Gravity mass instead of dark matter

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Abstract

We showed that the massive effect of gravitational potential is equivalent to dark matter that should be added to newtonian dynamic to solve the cosmological problems. We constructed mass formulae in concept of gravitational effects on the mass, which resolves some part of the missing matter the amount of mass effect that could be generated by the gravity was found to be 34% of the universe's mass. We proposed modified newtonian dynamics (MOND) as a consequence of mass effect of gravitational potential.

Keywords: Einstein equation, dark matter, modified newtonian dynamics.

Resumen

Mostramos que el efecto masivo del potencial gravitacional es equivalente al de la materia obscura que debería ser añadida a la dinámica newtoniana para resolver problemas cosmológicos. Construimos la formulación de la masa dentro del concepto de efectos gravitacionales sobre la masa, la cual resuelve alguna parte de la materia faltante dentro del efecto de masa faltante que podría ser generada por gravedad y que es encontrada ser del 34% de la masa del universo. Nosotros proponemos la dinámica newtoniana modificada (MOND) como una consecuencia del efecto de masa del potencial gravitacional.

Palabras clave: Ecuación de Einstein, material oscura, dinámica newtoniana modificada.

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I. INTRODUCTION

To solve the galaxy rotation problem, Mordehai Milgrom in 1983 proposed an empirical law to modified Newtonian dynamics (MOND). This problem began with the discovery of Oort, when he noticed, in 1932, a shortage of mass required to describe the velocity of stars in the solar neighborhood [1]. In 1933, Zwicky found the same problem by applying the virial theorem to the Coma cluster, based on the radial velocities of a few galaxies [2]. Later, Jaan Einasto and others, showed that a large fraction of mass, in addition to the observed luminous mass, was necessary to describe the dynamics of galaxies [3, 4, 5].

Milgrom noted that Newton's law for gravitational force has been verified only where gravitational acceleration is large, and suggested that for extremely low accelerations the theory may not hold. MOND theory posits that acceleration is not linearly proportional to force at low values. The basis of the modification is the assumption that, in the limit of small acceleration a very low characteristic acceleration, a_0, the acceleration of a particle at distance r from a mass M satisfies approximately the relation $a_N = a\,\mu\left({}^{a}/_{a_0}\right)$, where a_N is the Newtonian acceleration, a is the MOND acceleration of gravity, a_0 is constant acceleration and $\mu\left({}^{a}/_{a_0}\right)$ is the interpolation function [6, 7, 8].

The contradiction of MOND with the well-known theory and dark matter hypothesis, derived most of astrophysicists and cosmologists do not believe that Modified Newtonian Dynamics fits the evidence.

In this work we constructed mass formulae in concept of gravitational effects on the mass, which resolves some part of the missing matter. This was done by using the stress energy momentum tensor of a perfect fluid in Background of cosmic fluid with negative pressure. After that, we compare the mass term with the equivalent term in the relativistic Klein – Gordon equation.

II. THE MASS EFFECT OF GRAVITY (MEG)

As we know stress energy momentum tensor of a perfect fluid, in Background cosmic fluid with negative pressure $p = -\rho c^2$ is given by $T_{\mu\nu} = g_{\mu\sigma} g_{\nu\zeta}\left(\rho + \frac{p}{c^2}\right)c^2 - p g_{\mu\nu} \equiv \rho g_{\mu\nu}$ and Einstein equation is $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}$. Rewriting R.H.S of Einstein equation in terms of

Schwarzschild radius and rational stress energy momentum tensor yields,

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -4\pi R_s \widetilde{T_{\mu\nu}} \,. \qquad (1)$$

Where $R_s = \frac{2GM}{c^2}$ and $\widetilde{T_{\mu\nu}} = {T_{\mu\nu}}/{Mc^2}$.

Then, replacing Schwarzschild radius with Compton length (λ_c) $R_s = \alpha^2 \lambda_c$ in Einstein equation (1). That gives

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\frac{4\pi\alpha^2\hbar}{mc} \widetilde{T_{\mu\nu}} \,. \qquad (2)$$

Now Einstein equation in Background of cosmic fluid with negative pressure, can be written as the following

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -\frac{4\pi\alpha^2\hbar}{mc} \tilde{\rho} g_{\mu\nu} \,. \qquad (3)$$

Where $\tilde{\rho} = {\rho}/{Mc^2} \propto r^{-3}$, a is the scale (radius) of the system.

Solving Einstein equation in conformal flat space time with metric tensor, $g_{\mu\nu} = e^{\psi} \eta_{\mu\nu}$, where ψ is coordinate function. Here, we recourse the result in (John.A, et al, 1997) to write Einstein equation as

$$\frac{\partial^2 \psi}{\partial x^\mu \partial x_\nu} + \tfrac{1}{2} \delta_\nu^\mu \frac{\partial^2 \psi}{\partial x^\alpha \partial x_\alpha} - \tfrac{3}{2} \delta_\nu^\mu \psi = -\frac{4\pi\alpha^2\hbar}{mc} \tilde{\rho} (1 + 2\psi) \delta_\nu^\mu . \qquad (4)$$

Equation (4) implies that $\Box\psi - \frac{32\pi\alpha^2\hbar}{3mc} \tilde{\rho}\psi = \frac{16\pi\alpha^2\hbar}{3mc} \tilde{\rho}$. In static case, one has

$$\nabla^2 \psi - \frac{32\pi\alpha^2\hbar}{3mc} \tilde{\rho}\psi = \frac{16\pi\alpha^2\hbar}{3mc} \tilde{\rho} \,. \qquad (5)$$

By comparing equation (5) with Klein – Gordon equation

$$\nabla^2 \emptyset - \frac{m_\emptyset^2 c^2}{\hbar^2} \emptyset = \frac{1}{c^2} \frac{\partial^2 \emptyset}{\partial t^2} \qquad (6)$$

$m_\emptyset$ is the relativistic mass.

Hence, the equivalent mass expression is given as

$$m_\emptyset = \sqrt{\frac{16\pi\alpha^2\hbar^3}{3mc^3} \tilde{\rho}} \,. \qquad (7)$$

To calculate the equivalent mass we assumed the system is a sphere of radius *r* where $r \geq R_s$. At this end one finds

$$m_\emptyset = \sqrt{\frac{4\alpha^2\hbar^3}{mc^3} r^{-3}} \;\; = \frac{2\hbar}{c\, r} \sqrt{\frac{R_S}{r}} \;\; . \qquad (9)$$

For a complete treatment we should write equation (9) in two cases, microscopic scale ($m < m_p$) and macroscopic scale ($m > m_p$). Before this we should get the transformation between microscopic and macroscopic scale, this transformation can be done according to [9] $GM \rightarrow \frac{\hbar c}{m}$, where M refers to macroscopic mass and m is the microscopic mass.

First, we formed Planck length as a combination of two lengths, ${l_p}^2 = \frac{2Gm}{c^2} * \frac{\hbar}{mc}$. As we know the transformation keeps the relation invariant. Thus, the corresponding transformation will be,

$${l_p}^2 \rightarrow {R_s}^2 \rightarrow {\lambda_c}^2 \qquad (12)$$

According to equation (9) and transformation (12) one has, in microscopic scale

$$m_\emptyset = \frac{2\hbar}{c\, r} \sqrt{\frac{\lambda_c}{r}} . \qquad (13)$$

When $r \rightarrow \lambda_c$, one finds $m_\emptyset = 2m$. And in macroscopic scale

$$m_\emptyset = \frac{2mR_s}{r} \sqrt{\frac{R_s}{r}} . \qquad (14)$$

At the limit when, $\rightarrow R_s$, one finds $m_\emptyset = 2m$.

Equations (13) and (14) confirm that the mass $m_\emptyset = 2m$ at the critical radius and vanished when $r \rightarrow \infty$. Therefore, $m_\emptyset$ existence depends on the gravitational potential. Thus, this mass effect appears as an external mass interact gravitationally in the same manner of baryonic mass. The additional mass that could be added to the interaction at the critical radius is twice the baryonic mass. The standard interpretation of the observations of the cosmic structure, from galactic scales to the CMB, suggests that only 17% of the matter in the Universe is baryonic. If the baryonic matter distributed uniformly at the critical radius of the universe (Schwarzschild's radius) and interact gravitationally, thus one finds that, the universe has 34% mass due gravity in addition to the 17% baryonic mass. Hence, this shed light on 51% of the total mass of the universe.

III. MOND IS A CONSEQUENCE OF MEG

The acceleration according to equation (9) and Newton law, is given by

$$a = -\frac{Gm_\emptyset}{r^2} = -\left(\frac{l_p}{r}\right)^2 \frac{\sqrt{2GMa_0}}{r} . \qquad (15)$$

Where $a_0 = \frac{c^2}{r}$ and $l_p = \sqrt{\frac{2\hbar G}{c^3}}$ is the Planck length.

Equation (15) is the same equation which was proposed by Milgrom, in modified Newtonian Dynamics, with specific value of interpolation function $\mu\left({a}/{a_0}\right)$ [7].

MOND weak acceleration limit of gravity is $a = -\frac{\sqrt{GMa_0}}{r}$. This means that, according to equation (15) the weak limit of gravity is obtained when the interaction radius is equal to the critical radius, in this case $r = l_p$ requires the mass *M* to be equal to Planck mass (threshold mass between macroscopic and microscopic world).accordingly, the weak limit of gravity is obtained when the interaction radius is equal to Schwarzschild radius or Compton length in macroscopic scale and microscopic scale respectively.

To this end, equation (15) should be written in macroscopic scale and microscopic scale.

First in macroscopic scale, by substituting equation (14) in the weak equivalence principle, one has

$$a = -\left(\frac{R_s}{r}\right)^2 \frac{\sqrt{2GMa_0}}{r}. \qquad (16)$$

Second in microscopic scale, equation (15) reads

$$a = -\left(\frac{\lambda_c}{r}\right)^2 \frac{c\sqrt{\lambda_c a_0}}{r} \qquad (17)$$

Equations (16) and (17) satisfy that, the acceleration approaches a_0 at the limit when $r \to R_s$ and $r \to \lambda_c$ respectively. At this limit the transition occurs smoothly. This agrees with MOND (The transition occurs smoothly near the distance where the acceleration falls to $\sim a_0$, a constant $\sim 10^{-10}\ m/s^2$). On the other hand, $a_0 = \frac{c^2}{r}$ is constant due to the system e.g. for the universe where $r \sim 10^{26}\ m$ one finds $a_0 \sim 10^{-10}\ m/s^2$.

IV. CONCLUSIONS

Equations (13) and (14) explain the existence of extra matter in microscopic and macroscopic scales. The amount of 34% of missed mass in the universe is related to MEG. If the baryonic matter distributed uniformly at Schwarzschild's radius of the universe and interacts gravitationally.

The result that MOND is not a modification of Newtonian Dynamics rather than modification of non-baryonic mass relation (MEG) had shown by equation (15).

Equations (16) and (17) describe the acceleration in both macroscopic and microscopic scales which provide full treatment to MEG and its applications.

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