Incorrect Applications of Archimedes Principle

Selma Moğol, Yasin Ünsal
Department of Physics, Faculty of Education, Gazi University, 06500, Ankara, Turkey.

E-mail: yunsal@gazi.edu.tr

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Abstract
Archimedes' Principle is stated in some sources incorrectly. In this study, misperceptions arising from this situation and results of a survey applied to graduate students are discussed.

Keywords: Archimedes principle, Dip, Submerge, Fluid, Buoyant force.

Resumen
El principio de Arquímedes se indica incorrectamente en algunas fuentes. En este estudio se discuten, las percepciones erróneas que surgen de esta situación y los resultados de una encuesta aplicada a los estudiantes de postgrado.

Palabras clave: Arquímedes principio, Fondo, Sumergir, Fluido, Fuerza de empuje.

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I. INTRODUCTION

The **Archimedes principle** is one of the most fundamental concepts in fluid statics [1]. Archimedes Principle, which has a broad field of application in physics, is expressed in a commonly-used textbook [2] as follows: "*A body that is completely or partly dipped in a liquid is raised upward with a force equal to the weight of a displaced fluid.*" In other textbook [3], it is expressed as follows: "*A body that is completely or partially submerged in a liquid is raised upward with a force equal to the weight of a liquid that is displaced by the body.*" In another resource [4], it is expressed as: "*The buoyant force that is applied to a body which is completely or partially submerged in a fluid is equal to the weight of the fluid that is equal to the volume of the part of the body, which is submerged in the liquid.*"

All these three definitions have expressions that cause the students to misunderstand and make mistakes in practices. Namely; since the first definition has an expression as "a body that is completely or partially submerged in a liquid", the students calculate the **buoyant force** by taking the parts of body that remain within only the liquids and solve the problems accordingly, while applying the Archimedes Principle. In the second definition, it is once again stated that the buoyant force of a body that is partially or completely submerged in the liquid is equal to the weight of the liquid that is displaced with the body. The buoyant force, of the part of the body within another fluid, which for instance remains in the air, is not taken into consideration. In the third definition, the expression of "*a body that is completely or partially submerged in a fluid…*" is used and the expression of "it is equal to the weight of the fluid that is equal to the volume of the part of the body that is submerged in the fluid" is written for the definition of the buoyant force. Since the words "*dipped*" and "*submerged*" are used for only liquids, the students take only the parts of bodies that remain within the liquids into consideration while considering or calculating the buoyant force and neglect the parts of another fluid, which for instance remains in the air.

The reason of all these false definitions regarding the Archimedes Principle could be related with the fact that the meanings of words that are used in the strictest sense are neither emphasized nor considered. In the definitions, it is stated that the buoyant force is constituted by the liquid and contributions of the air and other fluids are neglected and excluded, even though it is not stated that they are neglected due to being small. This fact, on the other hand, prevents the person from thinking and calculating in an accurate and delicate way. We can write the accurate expression of the Archimedes Principle as follows: "*The buoyant force that is applied to a body within the fluid(s) by the fluid(s) is equal to the weight of fluid(s) that is displaced by the body.*" In other an expression [5]: "*A body submerged in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the body.*"

Now, as is seen in Figure (1), let's take a body within two different liquids. Where ρ_1, ρ_2 indicates the *densities* of the liquids. Where ρ_b is the density of the body. The relation between the densities of body and liquids can be given as ($\rho_1 > \rho_b > \rho_2$) in Figure (1).

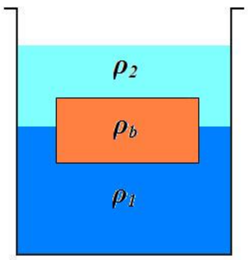

FIGURE 1. A body with ρ_b density in two different densities in the liquids.

In this case the buoyant force that affects such a body could be expressed as in equation (1).

$$F = \rho_1.V_{b_1}.g + \rho_2.V_{b_2}.g \, . \quad (1)$$

Where V_{b_1} indicates the *volume* of the part of the body that remains within the ρ_1- density liquid and V_{b_2} indicates the *volume* of the part of the body that remains within the ρ_2- density liquid. Additionally, where g is acceleration of gravity. On the other hand, the downward *weight* of body (W) is shown in equation (2).

$$W = \rho_b.V_b.g \, . \quad (2)$$

Where V_b indicate the total volume of the body. Equation (3) and equation (4) could be written, since the body is on a static balance.

$$W = B \, , \quad (3)$$

$$\rho_b.V_b = \rho_1.V_{b_1} + \rho_2.V_{b_2} \, . \quad (4)$$

In case that there is another fluid, for instance air instead of the fluid with a smaller density, it will be proper to write the equation (5) instead of equation (4).

$$\rho_b.V_b = \rho_1.V_{b_1} + \rho_a.V_{b_a} \, . \quad (5)$$

Where ρ_a indicates the *density* of **air**, V_{b_a} indicates the *volume* of the part of the body that remains in the air. In academical textbooks [2, 3, 4] and routine problem-solving applications, the second term $(\rho_a.V_{b_a})$ on the right of the equation (5) is neglected. Due to this negligence, the results of some problems come out wrong. For example, as Lan [6] discusses, let's consider the body in Figure (1) as **ice**. Since the *density* of ice (ρ_{ice}) is less than the density of water, a part of it remains in the **water** and another part in the air.

When the students are asked about what kind of a change will occur for the water level in case that the whole ice melts in the basin; they generally state that no change will occur due to the aforementioned negligence and by this way give the wrong answer. In this study, the reason of this wrong answer and results of a survey application performed with undergraduate students on this subject were emphasized.

II. A SURVEY APPLICATION REGARDING THE PROBLEM

During the single-question assessment test that was performed at the end of the subject of the buoyant force, 106 students having education in the undergraduate program of Physics Teaching and receiving the lesson of **Mechanics II** were asked a question; "*What kind of a state (increase / decrease / stationary) is observed for the water level when an ice cube that swims in a water-filled glass melts?*" and the answers "*The water level increases / decreases / remains unchanged.*" were received in a written format with their reasons. The answers given to the end-of-subject assessment question by students could be seen in Table I.

TABLE I. Distribution of Answers Given to the Assessment Question by the Students.

Answer	Number of Students (N)	Percentage (%)
Water level increases.	*5*	*4,71*
Water level remains unchanged.	*90*	*84,91*
Water level decreases.	*3*	*2,83*
No answer	*8*	*7,55*
Total	**106**	**100,00**

As is seen in Table I, a great majority of students (84,91%) indicated that the water level will not change, in other words, the glass will not overflow. Without any calculation, they made the interpretation of "*When the ice melts, its volume decreases and density increases. Thus, the water level remains unchanged*." as a comment for that.

Another interesting interpretation regarding the water level will not change is the erroneous interpretation that "*When a solid body melts within its own liquid, its volume will remain unchanged.*" Another student indicated that "*If the ice does not overflow within the water, then the water level has already increased as high as the volume of the ice, therefore there will be no change for the water level.*"

A widespread opinion among the students who think that the water level will increase and consequently the water in the glass will overflow is that "*Since the density of* *ice is smaller than that of water, it will swim on the surface of water; however, as it melts, there will be an increase in the volume of water in the basin and the water level together with the addition of the volumes of parts that are not submerged in water. The glass will overflow, since it is initially filled with water.*" However, in addition to this opinion, they stated that the knowledge regarding the fact that the volume of ice decreases as it melts is negligible.

On the other hand, students having the opinion that "*Water level decreases.*" explained the swimming of the ice with a chain of mistakes that it has a greater volume, this volume decreases as a result of melting and this condition will cause a decrease in the water level.

III. DISCUSSION AND CONCLUSION

As the stunning conclusions of the single-question survey application reveals, it is seen that students have a set of misperceptions about the applications of the Archimedes Principle. In this section, the source of this mistake will be emphasized.

As is calculated in Lan's article [6], if the answer of this question is found by taking the equation (5) into consideration, it is concluded that the water level is supposed to increase in case that the ice completely melts. Namely; let's consider a cylindrical basin and cylindrical ice bar. If the equation (5) is rewritten by using the sign "ice" representing the ice instead of the sign "b" representing the body and "f" representing any fluid instead of the sign "a" representing the air, the equation (6) will be obtained.

$$\rho_{ice}.V_{ice} = \rho_1.V_{ice_1} + \rho_f.V_{ice_f} \,. \tag{6}$$

When the ice bar melts, the water mass caused by the melting process of the ice becomes equal to the mass of the ice cube. In that case, the equation (7) could be written.

$$\rho_1.V_{melted-water} = \rho_{ice}.V_{ice} \,. \tag{7}$$

Here, ($V_{melted-water}$) indicates the volume of water caused by the melting process of the ice. If the equation (8) is rewritten by using the equation (6), the equations (8) and (9) could easily be obtained.

$$\rho_1.V_{melted-water} = \rho_1.V_{ice_1} + \rho_f.V_{ice_f} \,, \tag{8}$$

$$V_{melted-water} = V_{ice_1} + \frac{\rho_f}{\rho_1}.V_{ice_f} \,. \tag{9}$$

As is clearly seen in equation (9), expression of (10) could be written.

$$V_{melted-water} > V_{ice_1} \,. \tag{10}$$

By this way, an increase occurs in the water level for any fluid, after the melting process of the ice. In case that there is air instead of the small-density fluid in Figure 1, Ehrlich [7] indicated that the volume of water that occurs as a result of the melting process of the ice becomes equal to the volume of the part of ice that is submerged. Similarly, regarding the air, Nelson and Parker [8] indicated that the volume of water that occurs as a result of the melting ice cube is equal to the part of the ice that remains in the water. Thus regarding the air, Ehrlich, Nelson and Parker agree on the erroneous idea that the water level will remain unchanged after the melting process of the ice. But, in his study, Lan [6] calculated how much the water level would increase.

The amount of increase in the water level is given in equation (11), considering that we take the state of a cylindrical ice cube, which approximately has the same cross-sectional area within a cylindrical basin with a cross-sectional area of "A".

$$h_{rise} = \frac{\rho_f}{\rho_w}\left(\frac{\rho_w - \rho_{ice}}{\rho_w - \rho_f}\right).h_{ice} \,. \tag{11}$$

Let's calculate the amount of increase in the water level for the two different fluids. As the amount of increase in the water (hrise) could be written expression (12) in case that the fluid above is air, considering the density of air is 1.29 kg.m-3, water is 1000 kg.m-3 and ice is 917 kg.m-3.

$$h_{rise} \approx 1.07.10^{-4} h_{ice} \,. \tag{12}$$

As is seen, this result clearly shows us that there will be an increase in the water level.

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REFERENCES

[1] Chu, P. Y., Chuu, D. S., Han, C. S. and Yang, T. J., *Archimedes principle in a rotating compressible fluid*, Eur. J. Phys. **13,** 17-20 (1992).

[2] Serway, R. A. and Beichner, R. J., *Physics for Scientists and Engineers with Modern Physics,* (Palme Publishing, Ankara, 1995) p. 399 (Turkish translation from the third edition / Translator Editor: K. Çolakoğlu)

[3] Bueche, F. J. and Jerde, D. A., *Principles of Physics* (Palme Publishing, Ankara, 2000), p. 278 (Turkish translation from the sixth edition / Translator Editor: K. Çolakoğlu)

[4] Halliday, D. and Resnick, R., *Fundamentals of Physics* (Arkadaş Publishing, Ankara, 1992), p. 323 (Turkish translation / Translator Editor: C. Yalçın)
[5] Ridgely, C. T., *Archimedes' Principle in General Coordinates*, Eur. J. Phys. **31,** 491-499 (2010).
[6] Lan, B. A., *An ice-cube puzzle,* Phys. Educ. **36,** 75-76 (2001).
[7] Ehrlich, R., *Why Toast Lands Jelly-Side Down: Zen and the Art of Physics Demonstrations,* (Princeton, NJ, 1997) p. 101.
[8] Nelson, M. and Parker, P., *Advanced Level Physics (Heinemann*, Oxford, 1995).