Errors of observations and our understanding of Physics

D. C. Agrawal[1], V. J. Menon[2]
[1]*Department of Farm Engineering.*
[2]*Department of Physics, Banaras Hindu University Varanasi 221005, India*

E-mail: dca_bhu@yahoo.com

(Received 10 October 2009; accepted 17 December 2009)

Abstract
A recipe for calculating errors of observations in the undergraduate practical class is given along with the classification and importance of the same.

Keywords: Physical phenomenon, associated quantities, errors of observations.

Resumen
Se presenta una colección de errores de observaciones en clase práctica de licenciatura, asimismo se incluye su clasificación e importancia.

Palabras clave: Fenómeno físico, cantidades asociadas, errores de observaciones.

PACS: 01.30.L-, 01.50.-I, 01.50.Qb, 06.20.Dk. **ISSN 1870-9095**

I. INTRODUCTION

The broad aim of physics is to understand and explain various physical phenomena occurring in nature/laboratory through observation, experimentation and theoretical formulation. Well known examples of physical processes are the motion of planets around the sun, evaporation of water, sound emission from a tuning fork, refraction of light, attraction of iron by magnets, discharge of an electrical capacitor, decay of the pi meson, etc. Whenever we observe any physical phenomenon, or perform any experiment to measure some physical quantity or formulate a theory to explain the same, we always land up at a number or a set of numbers. For example, if you observe the motion of a planet around the sun you may be interested to quantify its angular velocity, radius of the orbit, time period of the revolution, etc and these quantities are obviously numbers. Similarly, you may easily convince yourself that the several other examples cited above can also be characterized by numbers. Now as will become clear in the sequel the outcomes of the measurements are never absolutely precise [1, 2] due to various limitations of the apparatus/experimenter/adopted-method and that is why one is neither able to reproduce it nor two or more measurements arrive at the same value. The variation in the values is caused due to the limitation of the instrument being used since it cannot measure a particular reading more precisely than the least count of the instrument; this is termed as errors of observations. The aim of the present paper is to focus attention on the classification, calculation and importance of these errors of observations. Our emphasis here will be to provide a working recipe to the undergraduate students which will enable them to carry out their laboratory work without a prior exposure to the statistical theory of errors as given by Gauss [3].

II. CLASSIFICATION

Suppose a group of five students is asked to find the density of a given copper wire using separate balances, meter scales and screw gauges. Let the numbers for the density (in gm/cm^3) reported by them be

$$8.2,\ 5.0,\ 8.39,\ 8.894,\ 9.1\ . \qquad (1)$$

The fact that these figures differ among themselves is to be attributed to the following classification of errors. The number 5.0 gm/cm^3 obviously arose from a mistake on the part of the observer since it is quite far from the standard value [4], viz. 8.96 gm/cm^3. Next the numbers 8.2 and 8.39 suffer from the so called instrumental error (or systematic error) which is associated with an improper calibration of the apparatus employed. Finally, the numbers 8.894 and 9.1 differ slightly from the standard value on account of what are known as errors of observations (or statistical errors). These arise due to the inherent limitations of the instruments as well as student's power of observing and judging as elaborated below.

In the case under study, the length measurement on the meter scale and diameter measurement on screw gauge have been done by observing scale readings, assuming that the mass of the wire has been obtained with a physical balance using the null method. Now an average student cannot have a too sharp observational ability and sometimes a scale may be read at an oblique angle rather than at right angle as desired. Therefore, a graduation should be reported by quoting the least count of the instrument as the error of observation [5, 6]. For example, in the case of copper wire of interest a student may conveniently report its length as 8.7 ± 0.1 cm using a meter scale. As far as the mass measurement via the null method using a physical balance is concerned it is convenient to take the lowest fractional weight used as the least count.

III. ERROR IN DENSITY

You may now ask that if the least counts in the measurement of length ℓ, radius r and mass m of the wire are denoted by $\delta\ell, \delta r$ and δm, respectively, then how to get the error of observation $\delta\rho$ associated with the density

$$\rho = \frac{m}{\pi r^2 \ell} \ . \tag{2}$$

To answer this, let us try a first guess by taking the logarithmic derivative

$$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} - \frac{2\,\delta r}{r} - \frac{\delta\ell}{\ell} \ . \tag{3}$$

Now, it may happen that in a given experiment the value of m, r and ℓ along with their δ's may be so arranged that by accident the right side of the above equation almost vanishes. Such an accident [7], obviously, is the outcome of the various minus signs involved in Eq. (3) and it by, by no means, implies that the density measurement was very precise. Hence, the above guess should be modified by taking the absolute magnitudes of each term on the right hand side so as to determine the maximum possible error that could have been committed in the given experiment. In other words, the correct expression for the desired error in density is

$$\delta\rho \ = \ \rho\left[\frac{\delta m}{m} + \frac{2\,\delta r}{r} + \frac{\delta\ell}{\ell}\right] . \tag{4}$$

As a numerical illustration, if the input mass, radius and length values along with their least counts were (0.78 ± 0.005) gm, (0.057 ± 0.001) cm and (8.7 ± 0.1) cm, then you may readily verify that

$$\delta\rho \ = 8.784\left[\frac{0.005}{0.78} + \frac{2\times 0.001}{0.057} + \frac{0.1}{8.7}\right],$$

$$= 8.784\,[0.0064 + 0.0351 + 0.0115]\ , \tag{5a}$$

$$= 8.784 \times 0.0530 \ = 0.47 \ \ \text{gm/cm}^3 \ . \tag{5b}$$

The physical significance of $\delta\rho$ is that it is a measure of the uncertainty in the value of the density in such a way that, with a fairly high degree of confidence [8], the unknown density would lie in the interval $\rho+\delta\rho$ and $\rho-\delta\rho$ *i.e.*, $8.3 < \rho < 9.3$ gm/cm^3. Following the guideline mentioned by Resnick and Halliday [9] we have kept more significant figures [*cf*. Eq. (5a)] in the intermediate steps than permitted in order to keep the precision in the final result. If it is rounded off at every step, this would result in reduced precision in the final result.

IV. RADIUS OF TUNGSTEN FILAMENT

Here is a case where the radius of wire has to be measured with more precision. The wire used in the fabrication of incandescent lamps is known as filament which is basically tungsten metal. Its radius is measured in mils which is related with centimeter as follows

$$1 \text{ mil} = 0.00254 \text{ cm} \ . \tag{6}$$

Some typical radii of various wattages of filament lamps [10] are listed in Table I.

TABLE I. Typical radii of various wattages of tungsten filament lamps.

Wattage (Watt)	Radius of the filament (mil)	Radius of the filament (cm)
10	0.32	0.000815
100	0.80	0.00203
1000	5.35	0.01359
10000	23.00	0.05842

The above table shows that one has to achieve precision up to five decimal places in fabricating the filaments for incandescent lamps. This precision cannot be achieved by any micrometer. The most satisfactory way to find the radius of such wires is to weigh a measured length. If w gm is the weight of the tungsten filament of length ℓ cm then

$$r = \sqrt{\frac{w}{\pi\rho\ell}} \ . \tag{7}$$

For applying this method [11] the density ρ of the tungsten metal should be known with high precision. This was determined using X-rays beam on single crystal of tungsten as 19.32 ± 0.02 gm/cm^3. Now let us examine the inherent precision of this method of finding the radius of the filament by taking the logarithmic derivative of the expression in Eq. (7)

$$\delta r = \frac{1}{2}\sqrt{\frac{w}{\pi\rho\ell}}.\left[\frac{\delta w}{w}+\frac{\delta\rho}{\rho}+\frac{\delta\ell}{\ell}\right]. \quad (8)$$

Let us consider a hypothetical case with the following data for this experiment

$$\textit{Weight of the filament}\left(w\right) = 0.095 \pm 0.001 \text{ gm},$$

$$\textit{Length of the filament}\left(\ell\right) = 40.3 \pm 0.1 \text{ cm},$$

$$\textit{Density of tungsten}\left(\rho\right) = 19.32 \pm 0.02 \text{ gm/cm}^3. \quad (9)$$

Substitution of these values in the expression (8) gives

$$\delta r = \frac{0.00623}{2}\left[\frac{0.001}{0.095}+\frac{0.02}{19.32}+\frac{0.1}{40.3}\right],$$

$$= 0.00311\left(0.01053+0.00104+0.00248\right),$$

$$= 0.00311\times 0.01405,$$

$$= 0.00004 \text{ cm}. \quad (10)$$

This is the desired precision required for the radius of tungsten filament in the fabrication of incandescent lamps as evident from some typical values given in Table I.

V. POWER RADIATED FROM TUNGSTEN FILAMENT

The examples cited so far were supported by hypothetical data. Now we report a case where the actual data are available for demonstrating the role of errors of observation. This is the value of power radiated W by tungsten filament per unit area over the temperature range 273-3655 deg. K obtained by Jones [11], Jones and Langmuir [12], Zwikker [11], and Forsythe and Worthing [11] and presented in Table II. In their experiments basically V-shaped tungsten filaments having length (ℓ) 50 cm, diameter (d) 2.61 mils and resistance $R = 7.243$ ohms were mounted in lamp bulbs and the energy radiated W' over the temperature range 273 to 3655 deg. K were measured. Thus the reported value of radiated power satisfies the relation

$$W = \frac{W'}{\pi\ell d} \text{ Watt/cm}^2. \quad (11)$$

TABLE II. The radiation data from tungsten filaments having length ℓ cm and diameter d cm in the temperature range 273 to 3655 deg. K. The results of experiments performed by Jones [11], Zwikker [11], Forsythe and Worthing [11], and Jones and Langmuir [12] quote the values of power radiated W in the units of Watts per square centimeter. The dimensions of the filaments were measured at 293 deg. K.

Temperature (K)	Jones W	Zwikker W	Forsythe and Worthing W	Jones and Langmuir W
273	0.0	…………	………..	
293	0.0	…………	…………	0.0
300	0.000016	…………….	-	0.000032
400	0.00197			0.00199
500	0.00970			0.0097
600	0.0304			0.0304
700	0.0764			0.076
800	0.169			0.169
900	0.331			0.3314
1000	0.602			0.6019
1100	1.030			1.026
1200	1.67	1.70	1.70	1.658
1300	2.58	2.70	2.60	2.566
1400	3.86	3.94	3.86	3.823
1500	5.54	5.52	5.61	5.516
1600	7.77	7.90	7.86	7.741
1700	10.6	10.7	10.73	10.59
1800	14.2	14.1	14.46	14.18
1900	18.6	18.6	18.6	18.61
2000	23.9	24.0	24.1	23.99
2100	30.3	30.5	30.4	30.46
2200	37.9	38.2	38.1	38.13
2300	46.8	47.2	47.0	47.17
2400	57.3	57.3	57.0	57.68
2500	69.2	69.4	69.2	69.81
2600	83.0	83.5	83.0	83.72
2700	98.8	100.5	98.9	99.54
2800	116.7	119.0	116.5	117.4
2900	137.2	139	136.5	137.6
3000	160.1	162	159.6	160.3
3100	186.1	189	184.2	185.6
3200	215.0	221	211	213.7
3300	247.6	254	242	245.0
3400	284.0	291	276	279.6
3500	325.0		314	317.7
3600	371.0			359.7
3655	399.4		376	382.6

The variations in the values of power radiated W Watts per square centimeter mentioned in columns 2, 3, 4 and 5 can be explained by calculating the errors associated with this particular method. For this we make use of the Planck's radiation formula and rewrite the expression (11) as

$$W = \frac{\sigma\pi\ell d\varepsilon T^4}{\pi\ell d} = \sigma\varepsilon T^4. \quad (12)$$

Here σ is the Stefan-Boltzman constant and ε is the emissivity of tungsten filament at temperature T. Now we take the logarithmic derivative of this expression

$$\frac{\delta W}{W} = \frac{\delta\varepsilon}{\varepsilon} + \frac{4\delta T}{T}, \quad (13)$$

which gives the error associated with value of W as

$$\delta W = W\left[\frac{\delta\varepsilon}{\varepsilon} + \frac{4\delta T}{T}\right]. \quad (14)$$

The values of ε reported by Jones and Langmuir [12] [*cf.* Table III] throughout temperature range 273-3655 deg K do not quote the errors associated with its values. So we will take the value of error in ε as one which enters due to rounding of the result. For example, the value of ε at 1200 deg. K is quoted as 0.141 which signifies that if its value were either 0.14051 or 0.14149, in both cases when it is rounded off to three significant figures its value will be 0.141. So the error that enters into the value of ε at 1200 deg. K will be ± 0.0005. The evaluation of error δT associated with the measurement of temperature has been carried out by Dmitriev and Kholopov [13] in the temperature range 900-3200 K but we take their values in the temperature 1200-3200 K [*cf.* Table III] since radiation measurement data of all the four experiments are available in this range.

TABLE III. Calculation of W and δW using the formulae (12) and (14) in the temperature range 1200-3200 deg. K. The corresponding experimental values of Planck's radiation are compared with range $W + \delta W$ to $W - \delta W$ and those experimental values which lie outside this range or superscripted by a star.

Temperature (K)	Emissivity ε	δT [*cf.* Ref. 12]	Theoretical value $W \pm \delta W$	Jones W	Zwikker W	Forsythe and Worthing W	Jones and Langmuir W
1200	0.141	2.0	1.66±0.02	1.67	1.70*	1.70*	1.658
1400	0.175	1.5	3.81±0.03	3.86*	3.94*	3.86*	3.823
1600	0.207	2.1	7.69±0.06	7.77*	7.90*	7.86*	7.741
1800	0.237	2.7	14.11±0.11	14.2	14.1	14.46*	14.18
2000	0.263	3.6	23.86±0.22	23.9	24.0	24.1*	23.99
2200	0.285	4.8	37.86±0.40	37.9	38.2	38.1	38.13
2400	0.304	6.0	57.19±0.67	57.3	57.3	57.0	57.68
2600	0.320	7.4	82.91±1.07	83.0	83.5	83.0	83.72
2800	0.334	9.1	116.4±1.7	116.7	119.0*	116.5	117.4
3000	0.346	10.8	158.9±2.5	160.1	162*	159.6	160.3
3200	0.357	12.9	212.3±3.7	215.0	221*	211	213.7

Now we turn to the calculation of theoretical values of W and δW by making use of the formulae (12) and (14) using the quoted values of ε and δT in Table III in the temperature (T) range 1200-3200 K and taking $\sigma = 5.67 \times 10^{-12}$ Watt/cm^{2}K^4 and $\delta\varepsilon = 0.0005$. The theoretical values so obtained are mentioned in column four of the Table III. The theoretical range $W + \delta W$ to $W - \delta W$ provide the width inside which the experimental results should lay otherwise either the errors δW associated with experimental parameters $\delta\varepsilon$ and δT are not true or something is wrong with the theory. This comparison has been carried out in Table III and

- a star is superscripted on those experimental values where it is outside the range but at higher side.
- two stars are superscripted on those experimental values where it is outside the range but at lower side.

This comparison leads us to the following observations.

- In the temperature range 1200-1600 deg. K the values [11] reported by Jones, Zwikker and Forsythe and Worthing lie outside this range whereas values obtained by Jones and Langmuir [12] are consistent with the theory.
- In the temperature range 1800-2000 the values reported by Forsythe and Wothing [11] only are not consistent with calculation.
- In the temperature range 2800-3200 only the Zwikker [11] findings lie beyond the range.
- In all the above observations the values are at higher side. Not a single case is seen where it is below the range.
- In the temperature range 2000-2600 the findings of all the four experiments are within the experimental error limit.

The above observations guide us that either there is something wrong with the theory or the values of errors $\delta\varepsilon$ and δT associated with the measurements of emissivity and temperature, respectively are not true. It is well known that the theory of Planck's radiation cannot be doubted and also we cannot comment on the reported values of emissivity. This leads we to conclude that the uncertainty δT in the measurement of temperature should have been somewhat large compared to the marginal values reported by Dmitriev and Kholopov [13] [*cf.* Table III]. In fact, Mee, Elkins, Fleenor, Morrison, Sherill and Seiber [14] have reported this uncertainty to be as large as ± 20.5 K. The calculations were repeated taking this value for δT and the resulted range $W+\delta W$ to $W-\delta W$ demonstrate that all the measured values by Jones [11], Zwikker [11], Forsythe and Worthing [11], and Jones and Langmuir [12] are now consistent with the theory. The values so obtained are not reported here and it is left as an exercise for the students.

VI. SINGLE AND MULTIPLE OSERVATIONS

The above was an example of the so called single observation composed of a set of variables measured once to determine a desired quantity in the laboratory. Such a situation often arises either because the student has to finish an experiment in one practical class of generally two hours duration or the researcher does it in his laboratory and the observations are not repeated. The latter case arises for example when temperature of a cooling object is recorded as a function of the time because at any given instant only one reading of the thermometer can be taken.

However, in many situations multiple observations are allowed, e.g. when the student is permitted to repeat the above mentioned density experiment or the thermal radiation measurements carried out by the four well known scientists over large sequence of the experiments. If s is the observable of our interest and $s_1 \pm \delta s_1, s_2 \pm \delta s_2, \ldots\ldots, s_n \pm \delta s_n$ are the individual values of the interest along with their errors, then it is convenient to introduce the mean value $\overline{s}$ of it and the associated error $\delta\overline{s}$ given by

$$\overline{s} = \sum_{i=1}^{n} \frac{s_i}{n} \tag{15}$$

$$\delta\overline{s} = \left[\frac{\sum_{i=1}^{n} (\delta s_i)^2}{n^2} \right]^{1/2} \tag{16}$$

Theoretically speaking, this $\overline{s}$ is more reliable estimate of the unknown quantity of our interest because the error of observations $\delta\overline{s}$ tends to zero as n goes to infinity provided the $\delta s_1, \delta s_2, \ldots\ldots \delta s_n$ are regarded as independent random variates. The above finding may be generalized to other observables to serve as readymade recipes to any student for calculating error of observations in the practical class even if the student has not been taught either the physical significance of the said observables or the formal theory of errors.

VII. UTILITY OF ERRORS

Knowledge of the possible error associated with a quantity is of great importance not only in academic line but also in day-to-day life. This fact will be elucidated under four sub-headings which will serve dual objectives: firstly, one will be able to refresh one's memory regarding some theoretical/experimental facts known to him/her and secondly one will enjoy learning some new facts which one might not have come across earlier.

A. Ascertaining relative importance of variables

● A glance at equations (4, 5) for the statistical error shows that every involved variable contributes its own relative error in additive manner with appropriate coefficient. These coefficients represent the powers carried by the concerned variables in the expression of the observable. For example, the coefficient 2 before $\delta r/r$ [*cf.* Equation (4)] arises because the density of the wire depends on the radius through r^2. In the present case the relative errors contributed by the mass, radius and length [cf. equation (5)] are 0.64%, 3.51% and 1.15%, respectively, totaling to 5.3%. Therefore, a student knows which variable gives maximum contribution to the error in his experiment and hence which variable should be measured with more sensitive instrument in order to increase the reliability of the final result.

● Referring back to the set of numbers equation (1) it is noticed that different students have quoted their results for density up to varying number of decimal digits. Naturally, one may ask a genuine question as to what should be the criterion to decide on the number of digits which should be retained after the decimal point. This criterion is fixed by the value of error of observations. To understand this let us look at the numerical illustration [*cf.* Equation (5)] in which the error of observation was found to be 0.47 which may be rounded off to 0.5. Therefore, the value of density of wire may be quoted up to one digit after the decimal place, viz 8.8 ± 0.5 gm/cm^3 because although there is uncertainty at the first decimal place the most probable digit in the ρ at the first place is 8. We say that final value of the desired quantity in the present experiment has two significant figures [15, 16, 17, 18, 19, 20].

B. Cases where theory needs refinement

● Next, knowledge of the error of observations can often tell us whether a proposed theory needs improvement. For example, consider the problem of determining the acceleration due to gravity *g*. Its theoretical value in terms [21] of the gravitational constant *G*, mass of the earth *M*, and mean radius of the earth *R* is known to be

$$g_{theory} = \frac{GM}{R^2} = \frac{6.67\times10^{-8}(gm\cdot cm^3\cdot s^{-2})5.98\times10^{27}(gm)}{\left(6.37\times10^{8}(cm)\right)^2}, \quad (17a)$$

$$=983 \text{ cm/s}^2 . \quad (17b)$$

● Suppose that its experimental measurement with the help of a Kater's pendulum at a given spot yields $g_{theory} = 975 \pm 5 cm\cdot s^{-2}$. Since g_{theory} lies outside the experimental error bars hence an improvement of theory is called for by incorporating corrections due to local hole/mass distribution around that spot, non-spherical shape of the earth, etc. As another example, let us take the case of spectrum of alkali atoms which should have been hydrogen-like but the precise measurements of the energy levels within an accuracy of 10^{-4} eV reveals a fine structure. To explain this experimental finding one has to incorporate the effect of spin-orbit coupling in the theory.

● As a historically interesting illustration [5], we may recall the observation made by Lord Rayleigh in 1894. Whereas a liter of nitrogen derived from the air weighed 1.2572 gm, an equal volume of nitrogen prepared from its compounds weighed only 1.2506 gm. This apparently a small difference was, however, beyond the limit of experimental error involved and a series of elaborate experiments attributed it to the presence of an unknown element viz argon in the air.

C. Cases where experiment needs refinement

● Let us look from another angle at the above example of determining *g*. If the measurements were performed using a simple pendulum the accuracy would have been poorer; for instance one could find $g_{\exp t} = (980 \pm 80)cm/s^2$. Since in this case $g_{\exp t}$ and g_{theory} have obvious overlap the experiment does not provide any evidence against the theory. However, the existence of terms representing finer details of the theory of *g* can be ascertained by doing more precise experiment using Kater's pendulum as mentioned earlier. As a further illustration consider the well known theoretical prediction that elementary particles and their anti-particles such as electron and positron will have exactly the same mass *i.e.* $m_e^- = m_e^+$. If an experiment could be devised to measure the relative mass difference $\left(m_e^- - m_e^+\right)/m_e^-$ and a non-vanishing value within an accuracy of one part in a million was found then it would lead to revision of the theory.

● As a last example we may mention the phenomenon of bending of light [22] by gravitational refractive effect produced by massive objects. Einstein's general theory of relativity predicts that the stellar light passing near the sun's edge would bend by 1.7''. To measure this tiny effect, and thereby to confirm the theory, experiments during solar eclipses had to be refined progressively so that the error of observation could be brought down to about one percent.

D. Utility in practical life

● The concept of error or uncertainty plays a role in everyday life as well. For instance, if a car manufacturer specifies that upon applying the brakes the vehicle will come to halt within a distance of $30 \pm 20\,m$ then you will clearly not go for such a car. On the other hand, if a parachute manufacturer tells that the landing speed of the parachute will be $1.5 \pm 0.5 m/s$ then you would like to purchase such an item. This is because you know from your experience that if you jump without any aids from a height *h*=1m then the landing speed, viz. $\sqrt{(2gh)} \approx 4m/s$ is safe enough for the human body.

It is hoped that this much background will be sufficient for motivating students to start physics practical in the laboratory even if the prior theory of the experiment or the distribution theory of errors has not been taught beforehand.

REFERENCES

[1] Taylor, J. R., *An Introduction to Error Analysis* (University Science Books, California 1982) Chapters 1, 2.

[2] Blair, J. M. and Eaten, B. G., *Laboratory Experiments for Elementary Physics* (Burgess Publishing Company, Minneapolis, 1983) pp. 1-22.

[3] Ref. 1. p. 108.

[4] Halliday, D. and Resnick, R., *Fundamentals of Physics,* (John Wiley, New York, 1988) Appendix D.

[5] Dutta, B. N. and Dey, K. K., *University Practical Physics* (Bookland Private Limited, Kolkata, 1975) Chapter 1.

[6] It is worth pointing out that in Reference 1, page 9, Fig. 1.2 the reading on the shown voltmeter is guessed by the corresponding author as 5.3 volts while the least count of the instrument depicted is 1 volt. Such an example would be misleading to the students since guess work should not be encouraged in physics laboratory. It would have been better if the reading was reported as 5 volts.

[7] Constantinides, P. A., *A simplified method for verifying the Stefan-Boltzmann law of radiation and determining the Stefan constant*, Am. J. Phys.**9**, 87-93 (1941). This paper describes a case of mutual cancellation of errors leading to the derivation of a somewhat incorrect radiation law.

[8] Ref. 1, pp. 15, 130.

[9] Ref. 4, p. 62.

[10] Incandescent Lamps Pamphlet TP-110R2 (General Electric Company, Nela Park, Ohio, 1984)

[11] Jones, H. A., *A temperature scale for tungsten*, Phys. Rev. **28**, 202-207 (1926).

[12] Jones, H. A. and Langmuir, I. *The characteristics of tungsten filaments as functions of temperature*, Gen. Elect. Rev. **30**, 354-361 (1927).

[13] Dmitriev, V. D. and Kholopov, G. K., *Photoelectric pyrometer for measuring the true temperature of tungsten*, Zhurnal Prikladnou Spektoskpopii **8**, 216-222 (1968).
[14] Mee, D. K. Elkins, J. E., Fleenor, R. M., Morrison, J. M., Sherrill, M. W. and Seiber, L. E., *Uncertainty of pyrometers in casting facility,* (Y-12 National Security Complex, US Department of Energy, Oak Ridge, Tennessee, 2001) Y/DX-2434
[15] Ref. 1, p.15.
[16] Ref. 2, pp. 1-21.
[17] Oberhofer, E. S., *The vernier caliper and significant figures*, Phys. Teach. **23**, 493 (1985).
[18] Logan, T. H., *Teaching significant figures*, Am. J. Phys. **32**, 258-259 (1964) [3] Ref. 1. p. 108.
[19] Obermiller, J. C., *Significant Figures Made Easy* (J. Weston Walch, Maine, 1988).
[20] Fornasini, P., *The Uncertainty in Physical Measurements* (Springer, New York, 2008).
[21] Ref. 4, pp A3, A4.
[22] Smorodinsky, Y., *Late light from mercury*, Quantum **4**, (November/December 1993) 40-43.