3)}$ and R_∞, have obtained for the perturbed system from the above theortical analysis, and only subharmonic bifurcation of $n=1$ will appear. In case of $\alpha>0$, when $m\to\infty$ (namely $k\to 1$), $h\to 0$, the following relation exists for each fixed Ω.

$$\lim_{m\to\infty}R_m^{(2)}(\Omega)=\lim_{m\to\infty}R_m^{(3)}(\Omega)=R_\infty(\Omega) \quad (24)$$

which means that in this case, the thresholds of subharmonic bifurcation will tend to the thresholds of chaotic motion, the nonlinear dynamic system enters chaotic motion in the horseshoe sense through subharmonic bifurcation of infinite times with f/η increasing gradually.

V. NUMERICAL RESULTS AND DISCUSSIONS

Numerical analyses are carried out only for the case of $\alpha>0$, that is, the case for parameters of static temperature components are greater than their critical values. It is instructive to examine the behavior of the chaotic threshold R_∞ as functions of the static temperatures parameters λ_0 and excitation frequencies parameters Ω or ϖ. A typical plot of R_∞ vs. ϖ for some fixed values of λ_0 is shown in Figure 1, from which one sees that $R_\infty(\varpi)$ graph exhibits a similar shape of parabola. Furthermore, R_∞ has a single minimum at $\varpi_{\min}$, the most chaotic frequency. This value can be computed exactly by solving the transcendental equation $\mathrm{d}R_\infty/\mathrm{d}\Omega=0$ for Ω, which gives the value of $\Omega=1.219132$ or $\varpi=1.219132\sqrt{\alpha}$ as the root of this transcendental equation, and further the minimum of R_∞ be found as $R_\infty(\Omega)=0.9479879$. Figure 1 also permits understanding how the parameter λ_0 influences the chaotic parameter region, for low values of excitation frequency, λ_0 does not affect R_∞ appreciably; for high values of excitation frequency, the probability of chaotic motion is increase with λ_0.

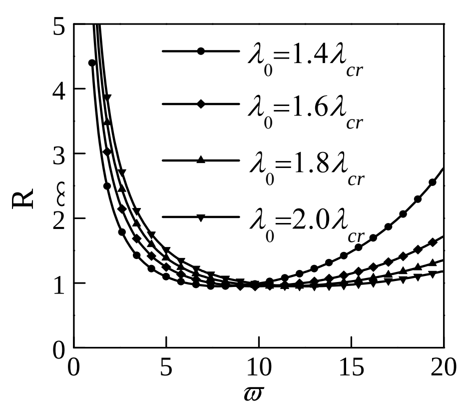

FIGURE 1. $R_\infty\sim\varpi$ graph.

Further investigations for equation (10) are developed by means of computer simulation with the application of Computer Algebra Systems Maple to find the possible chaotic solution [22]. The Poincaré map and phase portrait as well as time-displacement history technique are examined and the chaotic response is distinguished in this way from a regular one. A special group of dimensionless parameters include $\nu = 0.3$, $\lambda_0 = 1.2\lambda_{cr}$, $\varpi = 4.62$, $\lambda_t = 3.15$, $\mu = 0.2$ are taken as an example, and the criterion of Melnikov is satisfied in this case. The corresponding system features are numerically simulated with 6000 computation points and depicted in Figure 2. It is found from Figure 2 that the chaotic characteristic appears, the time-displacement histroy shown in Figure 2(a) is irregular, the phase portrait in Figure 2(b) is intertwisted, neither repeatable nor regular, the Poincaré map in Figure 2(c) reflects a complex chaotic attractor, thus we say that this is chaotic motion.

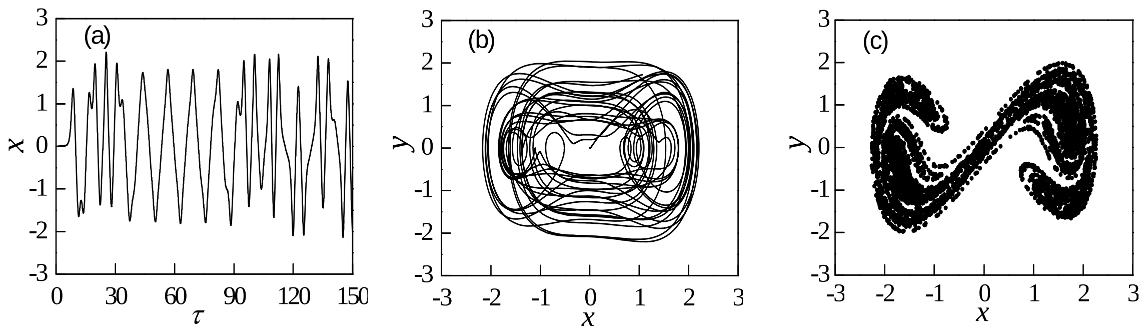

FIGURE 2. Chaotic motion: (a) time-displacement history; (b) phase portrait; (c) Poincaré map.

The results of numerical simulation illustrate that a large deflection motion of the heated bimetallic plate possess complex aperiodic behavior under various values of μ 、 λ 、 ϖ , more detailed numerical results will not be exhibited here.

VI. CONCLUSIONS

The basic equations governing the nonlinear vibration of bimetallic circular plates under uniformly distributed time-varying temperature are developed. These equations are similar to those of classical single-layered plates theory by redetermination of reference surface of coordinate. The critical conditions for occurrence of homoclinic and subharmonic bifurcations as well as chaos are studied theoretically by means of Melnikov function method. The chaotic motions are simulated numerically with the Maple program which shows the complex aperiodic behavior of bimetallic circular plates under various physical parameters.

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