Physics by Simulation: Teaching Circular Motion using Applets

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Abstract
We use web based simulation (freely available for use by everyone) to teach certain aspects of circular motion. In this article, we present the contents of a computer simulation exercise illustrating concepts of circular motion that a set of 47 students took. We discuss how the simulation and a lecture thereafter aided in student understanding of circular motion concepts. We have used carefully thought out instructions for the computer simulation in which the students play with various parameters in the simulation, attend a lecture on circular motion and solve a conceptual problem thereafter. We show that by using such simulation based activity in conjunction with lecture, student understanding of physical concepts can be greatly enhanced.

Keywords: Physics Education, Circular motion, Computer Simulation.

Resumen
Utilizamos la simulación Web (disponible gratuitamente para usarse por todos) para enseñar ciertos aspectos del movimiento circular. En este artículo, presentamos el contenido de la simulación por computadora para mostrar conceptos de movimiento circular que fue tomada por 47 alumnos. Discutimos cómo la simulación y la lectura del tema, ayudaron a los alumnos en la comprensión de conceptos del movimiento circular. Se empleó de manera cuidadosa el manejo de las instrucciones de la simulación para que el alumno pudiera variar diferentes parámetros de la simulación, poner atención a la lectura sobre el movimiento circular y después poder resolver un problema de manera conceptual. Mostramos que el uso de la simulación utilizada en conjunto con lecturas, puede mejorar la comprensión del alumno de manera altamente significativa.

Palabras clave: Física Educativa, Movimiento circular, Simulación por computadora.

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I. INTRODUCTION

In physics, we represent information about a physical system in many different ways: using words, equations, graphs, diagrams, and so on. Many students have considerable difficulty not only with creating these representations but in seeing how they express information about the system and how they are related to each other. Students in introductory physics classes treat equations as if they were only a way to calculate a variable or determine a number as a solution to an equation or a set of equations. However, physical equations have deeper meanings. They represent relationships between various observations and measurements. By setting up a simulation in which students can vary parameters and see the effect of these variations, the students' view of an equation is powerfully enriched [1].

Computer simulations can be immensely valuable tools when it comes to bridging the gap between teaching and the students' conceptual understanding of physical concepts [2, 3]. Simulations can also act as an effective means of stimulating curiosity in students. Educational research has demonstrated repeatedly that students learn much more effectively when they themselves are in control [4, 5, 6, 7]. Having simulations that students can use to explore a phenomenon on their own can produce more effective learning experiences [1, 2].

This study investigates the effectiveness of a combination of web-based computer simulation and a lecture thereafter in teaching circular motion. The paper is structured thus: we first introduce the details of the student body used for this study (Section II). We then present the simulation the students were asked to play with and the documentation to evaluate student observations from this simulation (Section III). We analyze this data to gauge the statistics of student understanding of the physical concepts from this exercise. We later present the theory contents of the lecture (section IV) that was intended to complement the student learning from the simulation. We then evaluate the effectiveness of the simulation and lecture combination by asking the students to solve a conceptual problem (Section V). We finally conclude by analyzing the students' solution to this problem statistically and show that this teaching method was indeed effective (Section VI).

II. STUDENT SAMPLE

The sample for this study included 49 students at Clarion University, Clarion, Pennsylvania. The students belonged to the class of PHY 251 (General Physics) course offered by Clarion University. This student group consists of people from various backgrounds such as chemistry, biology, geology, etc. (none of the students have had any other physics courses beyond high school). The class level of students ranged from freshman to senior undergraduate level.

A. Student preparation

Prior to working on this simulation, the students had taken a series of lectures covering introductory physics topics: Newton's laws of motion (equilibrium and non-equilibrium applications), the Normal Force and Gravitational Force, Static and frictional forces and Uniform circular motion (covering centripetal acceleration and centripetal force). The textbook followed for this course was "*Physics*" by Cutnell and Johnson [10]. These lectures were arranged to teach them how to draw free body force diagrams.

III. SIMULATION

The students were asked to visit the website http://www.mhhe.com/physsci/physical/giambattista/banked_curve/banked_curve.html. This web based freely available simulation published by McGraw-Hill publications on circular motion enables to students to explore the role of frictional force in the motion of a race car traveling down a banked circular road. A screen shot for this simulation is given below.

The students are asked to set the track incline (banking angle) to a constant value of 20^{o} and keep varying the speed (they start at a speed of 0 m/s and keep on increasing the speed by 1 m/s) and observe the speed at which the car crashes. The students were asked to make a table (Table I)

In Table I, the cases corresponding to μ =1.5, 1.2, 0.9 and 0.6 are straightforward. Lesser frictional force supports lesser speed in the centripetal term (mv^2/r, where m is the mass of the car, v is its speed and r is the radius of the circle) and hence cars have to travel slower to avoid a crash.

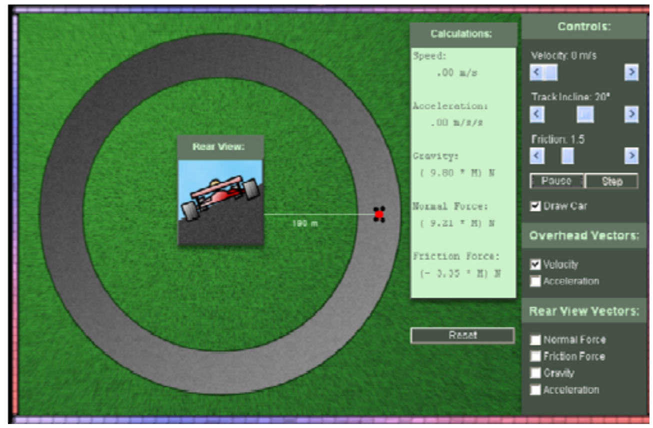

FIGURE 1. Screen shot of the simulation representing race car running on a circular track. The right side of the image shows the set parameters for this simulation.

TABLE I. Evaluation question handed out to students to evaluate their understanding of physical concepts from simulation shown in Figure 1.

Coefficient of friction, μ	Speed at which car crashes (m/s)
1.5 1.2 0.9 0.6 0.3	

However the coefficient of frictional force corresponding to μ =0.3 can be a little bit tricky. For this case, if the students start with a zero velocity, the car simply runs to the bottom of the incline, as the net force toward the center is greater than mv^2/r, the centripetal term. On the other hand, if one starts the car with a greater speed, the centripetal term mv^2/r is enough to balance the combined effect of the component of the car's weight acting down the incline as well as the frictional force (which acts towards the bottom of the incline in this case). If the speed is further increased, in this case beyond 38 m/s, the centripetal force outweighs the opposing forces and the car crashes towards the *top* of the incline. As shown in the data analysis in the next section, only some students could recognize this fact.

IV. SIMULATION DATA ANALYSIS

In order to evaluate student understanding of the physical concepts, we analyzed the table (Table I) completed by the students. The pie chart shown in figure 2 summarizes the answers to table 0 by the students.

We see that only 10.6% of the students recognized that there are two limits to the speed of the car as it moves on the inclined track, namely 0 m/s and 38 m/s. We decided to take this data as the starting point and went on to lecture the students about the forces acting on the car as the car moves along the race track with and without friction. The contents of this lecture are presented in the next section.

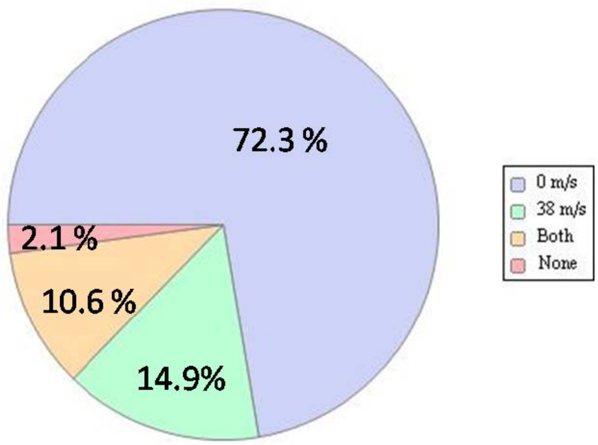

FIGURE 3. Data analysis to evaluate understanding of physical concepts by the students from simulation, by their answers to table I. It can be seen that only 10.6% of the students recognized the two cases of race car instability, based on simulation only.

II. THEORY

When a car travels without skidding around an unbanked curve, the static frictional force between tires and the road provides the centripetal force. The reliance on friction can be eliminated completely for a given speed, however if the curve is banked at an angle relative to the horizontal.

Figure 1 (a) shows a car going around a friction-free banked curve. The radius of the curve is r, where r is measured parallel to the horizontal. Figure 1 (b) shows the normal force $\vec{\mathbf{F}}_{\mathbf{N}}$ that the road applies on the car, the normal force being perpendicular to the road. Because the roadbed makes an angle θ with respect to the horizontal, the normal force has a component $F_N \sin\theta$ that points towards the center C of the circle and balances the centripetal term:

$$F_C = F_N \sin\theta = \frac{mv^2}{r}. \quad (1)$$

The vertical component of the normal force is $F_N \cos\theta$ and, since the car does not accelerate in the vertical direction, this component must balance the weight mg of the car. Therefore, $F_N \cos\theta = mg$. Dividing this equation by equation (1) shows that

$$\tan\theta = \frac{v^2}{rg}. \quad (2)$$

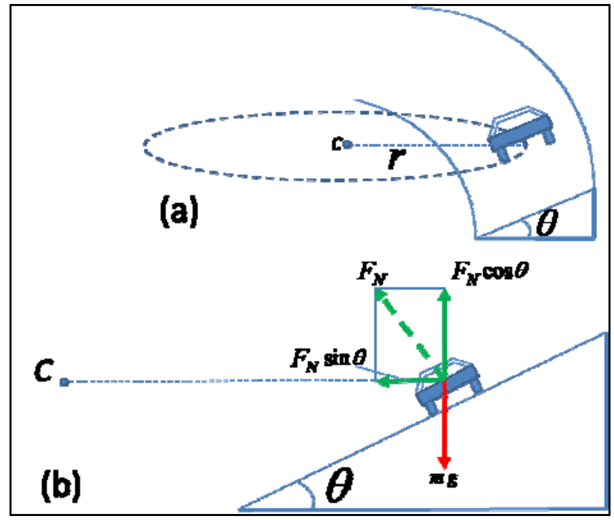

FIGURE 1. (a) Schematic of a car traveling on a frictionless banked road with banking angle θ, describing a circle with center C and radius r and (b) Free body force diagram showing forces acting on the car while describing the circle.

A. Banked curve with friction

The speed v in equation 2 represents the *only* speed with which the car can move on a banked incline. If the car tries to travel with a speed greater than v, then the centripetal term outweighs $F_N \sin\theta$ and the car tends to crash towards the shoulder *up the incline*. On the other hand, if the car tries to travel with a speed lower than v, then the force $F_N \sin\theta$ outweighs the centripetal term and the car tends to crash towards the *bottom of the incline*.

In the case of a car traveling on a banked road *with* friction, the scenario is different. As the car shows a tendency to move either to the top of the hill or towards the bottom of the hill, the frictional force will resist the movement.

So there is a maximum velocity $v_{\max}$ with the car *can* travel on a banked circular track and there is a minimum velocity $v_{\min}$ with which the car *should* travel in order to maintain its circular motion on the track. Let us explore these two velocities using Newton's laws.

B. To find $v_{\max}$

When the speed of the car approaches $v_{\max}$ it shows a tendency to move towards the top of the hill. So in this case the static frictional force acts *down the hill*. The free body force diagram on the car is shown in figure 4.

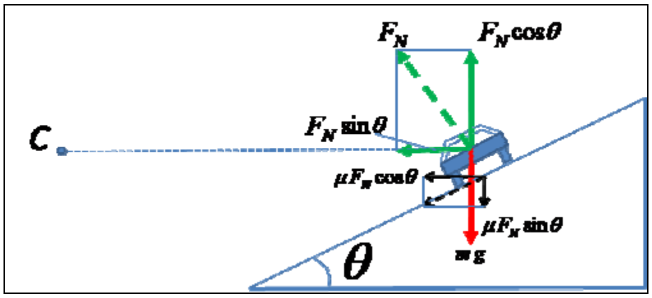

FIGURE 5. Free body force diagram of the car moving on a road with banking angle θ as the speed of the car approaches the speed v_{max}. The coefficient of static frictional force between the car and the road is μ. In this case the sum of the components of frictional force and normal force in the horizontal direction balance the centripetal term.

In this case the components of frictional force and the normal force acting down the hill balance the centripetal term. The condition for limiting equilibrium of horizontal and vertical forces yield:

$$\mu F_N \cos\theta + F_N \sin\theta = \frac{m v_{max}^2}{r}, \quad (3)$$

$$F_N \cos\theta - m g - \mu F_N \sin\theta = 0. \quad (4)$$

From equations (3) and (4) we find that the maximum speed that the car can have on the slope is given by:

$$v_{max} = \sqrt{g r \left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)}. \quad (5)$$

C. To find v_{min}

When the speed of the car approaches v_{min}, it shows a tendency to move towards the bottom of the hill. So the frictional force acts towards top of the hill. The free body force diagram for the car is shown in figure 6.

In this case the net force along horizontal direction toward the center provides the centripetal force. The conditions for limiting equilibrium of the horizontal and vertical forces yield:

$$F_N \sin\theta - \mu F_N \cos\theta = \frac{m v_{min}^2}{r}, \quad (6)$$

$$F_N \cos\theta - m g + \mu F_N \sin\theta = 0. \quad (7)$$

From equations 6 and 7 we find that the minimum speed with which the car *should* move along the circular track is given by:

$$v_{min} = \sqrt{g r \left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)}. \quad (8)$$

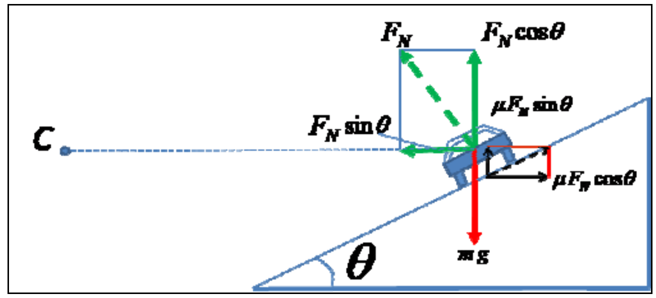

FIGURE 6. Free body diagram of the car as the car moves along the circular track with the minimum speed v_{min}. In this case the difference in the normal force and static friction components in the horizontal direction balance the centripetal term.

V. OUTCOMES OF THE STUDY

In order to evaluate the effectiveness of this pedagogy, the students were asked to solve a conceptual problem. The problem given to the students was:

Q. *Two curves of a highway have the same radii. However one is unbanked and the other is banked at an angle* θ. *A car can safely travel along the unbanked curve at a maximum speed* v_0 *under conditions when the coefficient of static friction between the tires and the road is* $\mu = 0.81$. *The banked curve is frictionless, and the car can negotiate it at the same maximum speed* v_0. *Find the angle* θ *of the banked curve.*

Solving this problem involves recognizing first of all the fact that on an unbanked road, frictional force provides the centripetal force, and the car traveling at maximum speed means maximum frictional force. Hence, $\frac{m v_0^2}{r} = \mu m g$ which implies

$$v_0 = \sqrt{\mu r g}. \quad (9)$$

In the case of banked curve without friction, speed with which the car should travel is given by substituting $\theta = 0^o$ in equation (5), which gives us:

$$v_0 = \sqrt{r g \tan\theta}. \quad (10)$$

Comparing equations (9) and (10) we have $\tan\theta = \mu = 0.81$ or $\theta = \tan^{-1}(0.81) = 39^o$.

Statistical analysis of the solutions given by the students (summarized in figure 8) showed that 73.5% of the students were able to solve the problem correctly based on the knowledge of their simulation and the following lecture. 12.2% of the students were able to correctly model the problem based on their knowledge just gained but couldn't arrive at the final right answer due to errors in their algebra/calculations. We note that 14.3% of the students were not able to model the problem correctly. This could mean that certain portions of the lecture were either not clear to students, or needed to be reinforced.

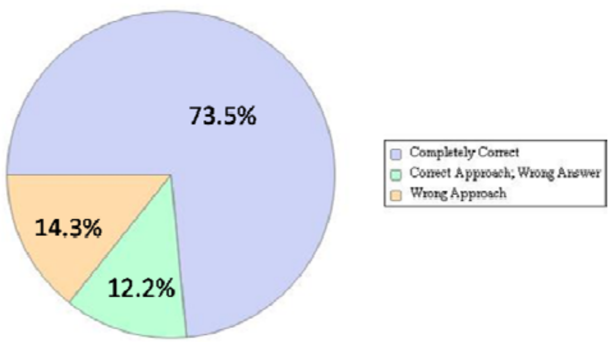

FIGURE 7. Data analysis to evaluate understanding of physical concepts by the students after they attended the lecture on circular motion. The contents of the lecture are described in section IV. The students were asked to solve the problem described in section V and their solutions were classified to be (i) completely correct, (ii) Correct approach, but wrong answer or (iii) Wrong approach.

VI. CONCLUSIONS

In conclusion, we have shown that computer simulations are an excellent way to focus students' understanding of principles of circular motion. The resources required for such pedagogical techniques are modest. For schools/universities which can afford a few computers for the entire class, using computer simulation in conjunction with lectures should be possible. Simulations also stimulate students' interest in the subject, when the computer simulation resembles a real world activity. By framing appropriate questionnaires complementing the computer simulation, student understanding of the physical concepts can be greatly enhanced.

It is interesting to ask the question as to how the students would have performed if they had *not* taken the simulation exercise prior to the lecture. This needs to be performed as a separate study by giving the same problem to two sets of students – half the students undergo the simulation exercise, whereas half do not, prior to the lecture. However this activity reported unambiguously establishes that the simulation exercise enhances the students' understanding of circular motion concepts.

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