Bubble’s Oscillations in Liquids

Bahar Aghazadeh
Ariaian Young Innovative Minds Institute, AYIMI, Tehran, Iran.

E-mail: info@ayimi.org

ISSN 1870-9095

(Received 28 October 2019, accepted 27 May 2020)

Abstract
When a container of liquid (e.g. water) oscillates vertically, it is possible that bubbles in the liquid move downwards instead of rising. As the bubble moves downward, the pressure of the water increases and the volume of the bubble will decrease. Then the added mass decreases and the bubble accelerate downward. But when the bubble moves upward, the volume and the added mass will increase, and pressure decreases so the acceleration will decrease. The point is that every time when the bubble moves up; because of low acceleration, it will not go to its initial place. It means in each oscillation the bubble effectively goes down. In this research, we are going to investigate this phenomenon.

Keywords: Liquid, Oscillation, Bubble.

Resumen
Cuando un recipiente de líquido (por ejemplo, agua) oscila verticalmente, es posible que las burbujas en el líquido se muevan hacia abajo en lugar de subir. A medida que la burbuja se mueve hacia abajo, la presión del agua aumenta y el volumen de la burbuja disminuirá. Luego, la masa agregada disminuye y la burbuja se acelera hacia abajo. Pero cuando la burbuja se mueve hacia arriba, el volumen y la masa agregada aumentarán y la presión disminuirá, por lo que la aceleración disminuirá. El caso es que cada vez que la burbuja sube; debido a la baja aceleración, no volverá a su lugar inicial. Significa que en cada oscilación la burbuja efectivamente desciende. En esta investigación vamos a investigar este fenómeno.

Palabras clave: Líquido, oscilación, burbuja.

I. INTRODUCTION

It was found in experiments in the late 1950s that, under certain circumstances, bubbles would remain suspended or even sink to the bottom of the tank. It has been suggested that this phenomenon could be a cause of early rocket failures due to collecting bubbles interfering with fuel sensors as they are cylindrical and vibrating vertically in the rockets and causing premature stage separation [1]. Bubble media are actively used in the processes of purification of melts by the passage of insoluble gas bubbles through them. Bubbles are believed to be insoluble in the liquid. The effects occurring at the interface between a bubble and the liquid and the kinetics of bubble merging are excluded from consideration [2].

II. EXPERIMENTAL SETUP

To do our experiments we used a 4-ohm speaker as the vibrating device which the dust cap is removed, a 30-watt amplifier, containers with 2.5 cm in diameter and different heights (6, 7.4, 10.5, 12.1 cm) and caps. To place the containers on the speaker, they were stuck to a cap and put in the cavity of the speaker (Fig. 1).

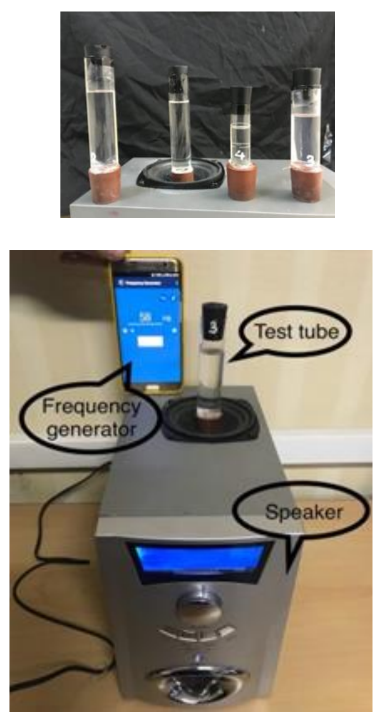

FIGURE 1. Experimental Setup.

III. METHODS AND THEORY

We consider the container oscillates with amplitude "A" and the frequency "ω" which the position of the container is $x = A \sin\omega t$ and its acceleration is $\ddot{x}$. The total acceleration of the bubble is the sum of gravity and the acceleration of the vibration (Eq. 1-3). The data analysis will be by tracking the bubbles (Fig. 2).

$$\text{Displacement}= x = A \sin\omega t \,. \tag{1}$$

$$\text{Sinusoid alacceleration}= \ddot{x} = A\omega^2 \sin\omega t. \tag{2}$$

$$\text{Total acceleration}= g + A\omega^2 \sin\omega t. \tag{3}$$

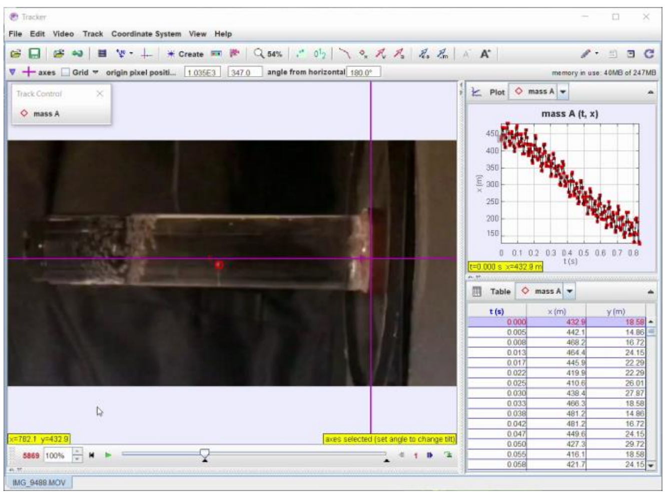

FIGURE 2. Tracking the bubbles in our experimental setup.

A. Volume of the bubble

The air inside the bubble is considered as an ideal gas so the relation between pressure, volume and temperature of the air is given by the ideal gas law. We assume the temperature of air is almost constant during vibration then the product of volume and pressure in each position is constant too (Eq. 4).

$$P_t V_b = \overbrace{nRT}^{\text{Const.}} = P_0 V_{b0} \,. \tag{4}$$

The secondary volume and pressure in each time and height, the pressure of a bubble in time (t) and height (x) is found (Eq. 5).

$$P_t = P_0 + \rho x (g + A\omega^2 \sin\omega t) + \frac{\sigma}{2r} . \tag{5}$$

By substituting this formula in equation (4), we can get the volume of the bubble (Eq. 6).

$$V_b = \frac{P_0 V_{b0}}{P_0 + \rho x (g + A\omega^2 \sin\omega t) + \frac{\sigma}{2r}} \,, \tag{6}$$

$\rho H_0 (g + A\omega^2)$ is the pressure exerted by the liquid which is very smaller than the pressure of atmosphere. So $\left(\frac{\rho H_0 (g + A\omega^2)}{P_0}\right)^2 \ll 1$. With Taylor expansion we can change the equation to the final equation of the bubble's volume (Eq. 7).

$$V_b = V_{b0} \left(1 - \gamma \frac{x}{H_0} - \gamma \frac{x}{H_0} W \sin\omega t - \frac{\sigma}{2r}\right). \tag{7}$$

B. Forces affecting the bubble

In order to get the equation of motion of the bubble, we have to draw the free body diagram of the bubble. There are three forces that act on the bubble, buoyancy, weight and drag force. The buoyancy force is always upward, the weight is downward and the drag force is always in opposite direction of the bubble's motion. (Eq. 8).

$$F_B = \rho V g \,. \tag{8}$$

Therefore, the equation of motion according to Newton's second law is (Eq. 9).

$$-F_{(x)} + (m - \rho V_b)(g + A\omega^2 \sin\omega t) = mab \,. \tag{9}$$

C. Drag force

We have two different drag forces in this problem. First, is the drag force between two fluids and the second one is the drag force between a fluid and a rigid body. In this problem we use the drag force between a fluid and a rigid body for the bubbles which are fluid. In this drag force we have no-slip condition on the surface and the last layer which is sticked to the object has the same velocity as the object does, and each layer that gets further from the object has lower velocity until it gets to zero. In the drag force between two fluids we don't have no-slip condition on the surface and the last layer of two fluids has a velocity against each other. But their perpendicular velocity against each other is zero if it wasn't then the pressure inside the bubble would be bigger than (Eq. 10) [1].

$$P_{in} - P_{out} = \frac{\sigma}{2r}, \tag{10}$$

and the bubble will separate into two parts to manage its inside and outside pressure. But the reason of using the drag force between a fluid and a rigid body is that, in very small bubbles the air inside it can't find a pattern to move so the air will be compressed and it will act like a rigid body so we use (Eq. 11 and 12) to measure the drag forcé (Fig. 3).

$$F_D = 6\pi\mu r V \,, \tag{11}$$

$$F_D = \frac{1}{2} C \rho V^2 \quad . \tag{12}$$

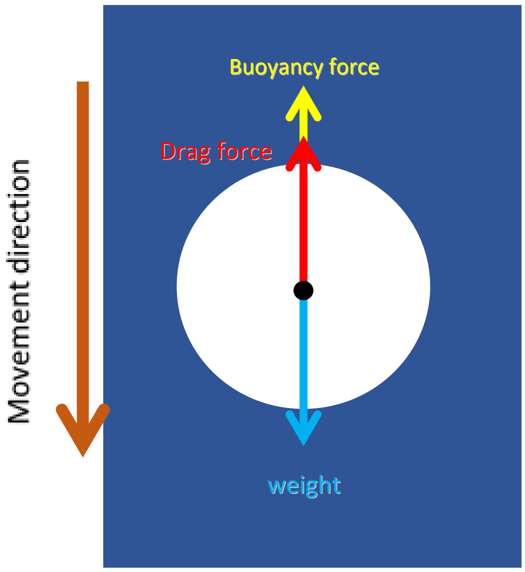

FIGURE 3. Forces affecting the bubble.

By comparing theory and experiment, we can find buoyancy and drag forces in this research (Eqs. 13 and 14) (Fig. 4).

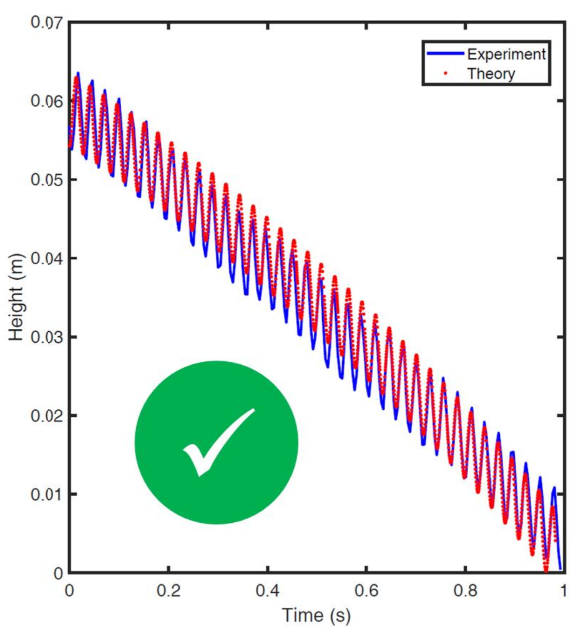

FIGURE 4. Comparing theory and experiment.

$$\text{Buoyancy Force} = 1.14 \times 10^{-6}\, N \,, \tag{13}$$

$$\text{Drag force} = 3 \times 10^{-6}\, N \,. \tag{14}$$

D. Added mass

The inertia of the bubble is not just depending on its own mass, while it depends on its volume too. This concept is added mass. Any object moving through a fluid needs to displace the fluid around it to occupy a new position. This means during its motion the object must not only dispense energy to move its own mass, but must also accelerate the fluid around its bulk. We can model the extra work done by the object as an extra amount of mass that is added to the object and moves with it through the fluid. The added mass is half of the mass of the liquid displaced by the bubble. Finally by considering all these parameters, the differential equation is written (Eqs. 15-17) [1]:

$$F = (m + m_0)\ddot{x} + \dot{m}_0 \dot{x} \,, \tag{15}$$

$$-F_{(x)} + (m - \rho V_b)(g + A\omega^2 \sin \omega t) = ma \,, \tag{16}$$

$$(m + m_0)\ddot{x} + \dot{m}_0 \dot{x} = -F_{(x)} + (m - \rho V_b)(g + A\omega^2 \sin \omega t) . \tag{17}$$

which gives us the equation of motion of the bubble. It is solved numerically by Matlab.

E. Different motions of the bubbles

There are three main different motions of the bubble, one is sinking, the other one is rising, and the last one is being trapped. In rising obviously the bubble once has sank. When the bubble sinks and hits the bottom of the tank (or in some special cases), it will gain some gases, so the volume will increase and the bubble will rise to the surface. In the case of being trapped, just like rising, increasing of bubble's volume and also the buoyancy force is not enough to counteract the effect of drag force and the added mass. So, the forces affecting the bubble will be zero in the middle of the container (Fig. 5).

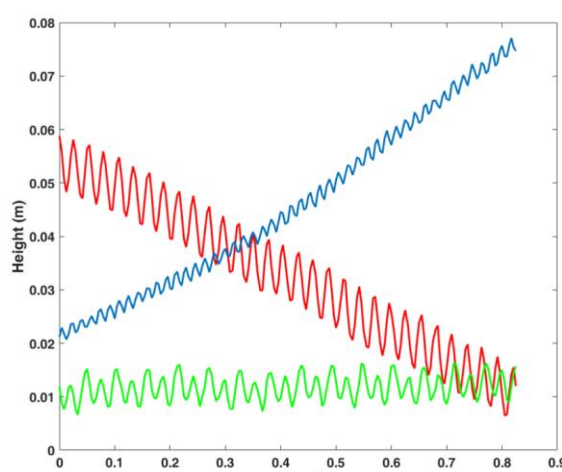

FIGURE 5. Different motions of the bubbles.

High viscosity of the liquid will increase the drag forcé which prevents the bubbles to sink or in very high viscosity, bubbles won't be even made.

As much as the viscosity and density of the liquid is lower the bubbles can move easier so it will sink sooner. So, bubbles sank sooner in alcohol than water (Fig. 6).

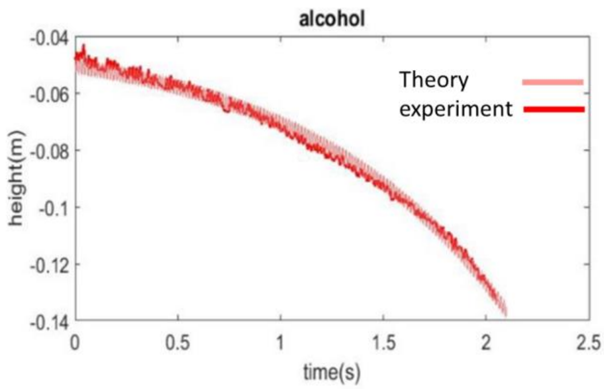

FIGURE 6. Motion of the bubble in different viscosity, comparing theory and experiment.

Density only affects the buoyancy force. If the density is higher, then the buoyancy force is higher too.

IV. CONCLUSIONS

In this research to investigate different parameters affect on the bubble movement in a liquid, several experiments have been done.

By Taylor expansion we could find volume of the bubbles. Three forces that act on the bubble, are buoyancy, weight and drag force which the buoyancy force is always upward, the weight is downward and the drag force is always in opposite direction of the bubble's motion. Different height of the bubble according to the effect of these three forces were compared in our experiment. By comparing theory and experiment, we found buoyancy and drag forces in this research too. Solving the equation of motion of the bubble has been done numerically by considering added mass model.

It is found as much as the viscosity and density of the liquid is lower the bubbles can move easier so it will sink sooner too.

ACKNOWLEDGEMENTS

This research is one of the IYPT 2019 problems which Team Iran participates in this great international physics tournament every year. I would like to thank Dr. Hossein Salari as the supervisor and AYIMI institute which is the organizer of several scientific activities and competitions. Iran national team is selected by PYPT (Persian Young Physicists' Tournament, AYIMI) to challenge with other teams from several countries in IYPT.

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