The Curzon-Ahlborn efficiency for three different energy converters

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Abstract
In this work we present three energy converters which have the same efficiency under certain optimum conditions of performance. The first one is the well-known Curzon-Ahlborn finite-time heat engine. The second one is an infinite-time cycle operating between two thermal bodies with finite heat capacities. The last case is a water mixer working as engine, which is designed with two stationary fluxes of water at different temperatures, T_1 and T_2, with only one exit, at temperature T, the specific heat capacity, c, is considered as a constant. The three converters under maximum power output, maximum work and maximum kinetic energy output, respectively, have the same Curzon-Ahlborn effiency, that is, $\eta_{CA} = 1 - \sqrt{T_2 / T_1}$, being T_1 and T_2 the extreme absolute temperatures involved in the performance of each converter.

Keywords: Thermodynamics, optimal operation, Finite-Time Thermodynamics.

Resumen
En este trabajo se presentan tres diferentes convertidores de energía que tienen la misma eficiencia bajo ciertas condiciones óptimas de operación. El primero se trata de la ampliamente conocida máquina de Curzon-Ahlborn operando a tiempo finito. El segundo se trata de una sucesión de ciclos de Carnot operando entre dos cuerpos térmicos con capacidad calorífica finita. El último caso es una mezcladora de agua trabajando como máquina, la cual está diseñada con la entrada de dos flujos estacionarios de agua a diferentes temperaturas, T_1 y T_2, con una sola salida, a temperatura T, con calor específico, c, que se toma como constante. Los tres convertidores bajo condiciones de máxima potencia de salida, máximo trabajo producido y máxima energía cinética obtenida, respectivamente, tienen la misma eficiencia conocida como de Curzon y Ahborn, cuya expresión es, $\eta_{CA} = 1 - \sqrt{T_2 / T_1}$, donde T_1 y T_2 son las temperaturas extremas absolutas involucradas en la operación de cada convertidor.

Palabras clave: Termodinámica, operación óptima, Termodinámica de Tiempo-Finito.

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I. INTRODUCTION

In 1975 Curzon and Ahlborn [1] introduced a Carnot-like thermal engine in which there is no thermal equilibrium between the working fluid and the thermal reservoirs at the isothermal branches of the cycle. These authors demonstrated that such an engine produces nonzero power (contrary to the Carnot engine), and that the power output can be optimized by varying the temperature of the cycle's isothermal branches. The efficiency under these conditions is

$$\eta_{CA} = 1 - \sqrt{\frac{T_2}{T_1}}\,, \qquad (1)$$

where T_1 and T_2 are the temperatures of hot and cold thermal reservoirs, respectively. This seminal paper led to the establishment of a new branch of irreversible thermodynamics, known as finite-time thermodynamics [2, 3, 4, 5], which considers a macroscopic system as a network of systems working in cycles and exchanging energy in an irreversible manner. Thermodynamic optimization in finite-time thermodynamics has been quite fruitful due to its ability to provide realistic bounds to the performance parameters of a large number of natural and artificial energy-converting systems [3, 4, 5, 6, 7, 8, 9, 10, 11]. Eq. (1) was obtained assuming that heat flows between thermal reservoirs and working fluid obey a Newton's cooling law, so this result depends on the type of heat transfer law. If a different one is used Eq. (1) is not obtained [12, 13, 14]. In this work we present an engine where no specific law is considered for heat transfer in an explicit way however we also obtain Eq. (1). Then the question is, for what kind of systems Eq. (1) is universal?. In the present work we briefly discuss three energy converters: A finite-time engine (CA-cycle), an infinite-time engine and a water mixer, where the CA-efficiency is obtained in spite of they seemingly differ in basic aspects.

II. CURZON AND AHLBORN THERMAL ENGINE

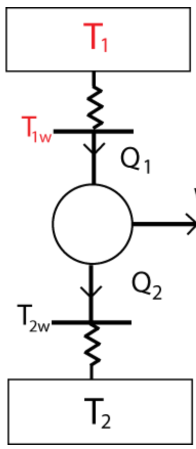

FIGURE 1. Curzon and Ahlborn thermal engine.

In figure 1, it is shown a schematic drawing of Curzon and Ahlborn (CA) engine. There are two thermal reservoirs at temperatures, T_1 and T_2 ($T_1 > T_2$). During the isothermal expansion of the working substance, the substance must be colder than the heat source, that is $T_{1W} < T_1$. The same happens for isothermal compression, in that branch the substance gets a T_{2W} ($>T_2$) temperature. As it was commented before, CA assumed that heat fluxes through the vessel containing the working substance are proportional to the gradient of temperatures. In the isothermal expansion we therefore have

$$\dot{Q}_1 = \alpha(T_1 - T_{1w}), \tag{2}$$

as the input heat.

And, over the isothermal compression, heat rejected to the heat sink is

$$\dot{Q}_2 = \beta(T_{2w} - T_2), \tag{3}$$

where, α and β are the thermal conductances, taken as constants, and the dots over $Q's$ mean derivative respect to time. The power (P) of the engine is then given by the expression

$$P = \frac{Q_1 - Q_2}{t}, \tag{4}$$

being t the cycle's period, and the efficiency of the cycle is,

$$\eta = \frac{P}{Q_1}. \tag{5}$$

Curzon and Ahlborn found that under maximum power conditions the efficiency is given by,

$$\eta_{CA} = 1 - \sqrt{\frac{T_2}{T_1}}.$$

This expression is widely known as efficiency of Curzon and Ahlborn.

III. INFINITE-TIME CYCLE

In the previous section, Eq. (1) has been obtained for the Curzon and Ahlborn engine, working at maximum power output, under a finite time operation. This efficiency seems to be a characteristic of finite-time cycles; however, it is possible to find Eq. (1) for reversible cycles working at infinite-time.

Let the next procedure be, we have two identical bodies with constant heat capacity, at T_1 and T_2 temperatures, respectively ($T_1 > T_2$), now we operate a thermal heat engine between both of them. If the bodies remain at constant pressure and there are not phase transitions,

$$W = C_p(T_1 + T_2 - 2T_f), \tag{6}$$

T_f is the final temperature of both bodies. In figure 2, it is shown a diagram of this process in an entropy *vs.* temperature diagram.

After n Carnot's cycles (small rectangles in the figure) the bodies at T_1 and T_2, initial temperatures acquire the same final temperature T_f, at this state it is not possible to obtain any more work from this device. If we want to get maximum work, then it is necessary that universe entropy production vanishes, *i.e.* $\Delta S_U = 0$.

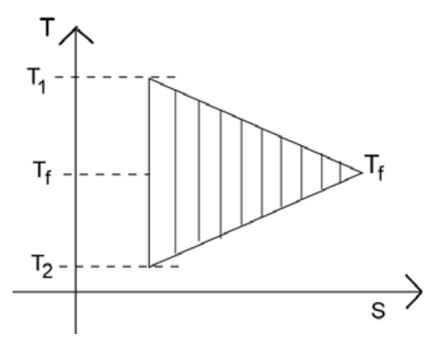

FIGURE 2. T-S diagram for heat thermal engine, working between two thermal bodies at constant pressure.

Thus,

$$\Delta S_u = C_p \ln\frac{T_f}{T_1} + C_p \ln\frac{T_f}{T_2} = 0. \quad (7)$$

If Eq. (7) is solved for T_f we straightforward get that,

$$T_f = \sqrt{T_1 T_2} \ . \quad (8)$$

Substituting Eq. (8) in (6), we obtain the maximum work

$$W_{\max} = C_p\left(T_1 + T_2 - 2\sqrt{T_1 T_2}\right). \quad (9)$$

From efficiency definition, at maximum work, we have

$$\begin{aligned} \eta &= \frac{W_{\max}}{Q_{in}}, \\ &= \frac{W_{\max}}{C_p(T_1 - T_f)}, \\ &= \frac{W_{\max}}{C_p(T_1 - \sqrt{T_1 T_2})}. \end{aligned} \quad (10)$$

That is, the Curzon and Ahlborn efficiency,

$$\begin{aligned} \eta &= \frac{C_p\left(T_1 + T_2 - 2\sqrt{T_1 T_2}\right)}{C_p\left(T_1 - \sqrt{T_1 T_2}\right)}, \\ &= 1 - \sqrt{\frac{T_2}{T_1}}, \end{aligned}$$

is obtained by means of a sequence of reversible Carnot cycles. Now, it is important be aware that the entire cycle plotted in Fig. 2 is an irreversible cycle.

IV. WATER POWERED MACHINE

In figure 3 it is shown a self-contained machine with only two equal steady streams of hot and cold water at temperatures T_1 and T_2. Its only output is a single high-speed jet of water. The heat capacity per unit mass of water, C, may be assumed to be independent of temperature.

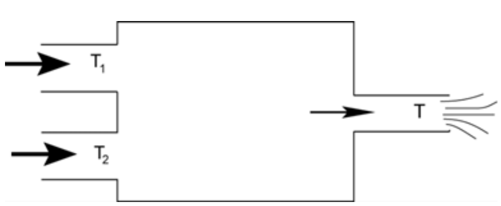

FIGURE 3. Water mixer.

The machine is in a steady state and the kinetic energy in the incoming streams is negligible. The heat intake per unit mass of water is

$$Q = \frac{\left[C(T_1 - T) - C(T - T_2)\right]}{2}. \quad (11)$$

Since the machine is in a steady state, due to energy conservation,

$$\frac{v^2}{2} = Q\,, \quad (12)$$

where $v^2/2$ is the kinetic energy per unit mass. Giving

$$v = \sqrt{C(T_1 + T_2 - 2T)}\ . \quad (13)$$

Since the entropy increase is always positive, we therefore have,

$$\Delta S = \frac{1}{2}C\left[\ln\frac{T}{T_1} + \ln\frac{T}{T_2}\right] \geq 0, \quad (14)$$

From this we have

$$T \geq \sqrt{T_1 T_2}\ . \quad (15)$$

Therefore, the maximum possible speed of the jet is given by

$$v \leq v_{\max} = \sqrt{C(T_1 + T_2 - 2\sqrt{T_1 T_2})}\ . \quad (16)$$

On the other hand, the efficiency for this energy converter is,

$$\begin{aligned} \eta &= \frac{E_{out}}{Q_1}, \\ &= \frac{v^2/2}{\frac{C}{2}\left(T_1 - \sqrt{T_1 T_2}\right)}, \\ &= \frac{C(T_1 + T_2 - 2\sqrt{T_1 T_2})}{C(T_1 - \sqrt{T_1 T_2})}. \end{aligned} \quad (17)$$

And, finally we get

$$\eta = 1 - \sqrt{\frac{T_2}{T_1}}. \quad (1)$$

That is, the same expression for the efficiency found by Curzon and Ahlborn, Eq. (1), but now we have a transformation of heat into maximum kinetic energy in a steady state flux and there is not a thermodynamic cycle by itself.

V. CONCLUSIONS

We present three different thermal engines where the efficiency of Curzon and Ahlborn is obtained, in one case, under maximum power output, another at maximum work and the last one at maximum kinetic energy output. Our first engine, proposed by Curzon and Ahlborn undergoes an irreversible global cycle in a finite time, but the inner cycle is reversible (endoreversibility condition). The second engine undergoes an overall irreversible process, with a sequence of reversible cycles and, finally the water mixer, which is not a cycle rather a steady process where heat is transformed into kinetic energy. The results suggest that the efficiency of Curzon and Ahlborn, is a general feature of thermal engines, without having the kind of universality of the Carnot's efficiency. On the other hand, is well known that Eq. (1) depends on the type of heat transfer law between working fluid and thermal baths (see figure 1), but in the others last two thermal engines, there is not an explicit heat law, consequently, efficiency is independent of any heat transfer law. This possible paradox until now has not been explained within the context of finite-time thermodynamics.

ACKNOWLEDGEMENTS

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