Thevenin's theorem: An easy proof suitable for undergraduate teaching

Anup Kumar Bandyopadhyay
Retired Professor, Department of Electronics and Telecommunication Engineering
Jadavpur University, Kolkata – 700032, India.

E-mail: akbandyopadhyay@gmail.com

(Received 11 July 2014, accepted 20 May 2015)

Abstract

A simple proof of Thevenin's theorem suitable for undergraduate teaching is presented. A modified node equation technique is developed for this purpose.

Keywords: Circuit Theory, Circuit Theorem, Thevenin's Theorem.

Resumen

Se presenta una sencilla demostración del teorema de Thevenin, adecuada para la enseñanza de pregrado. Se ha desarrollado para este propósito Una técnica de ecuación modificada de nodo.

Palabras clave: Teoría de Circuitos, Circuito teorema, Teorema de Thevenin.

PACS: 01.40.-d, 01.40.gb, **ISSN 1870-9095**

I. INTRODUCTION

Proof of Thevenin's theorem using algebraic manipulation of circuit equations is not available so far as my knowledge goes.

The proof methods that are available in literature contain lots of reasoning expressed in natural language and therefore become difficult to use them in undergraduate teaching. In this paper we present a proof of Thevenin's theorem using algebraic method. For this purpose we develop the modified node equation representation technique applicable to a general electrical circuit.

II. MODIFIED NODE EQUATION TECHNIQUE

Assume that the circuit has $n+1$ nodes of which node #0 is labeled as reference node. The circuit contains LRCM passive elements, independent voltage and current sources and linear controlled sources. The following algorithm would augment the circuit so that node equation technique may be used for its analysis. The underlying concept is to replace all the sources other than the independent current sources by unknown current sources. With these replacements included, one can easily obtain n node equations associated with n nodes. To solve the circuit, however, one would require $n+m$ number of equations, where, m is the total number of active sources that are replaced by unknown current generators in the augmented circuit. The extra m equations are readily obtained from the specifications of the replaced sources.

A Augmentation algorithm

- Replace all independent voltage sources, controlled current sources and controlled voltage sources by unknown current sources.
- Obtain the set of n node equations corresponding to n nodes of the circuit.
- For an independent voltage source equate the output voltage with the difference between the terminal node voltages to get an extra equation.
- For a controlled source express the control law in terms of the node voltages and the circuit parameters to get the necessary extra equation.

  The technique is illustrated by the following example.

B Example

Consider the circuit shown in Fig 1. It contains one independent current source I_1, two independent voltage sources E_1 and E_2 and one controlled voltage source VC. The independent and controlled voltage sources are replaced by unknown current sources J_1 through J_3 as shown in Fig 2.

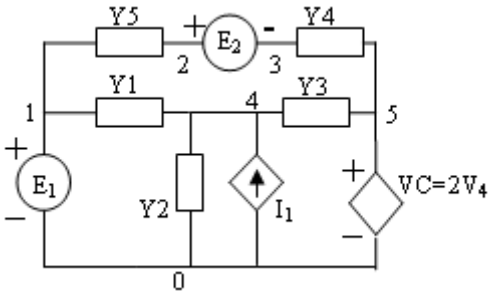

FIGURE 1. The example circuit.

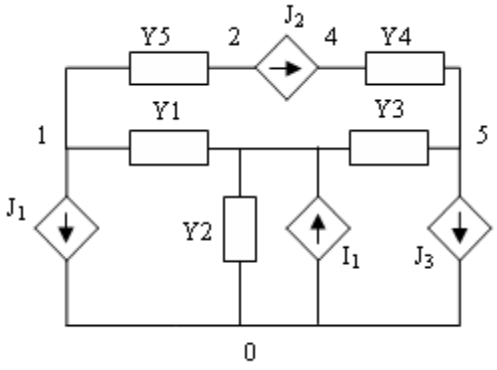

FIGURE 2. Modified example circuit.

Complete set of equations representing the circuit is as follows:

$$\begin{bmatrix} Y1+Y5 & -Y5 & 0 & -Y1 & 0 & +1 & 0 & 0 \\ -Y5 & Y5 & 0 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & Y4 & 0 & -Y4 & 0 & -1 & 0 \\ -Y1 & 0 & 0 & Y1+Y2+Y3 & -Y3 & 0 & 0 & 0 \\ 0 & 0 & -Y4 & -Y3 & Y3+Y4 & 0 & 0 & +1 \\ +1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & +1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & +1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ J_1 \\ J_2 \\ J_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ I_1 \\ 0 \\ E_1 \\ E_2 \\ 0 \end{bmatrix}$$

The above set of equations will be referred to as modified node equations. The advantage of this technique is that it can handle all types of linear sources.

III. THEVENIN'S THEOREM

Consider two one port networks *A* and *B* that are connected as shown in Fig 3.

Network **A** may contain only linear passive elements and linear independent and controlled sources. On the other hand there is no restriction of linearity for network B. However, following constraint exist:

1. There is no electromagnetic coupling between network *A* and *B*.
2. There exists no source at network *A* that is controlled by any voltage or current in network *B*.

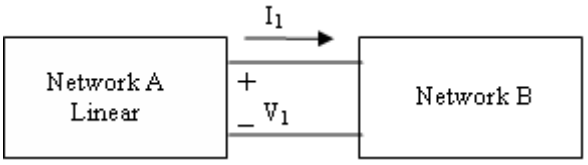

FIGURE 3. Example configuration.

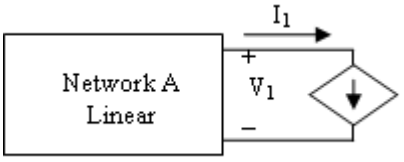

FIGURE 4. Example configuration with network B replaced by a current source.

Under this condition Thevenin's theorem states that the network *A* may be replaced by a voltage source V_θ connected in series with an impedance Z_θ where $\underline{V_\theta}$ is the open circuit voltage measured at the port of *A* and Z_θ is the driving point impedance of *A* with all the independent sources in *A* reduced to zero

A. Proof

There exists no coupling between the circuits *A* and *B*. We can consider network *A* separately with an output current I_1 flowing away from its port as shown in Figure (4). The port current I_1 and the port voltage V_1 will be the solution of the pair of equations relating V_1 and I_1 in *A* and in *B* respectively.

Thevenin's theorem aims at deriving the relationship valid for the first network that is network *A*. In general *A* can be represented by the modified node equation in the following form:

$$\begin{bmatrix} Y & U \\ C & Q \end{bmatrix} \begin{bmatrix} V \\ J \end{bmatrix} = \begin{bmatrix} I \\ E \end{bmatrix}.$$

Let:

$$Y_M = \begin{bmatrix} Y & U \\ C & Q \end{bmatrix}.$$

Also:

$$Y_M^{-1} = Z^M.$$

And an element of Z^M is denoted by z_{ij}^M.

Since there is no coupling between *A* and *B* all the elements of Z^M must be constant. Considering one terminal of the port of *A* as node #1 and the other as the reference node we get:

$$V_1 = -z_{11}^M I_1 + \sum_{i=2}^{n} z_{1i}^M I_i + \sum_{j=n+1}^{n+m} z_{1j}^M E_j .$$

From which one gets:

$$V_1\big|_{I_1=0} = V_\theta = \sum_{i=2}^{n} z_{1i}^M I_i + \sum_{j=n+1}^{n+m} z_{1j}^M E_j .$$

And:

$$z_{11}^M = Z_\theta$$

Where Z_θ is the driving point impedance of the network *A* when all the independent sources are made equal to zero.

Hence the theorem is proved.

VI. CONCLUSIONS

A modified node equation technique has been developed that can include any type of linear independent and controlled sources. The technique is applied to prove Thevenin's theorem. Strength of the technique lies in its simplicity.

REFERENCES

[1] Skilling, H. H., *Electrical engineering circuits*, (Wiley, New York, 1960).