Inextensible flows of binormal bishop spherical images according to type-2 bishop frame in E^3

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Abstract

In this paper, we study inextensible flows of binormal Bishop spherical images in Euclidean space $\mathbf{E^3}$. Using the type-2 Bishop frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in Euclidean space $\mathbf{E^3}$.

Keywords: Type-2 Bishop frame, Energy, Curvatures, Flows.

Resumen

En este trabajo se estudian los flujos inextensibles de imágenes binormales esféricas de Obispo en el espacio Euclidiano $\mathbf{E^3}$. Utilizando el marco de Obispo tipo 2 de la curva dada, se presentan las ecuaciones en derivadas parciales. Damos algunas caracterizaciones de curvaturas de una curva en el espacio Euclidiano $\mathbf{E^3}$.

Palabras-clave: Marco Obispo tipo-2, Energía, curvaturas, flujos.

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I. INTRODUCTION

Construction of fluid flows constitutes an active research field with a high industrial impact. Corresponding real-world measurements in concrete scenarios complement numerical results from direct simulations of the Navier-Stokes equation, particularly in the case of turbulent flows, and for the understanding of the complex spatio-temporal evolution of instationary flow phenomena. More and more advanced imaging devices (lasers, highspeed cameras, control logic, etc.) are currently developed that allow to record fully timeresolved image sequences of fluid flows at high resolutions. As a consequence, there is a need for advanced algorithms for the analysis of such data, to provide the basis for a subsequent pattern analysis, and with abundant applications across various areas, [7, 8, 10, 11].

In this paper, we study binormal Bishop spherical images in Euclidean space E^3. Using the type-2 Bishop frame of the given curve, we present partial differential equations. We give some characterizations for curvatures of a curve in Euclidean space E^3.

II. PRELIMINARIES

Assume that $\{\mathbf{T},\mathbf{N},\mathbf{B}\}$ be the Frenet frame field along α. Then, the Frenet frame satisfies the following Frenet-Serret equations:

$$\begin{aligned} \nabla_{\mathbf{T}}\mathbf{T} &= \kappa\mathbf{N}, \\ \nabla_{\mathbf{T}}\mathbf{N} &= -\kappa\mathbf{T} + \tau\mathbf{B}, \qquad (2.1) \\ \nabla_{\mathbf{T}}\mathbf{B} &= -\tau\mathbf{N}, \end{aligned}$$

where κ is the curvature of α and τ its torsion and

$$g(\mathbf{T},\mathbf{T}) = 1,\ g(\mathbf{N},\mathbf{N}) = 1,\ g(\mathbf{B},\mathbf{B}) = 1,$$

$$g(\mathbf{T},\mathbf{N}) = g(\mathbf{T},\mathbf{B}) = g(\mathbf{N},\mathbf{B}) = 0.$$

The Bishop frame or parallel transport frame is an alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative, [1]. The Bishop frame is expressed as

$$\begin{aligned} \nabla_{\mathbf{T}}\mathbf{T} &= k_1\mathbf{M}_1 + k_2\mathbf{M}_2, \\ \nabla_{\mathbf{T}}\mathbf{M}_1 &= -k_1\mathbf{T}, \qquad (2.2) \\ \nabla_{\mathbf{T}}\mathbf{M}_2 &= -k_2\mathbf{T}, \end{aligned}$$

Where

$$g(\mathbf{T},\mathbf{T}) = 1,\ g(\mathbf{M}_1,\mathbf{M}_1) = 1,\ g(\mathbf{M}_2,\mathbf{M}_2) = 1,$$

$$g(\mathbf{T},\mathbf{M}_1)=g(\mathbf{T},\mathbf{M}_2)=g(\mathbf{M}_1,\mathbf{M}_2)=0.$$

Here, we shall call the set $\{\mathbf{T},\mathbf{M}_1,\mathbf{M}_2\}$ as Bishop trihedra, k_1 and k_2 as Bishop curvatures and $\mho(s)=\arctan\frac{k_2}{k_1}$, $\tau(s)=\mho'(s)$ and $\kappa(s)=\sqrt{k_1^2+k_2^2}$.

Bishop curvatures are defined by

$$k_1=\kappa(s)\cos\mho(s),$$

$$k_2=\kappa(s)\sin\mho(s).$$

Let α be a unit speed regular curve and (2.1) be its Frenet-Serret frame. Let us express a relatively parallel adapted frame:

$$\nabla_{\mathbf{T}}\mathbf{\Pi}_1=-\varepsilon_1\mathbf{B},$$

$$\nabla_{\mathbf{T}}\mathbf{\Pi}_2=-\varepsilon_2\mathbf{B}, \tag{2.3}$$

$$\nabla_{\mathbf{T}}\mathbf{B}=\varepsilon_1\mathbf{\Pi}_1+\varepsilon_2\mathbf{\Pi}_2,$$

Where

$$g(\mathbf{B},\mathbf{B})=1,\ g(\mathbf{\Pi}_1,\mathbf{\Pi}_1)=1,\ g(\mathbf{\Pi}_2,\mathbf{\Pi}_2)=1,$$

$$g(\mathbf{B},\mathbf{\Pi}_1)=g(\mathbf{B},\mathbf{\Pi}_2)=g(\mathbf{\Pi}_1,\mathbf{\Pi}_2)=0.$$

We shall call this frame as Type-2 Bishop Frame. In order to investigate this new frame's relation with Frenet--Serret frame, first we write

$$\tau=\sqrt{\varepsilon_1^2+\varepsilon_2^2}. \tag{2.4}$$

The relation matrix between Frenet-Serret and type-2 Bishop frames can be expressed

$$\mathbf{T}=\sin\mathrm{A}(s)\mathbf{\Pi}_1-\cos\mathrm{A}(s)\mathbf{\Pi}_2,$$

$$\mathbf{N}=\cos\mathrm{A}(s)\mathbf{\Pi}_1+\sin\mathrm{A}(s)\mathbf{\Pi}_2,$$

$$\mathbf{B}=\mathbf{B}.$$

So by (2.4), we may express

$$\varepsilon_1=-\tau\cos\mathrm{A}(s),$$

$$\varepsilon_2=-\tau\sin\mathrm{A}(s).$$

By this way, we conclude

$$\mathrm{A}(s)=\arctan\frac{\varepsilon_2}{\varepsilon_1}.$$

The frame $\{\mathbf{\Pi}_1,\mathbf{\Pi}_2,\mathbf{B}\}$ is properly oriented, and τ and $\mathrm{A}(s)=\int_0^s\kappa(s)ds$ are polar coordinates for the curve α. We shall call the set $\{\mathbf{\Pi}_1,\mathbf{\Pi}_2,\mathbf{B},\boldsymbol{\varepsilon}_1,\varepsilon_2\}$ as type-2 Bishop invariants of the curve α, [17].

Definition 2.1. *Let α be a regular curve in $\mathbb{E}^3$. If we translate of the third vector field of type-2 Bishop frame to the center O of the unit sphere $\mathbb{S}^2$, we obtain a spherical image ϕ. This curve is called binormal Bishop spherical image or indicatrix of the curve α*, [17].

III. INEXTENSIBLE FLOWS OF BINORMAL BISHOP SPHERICAL IMAGE ACCORDING TO NEW TYPE-2 BISHOP FRAME

Let $\alpha(u,t)$ is a one parameter family of smooth curves in $\mathbf{E^3}$.

The arclength of α is given by

$$s(u)=\int_0^u\left|\frac{\partial\alpha}{\partial u}\right|du, \tag{3.1}$$

where

$$\left|\frac{\partial\alpha}{\partial u}\right|=\left|\left\langle\frac{\partial\alpha}{\partial u},\frac{\partial\alpha}{\partial u}\right\rangle\right|^{\frac{1}{2}}. \tag{3.2}$$

The operator $\frac{\partial}{\partial s}$ is given in terms of u by

$$\frac{\partial}{\partial s}=\frac{1}{v}\frac{\partial}{\partial u},$$

where $v=\left|\frac{\partial\alpha}{\partial u}\right|$ and the arclength parameter is $ds=vdu$.

Any flow of α can be represented as $\{\mathbf{\Pi}_1,\mathbf{\Pi}_2,\mathbf{B}\}$

$$\frac{\partial\alpha}{\partial t}=\mathrm{b}_1\mathbf{\Pi}_1+\mathrm{b}_2\mathbf{\Pi}_2+\mathrm{b}_3\mathbf{B}, \tag{3.3}$$

where $\mathrm{b}_1,\mathrm{b}_2,\mathrm{b}_3\in C^\infty(\mathbb{E}^3)$.

Definition 3.1. ([10]) *The flow $\frac{\partial\alpha}{\partial t}$ in $\mathbb{E}^3$ are said to be inextensible if*

$$\frac{\partial}{\partial t}\left|\frac{\partial\alpha}{\partial u}\right|=0. \tag{3.4}$$

Lemma 3.2. *Let $\frac{\partial\alpha}{\partial t}$ be a smooth flow of the curve α according to new type-2 Bishop frame. The flow is inextensible if and only if*

$$(\frac{\partial b_1}{\partial u}+b_3\nu\varepsilon_1)\sin A=(\frac{\partial b_2}{\partial u}+b_3\nu\varepsilon_2)\cos A, \qquad (3.5)$$

where $b_1, b_2, b_3 \in C^\infty(E^3)$.

Theorem 3.3

$$\frac{\partial \mathbf{\Pi}_1}{\partial t}=[p_1-\cos A(\frac{\partial b_1}{\partial s}+b_3\varepsilon_1+\frac{\partial A}{\partial s}\cos A)]\mathbf{\Pi}_2+p_2\mathbf{B},$$

$$\frac{\partial \mathbf{\Pi}_2}{\partial t}=[p_3+(\frac{\partial b_2}{\partial s}+b_3\varepsilon_2-\frac{\partial A}{\partial s}\sin A)\sin A]\mathbf{\Pi}_1+p_4\mathbf{B},$$

$$\frac{\partial \mathbf{B}}{\partial t}=[p_5+(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\sin A]\mathbf{\Pi}_1 +[p_6-(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\cos A]\mathbf{\Pi}_2,$$

where $p_1, p_2, p_3, p_4, p_5, p_6 \in C^\infty(E^3)$.

Theorem 3.4.

$$\frac{\partial}{\partial t}\mathbf{T}^\psi=[\frac{\partial}{\partial t}(\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})+(\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})[p_3+ (\frac{\partial b_2}{\partial s}+b_3\varepsilon_2-\frac{\partial A}{\partial s}\sin A)\sin A]]\mathbf{\Pi}_1 +[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})+(\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})[p_1- \cos A(\frac{\partial b_1}{\partial s}+b_3\varepsilon_1+\frac{\partial A}{\partial s}\cos A)]]\mathbf{\Pi}_2 +[p_2(\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})+p_4(\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})]\mathbf{B},$$

$$\frac{\partial \mathbf{N}^\psi}{\partial t}=[\frac{\partial}{\partial t}(\frac{1}{\kappa^\psi}\frac{\varepsilon_2^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2}))+ (\frac{1}{\kappa^\psi}\frac{\varepsilon_1^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1}))[p_3 +(\frac{\partial b_2}{\partial s}+b_3\varepsilon_2-\frac{\partial A}{\partial s}\sin A)\sin A]-(\frac{1}{\kappa^\psi})[p_5+(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1 -b_2\varepsilon_2)\sin A]]\mathbf{\Pi}_1+[\frac{\partial}{\partial t}(\frac{1}{\kappa^\psi}\frac{\varepsilon_1^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1})) +(\frac{1}{\kappa^\psi}\frac{\varepsilon_2^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2}))[p_1-\cos A(\frac{\partial b_1}{\partial s}+b_3\varepsilon_1 +\frac{\partial A}{\partial s}\cos A)]-(\frac{1}{\kappa^\psi})[p_6-(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\cos A]]\mathbf{\Pi}_2 +[p_2(\frac{1}{\kappa^\psi}\frac{\varepsilon_2^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2}))-\frac{\partial}{\partial t}(\frac{1}{\kappa^\psi})+ p_4(\frac{1}{\kappa^\psi}\frac{\varepsilon_1^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1}))]\mathbf{B},$$

$$\frac{\partial \mathbf{B}^\psi}{\partial t}=[[\frac{1}{\kappa^\psi}\frac{\varepsilon_1^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1})- \frac{1}{\kappa^\psi}\frac{\varepsilon_2^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2})]][p_5 +(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\sin A]-\frac{\partial}{\partial t}(\frac{1}{\kappa^\psi}\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}}) +(\frac{1}{\kappa^\psi}\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})[p_3+(\frac{\partial b_2}{\partial s}+b_3\varepsilon_2-\frac{\partial A}{\partial s}\sin A)\sin A]]\mathbf{\Pi}_1 +[\frac{\partial}{\partial t}(\frac{1}{\kappa^\psi}\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})+[\frac{1}{\kappa^\psi}\frac{\varepsilon_1^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1}) -\frac{1}{\kappa^\psi}\frac{\varepsilon_2^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2})]][p_6-(\frac{\partial b_3}{\partial s}-b_1\varepsilon_1-b_2\varepsilon_2)\cos A] -(\frac{1}{\kappa^\psi}\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})[p_1-\cos A(\frac{\partial b_1}{\partial s}+b_3\varepsilon_1+\frac{\partial A}{\partial s}\cos A)]]\mathbf{\Pi}_2 +[\frac{\partial}{\partial t}[\frac{1}{\kappa^\psi}\frac{\varepsilon_1^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1})-\frac{1}{\kappa^\psi}\frac{\varepsilon_2^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2})] -(\frac{1}{\kappa^\psi}\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})p_2+p_4(\frac{1}{\kappa^\psi}\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})]\mathbf{B},$$

where $p_1, p_2, p_3, p_4, p_5, p_6, b_1, b_2, b_3$ *are smooth functions of time and arc length.*

Proof. Using definition of ψ, we have

$$\mathbf{T}^{\psi}=(\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})\mathbf{\Pi}_1+(\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})\mathbf{\Pi}_2.$$

Since

$$\frac{\partial}{\partial t}\mathbf{T}^{\psi}=[\frac{\partial}{\partial t}(\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})+(\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]]\mathbf{\Pi}_1$$

$$+[\frac{\partial}{\partial t}(\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})+(\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]]\mathbf{\Pi}_2+[\mathrm{p}_2(\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})+\mathrm{p}_4(\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})]\mathbf{B}.$$

Using the (2.3) equations, we have

$$\mathbf{N}^{\psi}=(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2}))\mathbf{\Pi}_1+(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1}))\mathbf{\Pi}_2-\frac{1}{\kappa^{\psi}}\mathbf{B}.$$

Then,

$$\frac{\partial \mathbf{N}^{\psi}}{\partial t}=[\frac{\partial}{\partial t}(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2}))+(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1}))[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]-(\frac{1}{\kappa^{\psi}})[\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]]\mathbf{\Pi}_1+[\frac{\partial}{\partial t}(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1}))+(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2}))[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1$$

$$+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]-(\frac{1}{\kappa^{\psi}})[\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]]\mathbf{\Pi}_2+[\mathrm{p}_2(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2}))-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\psi}})+\mathrm{p}_4(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1^3}{(\varepsilon_1^2+\varepsilon_2^2)^2}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1}))]\mathbf{B}.$$

On the other hand,

$$\mathbf{B}^{\psi}=[\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1})-\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2})]\mathbf{B}-(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})\mathbf{\Pi}_1+(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})\mathbf{\Pi}_2.$$

This implies

$$\frac{\partial \mathbf{B}^{\psi}}{\partial t}=[[\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1})-\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2})]][\mathrm{p}_5+(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\sin\mathrm{A}]-\frac{\partial}{\partial t}(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})$$

$$+(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})[\mathrm{p}_3+(\frac{\partial \mathrm{b}_2}{\partial s}+\mathrm{b}_3\varepsilon_2-\frac{\partial \mathrm{A}}{\partial s}\sin\mathrm{A})\sin\mathrm{A}]]\mathbf{\Pi}_1$$

$$+[\frac{\partial}{\partial t}(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})+[\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1})$$

$$-\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2})][\mathrm{p}_6-(\frac{\partial \mathrm{b}_3}{\partial s}-\mathrm{b}_1\varepsilon_1-\mathrm{b}_2\varepsilon_2)\cos\mathrm{A}]$$

$$-(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})[\mathrm{p}_1-\cos\mathrm{A}(\frac{\partial \mathrm{b}_1}{\partial s}+\mathrm{b}_3\varepsilon_1+\frac{\partial \mathrm{A}}{\partial s}\cos\mathrm{A})]]\mathbf{\Pi}_2$$

$$+[\frac{\partial}{\partial t}[\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_2}{\varepsilon_1})-\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2^4}{(\varepsilon_1^2+\varepsilon_2^2)^{\frac{5}{2}}}\frac{\partial}{\partial s}(\frac{\varepsilon_1}{\varepsilon_2})]$$

$$-(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_2}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})\mathrm{p}_2+\mathrm{p}_4(\frac{1}{\kappa^{\psi}}\frac{\varepsilon_1}{\sqrt{\varepsilon_1^2+\varepsilon_2^2}})]\mathbf{B}.$$

Then, we obtain the theorem. So, theorem is proved.

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