The Coiling Factor in the Tungsten Filament Lamps

D. C. Agrawal

Department of Farm Engineering,
Banaras Hindu University,
Varanasi 221005, India.

E-mail: dca_bhu@yahoo.com

Abstract

Papers on tungsten lamps appearing in the pedagogic journals have been essentially assuming straight wire filaments however, in actual practice it is either single coiled or double coiled. This paper reports for the first time the D.D. Van Hover expression for the value of resistance of a coil and Milan R. Vukcevich theory based on coiled-ribbon filament for the radiation energy value from a coil. These expressions are applied to the coiled-wire filaments lamps of 6, 10, 25, 40, 60, 100, 200, 300, and 500 watts to estimate the values of pitch ratio, mandrel ratio and the temperature of the corresponding coil. For simplicity only the single coiled filament lamp cases are considered.

Keywords: Tungsten filament lamps, 6-500 watts, coiling factors, pitch ratio, mandrel ratio, temperature of coiled filament.

Resumen

Documentos sobre las lámparas de tungsten que aparecen en las revistas pedagógicas que han sido básicamente, asumiendo sin embargo filamentos de alambre rectos, en la práctica actual esto es bobina sola o bobina doble. Este documento informa para la primera vez de D.D. Van Hover expresión para el valor de resistencia de una bobina y la teoría de Milan R. Vukcevich basada en bobina-cinta de filamento para el valor de energía de la radiación de una bobina. Estas expresiones son aplicadas a las bobinas-filamentos de alambre enrollado de 6, 10, 25, 40, 60, 100, 200, 300 y 500 watts para estimar los valores de la relación de campo, la relación de Mandrel y la temperatura de la bobina correspondiente. Por simplicidad solo la lámpara de filamento de bobina simple son casos considerados.

Palabras clave: Lámparas de filamento de Tungsten, 6-500 watts, factores de bobinado, relación de campo

I. INTRODUCTION

The tungsten filament lamps discussed in the pedagogic journals [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] have been assuming filaments to be essentially a straight wire. However in actual practice it is either single coiled or double coiled. The expression for the resistance of a coil as well as theory of the radiation emanating from a coil are quite complex. The corresponding literatures are published by General Electric Company either as internal reports [15, 16, 17], manuals [18] or books [19] which are not readily available to the students and teachers. The aim of the present paper is to bring these theoretical formulae along with its application part to the readers of pedagogic journals. The next section describes the theoretical formulation and in Section III the application part will be discussed. The Section IV will discuss the main conclusions of the present paper. For simplicity only the single coiled filament lamp cases are considered.

II THEORY

A. Notations & Parameterization

Consider at room temperature T_0 a filament in the form of a metallic wire having length L_0, diameter D_0, radius r_0, emissivity ε_0 and resistivity ρ_0. The corresponding quantities at a general temperature are denoted by the same symbol but without the zero subscript. Suppose to a bulb having this filament an electrical voltage V is applied then within a time of the order of 0.1 s the current shoots to its normal value and the temperature increases to a steady value T, typically of the order of 3000 K. Many intrinsic properties of the metal tungsten are quite sensitive functions of the temperature and it will be convenient to parameterize them in a manner described below.

The length L, emissivity ε, and resistivity ρ can be parameterized as functions of temperature T as

$$L = L_0 \left(\frac{T}{T_0}\right)^{\beta_L} ; \; \beta_L = \frac{a_L \cdot T}{1000} + b_L , \qquad (1a)$$

$$\varepsilon = \varepsilon_0 \left(\frac{T}{T_0}\right)^{\beta_\varepsilon} ; \; \beta_\varepsilon = \frac{a_\varepsilon \cdot T}{1000} + b_\varepsilon , \qquad (1b)$$

$$\rho = \rho_0 \left(\frac{T}{T_0}\right)^{\beta_\rho} ; \; \beta_\rho = \frac{a_\rho \cdot T}{1000} + b_\rho . \qquad (1c)$$

Here for the first time the exponents are also considered to be functions of temperatures in contrast to constant values presumed in all the previous attempts.

B. Straight Wire Filament

The resistance R_{WIRE} of the straight wire tungsten filament will depend on the operating temperature T of the filament as follows

$$R_{WIRE} = \rho_0 \frac{L_0}{\pi \cdot r_0^2} \left(\frac{T}{T_0}\right)^{\beta_\rho - \beta_L} . \qquad (2)$$

If the lamp is of P watts its major portion goes into Planck's radiation P_{PLANCK} followed by gas loss P_{GAS}, end loss P_{END} and bulb and base loss P_{BULB} Gas losses result from the flow of filling gas in a convection stream past the filament rising to the top of the bulb and circulating down the sides. The vacuum lamps, of course, have no significant gas loss. End losses are the summation of losses that take place in the internal lamp parts. The filament support wires and the lead wires conduct heat away from the filament to the much cooler base. The bulb and base losses correspond to the direct filament radiation absorbed and heat conducted to the bulb through the gases. These facts can be summed up as

$$P = \frac{V^2}{R_{WIRE}} = 2\pi \cdot L \cdot r \cdot \sigma \cdot \varepsilon \cdot T^4 + P_{GAS} + P_{END} + P_{BULB} . \qquad (3)$$

Here σ is the Stefan-Boltzmann constant.

C. Coiled-Wire Filament

However in actual practice the tungsten filament inside the lamp is not in the shape of a straight wire rather it is either once coiled or coiled-coil so that about a half meter long wire could be fixed in the available space of around five centimeters inside the lamp. Before discussing how coiling affects the resistance and radiation formulae let us mention two parameters - pitch ratio and mandrel ratio which characterize a coil shown in the Fig. 1.
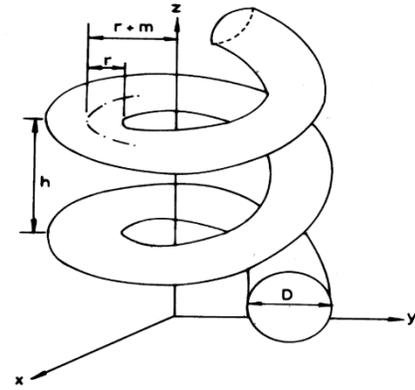

FIGURE 1. The diagram of a wire-coil. Here h is the distance between the successive coils, m is the inner hallow diameter of the coil, and D ($2 \cdot r$) is the diameter of the wire.

Pitch Ratio: $\qquad K_p = \frac{h}{2 \cdot r} . \qquad (4)$

Mandrel Ratio: $\qquad K_m = \frac{m}{2 \cdot r} . \qquad (5)$

Here h is the distance between the successive coils and m is the inner hallow diameter of the coil. The coiling affects the expressions derived above for straight wires in two ways. Firstly the Planck's radiation from the inside of the coil is partly obstructed by the other portions of the coil. A fraction of this obstructed radiation will be absorbed thereby increasing the temperature of the coil. Furthermore, the tighter the coil the more radiation will be obstructed and absorbed and the coil temperature will increase even more. Secondly the coiling also changes the expression for the resistance R_{COIL} which is marginally different from that of the straight wire R_{WIRE}. These points are discussed below one by one.

D. Radiation Coefficient δ

As the first step toward quantitative understanding of the effects of coiling Milan R Vukcevich [19] introduced a parameter called the radiation coefficient of coil δ - a correction factor which takes care of coiling and it is defined as

$$\delta = \left(\frac{Radiation \; \mathrm{int}\, ensity from coiled wire}{Radiation \; \mathrm{int}\, ensity from straight wire}\right)_{D,L,T} . \qquad (6)$$

Here the subscript D stands for the diameter of the wire, L for the length, and T for the temperature which signifies that the straight wire and the uncoiled coil have the same dimensions and that they radiate at the same temperature. The modified Planck's radiation formula from a coil becomes

$$P_{PLANCK} = 2\pi \cdot L \cdot r \cdot \varepsilon \cdot \delta \cdot T^4 . \qquad (7)$$

The radiation coefficient δ for coils is always less than one reminding the fact that some radiation did not escape from the inside of the coil.

The earlier attempts [20, 21] to find an expression for the radiation coefficient δ were not consistent with the experimental observations. Fortunately, Vukcevich [19] was successful by considering a geometrically simpler case based on thin ribbon-coil to arrive at an expression which is quite successful in explaining the experimental results for real wire-coils. The shape of a wire-coil vis-a-vis a ribbon-coil is shown in Fig. 2 and may be described by the same parameters used to characterize the wire-coils. The procedures adopted by him in finding the expression for radiation coefficient δ may be briefly described as follows.
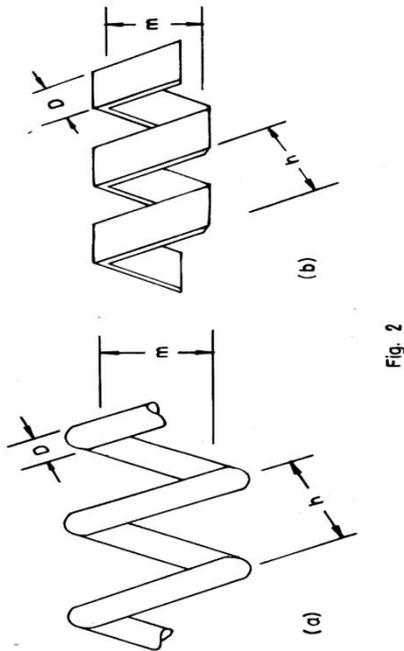

FIGURE 2. Definitions for (a) wire and (b) ribbon coils.

E. Derivation of Radiation Coefficient

The total radiation from this ribbon-coil will be the sum of radiation from the "outside" and "inside" portions. All the radiation coming from the outside surface will leave the ribbon without any hindrance. Hence for this surface the radiation coefficient is simply

$$\delta_O = 1, \tag{8}$$

where the subscript "O" refers to the outside. The amount of radiation leaving the inside of the ribbon is directly proportional to the area of the ribbon cylinder which is not covered by the ribbon i.e. $(h - 2r)$. The radiation which comes from the inside without any reflection, is then given by

$$\delta_I = \frac{h - 2 \cdot r}{h} = \frac{K_p - 1}{K_p}, \tag{9}$$

where the subscript "I" refers to the inside. The third component of the radiation, which also comes from within the ribbon, is the radiation which escapes after one or more reflections off the inside surface. Let us proceed as follows to calculate it.

1. The probability that a photon from the inside of the ribbon did not immediately escape is equal to $(1-\delta_I)$.
2. The probability that a photon hits the ribbon and gets reflected will be equal to the product of the probabilities i.e. $r_F(1-\delta_I)$. Here

$$r_F = 1 - \varepsilon, \tag{10}$$

is the reflectivity and ε is the emissivity of the ribbon surface.

3. The probability that a photon is still within the ribbon after n reflections is given by $[r_F(1-\delta_I)]^n$.
4. Finally, the probability that a photon leaves the inside of the ribbon after n number of reflections is equal to $\delta_I[r_F(1-\delta_I)]^n$.

Assuming diffuse scattering, the total probability of escape of radiation from within the ribbon is given by the sum of probabilities of escapes of photons which were reflected any number of times, from one to infinity

$$\delta_R = \sum_{n=1}^{\infty} \delta_I [r_F(1-\delta_I)]^n. \tag{11}$$

The above sum can be rewritten and then solved using the standard, tabulated series expansion

$$\delta_R = \delta_I r_F (1-\delta_I) \sum_{m=0}^{\infty} [r_F(1-\delta_I)]^m = \frac{\delta_I r_F (1-\delta_I)}{1 - r_F(1-\delta_I)}. \tag{12}$$

The overall radiation coefficient of this ribbon-coil can be obtained by summing all the three δ's derived above, after each has been multiplied by the appropriate fraction of the total ribbon surface

$$\delta = \frac{A_O}{A} \delta_O + \frac{A_I}{A} (\delta_I + \delta_R), \tag{13}$$

where A is the total surface area and A_O and A_I are the outside and the inside surfaces, respectively.

Implicit in this derivation is a large width-to-thickness ratio of the ribbon so that its sides can be neglected. This makes the following approximation sufficiently accurate.

$$A = A_O + A_I. \quad (14)$$

Substitution of (8, 9) and (12) into (13) yields

$$\delta = \frac{A_O}{A} + \frac{A_I}{A}\left[\frac{K_p - 1}{K_p - r_F}\right], \quad (15)$$

for the radiation coefficient of a ribbon coil. For a thin ribbon one may write

$$A_O = A_I, \quad (16)$$

which together with expression (14) can be used to reduce (15) as

$$\delta = \frac{1}{2}\left(1 + \frac{K_p - 1}{K_p - r_F}\right). \quad (17a)$$

According to Vukcevich [19] this simple formula for coils of thin ribbons predicts delta values which are systematically 2 to 5 percents lower than the measurements made on actual tungsten wire-coils. This discrepancy was taken care by multiplying the above δ by 1.035 termed as "the fudge factor" which modifies the above δ as

$$\delta = \frac{1.035}{2}\left(1 + \frac{K_p - 1}{K_p - r_F}\right). \quad (17b)$$

As far as the mandrel ratio is concerned its effect can be understood as follows. An actual coil is not a coiled thin ribbon; rather it has the same size in all directions perpendicular to its axis. The main effect being the area of the outside surface is always larger than the area of the inside surface. This is obvious in the case of a thick coiled square ribbon. The tighter the coil the more the inside becomes smaller relative to the outside surface area. The expression for these areas can be derived by imagining the entire surface of the wire coil to be divided into thin strips parallel to the wire axis. The length of the outermost strip relative to the length of the innermost strip is equal to $(2D + m)/m$. Integration around the perimeter and between these two extremes and the middle strip, yields the areas of the outside and the inside surfaces as

$$\frac{A_0}{A} = \frac{1}{2} + \frac{1}{2\pi(K_m + 1)}, \quad (18a)$$

$$\frac{A_I}{A} = \frac{1}{2} - \frac{1}{2\pi(K_m + 1)}. \quad (18b)$$

Substitution of these values in (15) provides the expression for the radiation coefficient of a wire coil as

$$\delta = \frac{1}{2}\left[1 + \frac{1}{\pi(K_m + 1)} + \left(1 - \frac{1}{\pi(K_m + 1)}\right)\frac{K_p - 1}{K_p - r_F}\right]. \quad (19)$$

Although the effect of mandrel size K_m is small the larger value of its size decreases the value of the radiation coefficient. The mandrel ratio values are preferred in the range 1.5 and 3.0 but the entire range of possible values – from vanishing size to an infinitely large mandrel – the radiation coefficient does not change more than 15 percent. The role of pitch ratio and mandrel ratio in the resistance of coil is described below.

F. Resistance of a Wire-Coil

The effect of coiling on the electrical resistance of a tungsten filament is important and this problem was solved by D. D. Van Horn [15, 19] (General Electric). This derivation is very tedious and it is beyond the scope of this pedagogic journal. However the result is worth quoting which is

$$R_{COIL} = \zeta \cdot R_{WIRE}; \quad \zeta = \frac{1}{2}\left[1 + \sqrt{1 - \frac{\pi^4(K_m + 1)^2}{\left[\pi^2(K_m + 1)^2 + K_p^2\right]^2}}\right]. \quad (20)$$

This formula is applicable to single coiled filaments. The typical values of ζ are around 0.99 causing approximately one percent change in the resistance and this has observable effect on the filament temperature. This is consistent with the fact that coiling causes an increase in the temperature of the filament as a part of the radiation is blocked.

G. Temperature of the Coil

The input power P of a lamp satisfies the following relation in terms of applied voltage V and the resistance of the coil R_{COIL}.

$$P = \frac{V^2}{R_{COIL}} = \frac{V^2}{\zeta \cdot R_{WIRE}} = \frac{V^2}{\zeta}\frac{\pi \cdot r_0^2}{\rho_0 L_0}\left(\frac{T_0}{T}\right)^{\beta_\rho - \beta_L}. \quad (21)$$

The output power W of the lamp is the sum of Planck's radiation from the coil, gas losses, end losses and bulb and base losses

$$W = 2\pi \cdot L \cdot r \cdot \varepsilon \cdot \sigma \cdot \delta \cdot T^4 + P_{GAS} + P_{END} + P_{BULB} \quad (22)$$

Equating the input and output powers gives

$$\frac{V^2}{\zeta}\frac{\pi \cdot r_0^2}{\rho_0 L_0}\left(\frac{T_0}{T}\right)^{\beta_\rho - \beta_L} = 2\pi \cdot r \cdot L \cdot \varepsilon \cdot \sigma \cdot \delta \cdot T^4 + P_{GAS} + P_{END} + P_{BULB} \quad (23)$$

Substituting the temperature dependence of length, radius and emissivity expressions from Eq. (1) we arrive at

$$\frac{V^2}{\zeta}\frac{\pi \cdot r_0^2}{\rho_0 L_0}\left(\frac{T_0}{T}\right)^{\beta_\rho-\beta_L} = 2\pi \cdot r_0 \cdot L_0 \cdot \varepsilon_0 \cdot \sigma \cdot \delta \cdot T^4 \cdot \left(\frac{T}{T_0}\right)^{2\beta_L+\beta_\varepsilon} + P_{GAS} + P_{END} + P_{BULB} \quad (24)$$

This is the final equation which will be used in the sequel to estimate the temperature, pitch ratio and mandrel ratio of the single coiled filament lamps of various powers.

III. NUMERICAL ILLUSTRATIONS

Before proceeding for the actual calculation of the temperature of coiled filaments of various wattages of lamps the length, emissivity and resistivity data reported by Jones and Langmuir [21] in the temperature range, room temperature $T_0 = 293K$ to the melting point $T = 3655K$, were parameterized using the expressions (1a, b, c). The corresponding results are

$$a_L = 0.00229, b_L = -0.00017, RMSError = 0.00015 \quad (25a)$$

$$\varepsilon_0 = 0.01450, a_\varepsilon = -0.13222, b_\varepsilon = 1.7650, RMSError = 0.0037 \quad (25b)$$

$$\rho_0 = 5.762 \times 10^{-6}, a_\rho = 0.00169, b_\rho = 1.1867, RMSError = 0.1640 \quad (25c)$$

The low values of the RMS errors signify that the corresponding data are very well reproduced in each case with the parameters being quoted here. The operating data of the tungsten filament lamps are borrowed from the General Electric Company's manual [18] titled *INCANDESCENT LAMPS*. These are the power of the lamp P, length of the straight wire tungsten filament L_0, its diameter $D_0 = 2 \cdot r_0$, the gas loss P_{GAS}, the end loss P_{END}, and the bulb and base loss P_{BULB} for the lamps of 6, 10, 25, 40, 60, 100, 200, 300, and 500 watts [cf. Table I]. For all these lamps which are single coiled the temperature, pitch ratio and mandrel ratio were estimated through the following steps one by one.

1. The temperature T of a straight wire filament ($\zeta = 1$) was obtained using the expression (21).
2. Next, the value of pitch ratio was ascertained by demanding that the power of the lamp P at the temperature T obtained in the step 1 is reproduced numerically through matching the expression

$$P = 2\pi \cdot r_0 \cdot L_0 \cdot \varepsilon_0 \cdot \sigma \cdot \delta \cdot T^4 \cdot \left(\frac{T}{T_0}\right)^{2\beta_L+\beta_\varepsilon} + P_{GAS} + P_{END} + P_{BULB}.$$

   Here the only unknown parameter, the radiation coefficient δ which is function of the pitch ratio K_p, was calculated from (17b). The condition that the right hand side of this expression should be equal to the input power of the lamp fixes the pitch ratio K_p in the range 1.0 to 3.0. The value of reflectivity r_F in (17b) is given by Eq. (10).
3. The above step 2 was again repeated to find the value of mandrel ratio K_m by taking the expression (19) [instead of (17b)] for δ in which the fudge

factor (1.035) has been dropped but it depends on mandrel ratio as well. The value of mandrel ratio comes out to be greater than 1.0.

4. Once the values of pitch ratio and mandrel ratio are known the factor ζ [cf. Eq. (20)] is calculated and the resistance of the coil is obtained via the relation $R_{COIL} = \zeta \cdot R_{WIRE}$.
5. The above four steps 1→4 are repeated till the values of the temperature T of the coil, its pitch ratio K_p and mandrel ratio K_m stabilize [cf. Table II].

The next section discusses the conclusions of the present work.

TABLE I. Operating data on standard lamps. All the lamps quoted here are single coiled except 60 and 100 watts lamps which are double coiled.

Lamp watt P	Length L_0 cm	Diameter D_0 cm	Gas Loss P_{GAS}%	End Loss P_{END}%	Bulb & Base Loss P_{BULB}%
6	37.084	0.001143	---------	1.5	5.5
10	43.180	0.001626	---------	1.5	5.0
25	56.388	0.003048	---------	1.5	4.5
40	38.100	0.003302	20.0	1.6	7.1
60	53.340	0.004572	13.5	1.2	4.5
100	57.912	0.006350	11.5	1.3	5.2
200	63.500	0.009652	13.7	1.7	7.2
300	72.390	0.012700	11.6	1.8	6.8
500	87.376	0.018034	8.8	1.8	7.1

TABLE II. Theoretically estimated values of temperature T, pitch ratio K_p, mandrel ratio K_m, radiation coefficient δ and the correction factor ζ for the standard lamps. Although 60 and 100 watts lamps are double coiled the reported values are based on single coiled.

Lamp watt P	Temperature of the coil T	Pitch ratio K_p	Mandrel ratio K_m	Radiation coefficient δ	Correction factor ζ
6	2425	1.134	2.204	0.6865	0.9757
10	2423	1.151	2.557	0.6949	0.9803
25	2553	1.020	5.000	0.5545	0.9930
40	2725	1.104	3.369	0.6464	0.9869
60	2553	1.498	1.071	0.8354	0.9443
100	2691	1.186	3.157	0.7047	0.9856
200	2804	1.127	4.010	0.6596	0.9900
300	2829	1.120	4.266	0.6528	0.9910
500	2841	1.104	3.797	0.6424	0.9890

IV. DISCUSSIONS AND CONCLUSIONS

In the last couple of decades all the attempts to discuss the physics of electric lamps for the students and teachers have assumed the tungsten filaments as straight wires. Based on this assumption the observables being studied are temperature [1, 2, 3, 4, 5, 6, 9] and color [3] of filament, thermal expansion of the filament [23], luminous flux [2, 4], efficiency and efficacy of the lamp [4], switching time

[7], exponent-rules for vacuum as well as gas-filled lamps [8, 13], mortality statistics and life of the lamp [24, 25], mass loss [26, 27] and the thumb-rule for the replacement of the lamps in an organization [28]. These observables should be reexamined in view of the fact that the actual filament is either single coiled or coiled coil. In the present paper the two available methods to determine theoretically the filament temperatures have been reexamined in the light of coil geometry. The first method being finding the temperature of the wire filament by adopting the following relation obtained via (2) and (3)

$$T = T_0 \left[\frac{V^2}{P} \frac{\pi \cdot r_0^2}{\rho_0 \cdot L_0} \right]^{\frac{1}{\beta_\rho - \beta_L}}, \qquad (26)$$

while the second method assumes that the total input electrical power goes predominantly into the radiative channel via Stefan's law. Most of the time the two temperatures so obtained by these methods do not match [2] and also the calculated value of Stefan's radiation based on the temperature of wire-filament from the first method predict [2] a larger value than the input power. Because of this difficulty the thermal losses were also not included in all these studies. These difficulties can be resolved by taking into consideration - the coiled geometry of the filament.

The availability of data on coiled filaments guided Milan R Vukcevich [19] (General Electric) in working out an approximate theory for a ribbon-coil in place of wire-coil. The predictions of this theory are quite satisfactory with experimental results on wire-coil. The coiling factor modifies the radiation coefficient given by the expression (19) and it depends on the parameters pitch ratio and mandrel ratio which characterize a coil. The value of the radiation coefficient is unity in the case of straight wire; however its value is less than one for coils. The coiling factor also modifies its resistance [cf. Eq. (20)]. The ribbon-coil theory is reproduced here with the hope that the students, teachers and researchers in the fields of physics, mathematics, statistics and electrical engineering will not be only benefited by this but they may also come forward with a theory for actual wire-coil.

The approximate theory of Vukcevich has been applied to the lamps of 6, 10, 25, 40, 60, 100, 200, 300, and 500 watts to deduce the values of the temperature of the coil, its pitch and mandrel ratios. This is achieved by matching the input power of the lamp and output power of the coil which is sum of the Planck's radiation, gas losses, end losses and bulb and base losses from the coil. The pitch ratio for these cases comes out to be in the range 1.02 to 1.5, mandrel ratios are in the range 4.2 to 5.0, the radiation coefficient lies in between 0.64 to 0.84 whereas the theory of Van Horn on the resistance of coil shows a change of resistance by.1 to 5 percent. As pointed out earlier the coiling data are not being provided by the manufacturers of the lamps however Vukcevich has quoted the following coiling parameters for only one vacuum lamp of 6 watts.

Pitch ratio K_p=1.4, Mandrel ratio K_m=3.0, Temperature
$$T = 2390K \qquad (27)$$

The calculations based on this and the values of the length, diameter, and losses mentioned in Table 1 show that these values correspond to input power $P = 5.83$ watts whereas the output power $W = 6.55$ watts. There is some appreciable difference between the calculated input and output powers which may be attributed to the fact that the ribbon-coil theory does need refinement and secondly the values of the losses reported need improvement.

The 60 and 100 watts lamps are basically coiled coil types but here the calculations were done under the assumption that they are coiled only once. Perhaps this is the reason that for 60 watts lamp all the parameters –pitch ratio, mandrel ratio, and radiation coefficient and correction factor in the resistance of the coil – are quite different from corresponding parameters for other lamps. However, all the calculated parameters for 100 watts lamps are consistent although this is also coiled coil one.

In the last it is worth pointing out that the ribbon-coil theory is quite satisfactory as far as predicting the wire-coil lamp parameters are concerned for manufacturing them. These lamps are likely to be removed from the society in near future due its unaffordable efficiency however; it will always remain a good teaching and research material in the subjects of physics, mathematics, statistics and electrical engineering.

REFERENCES

[1] Zanetti, V., *Temperature of incandescent lamps*, Am.J.Phys. **53**, 546-548 (1985).
[2] Harvey, S. Leff., *Illuminating physics with light bulbs*, Phys. Teach **28**, 30-35 (1990).
[3] Wagner, W. S., *Temperature and color of incandescent lamps*, Phys. Teach **29**, 176-177 (1991).
[4] Agrawal, D. C., Leff, H. S. and Menon, V. J., *Efficiency and efficacy of incandescent lamps,* Am. J. Phys. **64**, 649-654 (1996).
[5] Agrawal, D. C. and Menon, V. J., *Lifetime and temperature of incandescent lamps*, Phys. Educ. **33**, 55-58 (1998).
[6] MacIsaac, D., Kanner, G. and Anderson, G., *Basic physics of incandescent lamp (lightbulb)*, Phys. Teach **37**, 520-525 (1999).
[7] Menon, V. J. and Agrawal, D. C., *Switching time of a 100 watt bulb*, Phys. Educ. **34**, 34-36 (1999).
[8] Agrawal, D. C. and Menon, V. J., *Light bulb exponent-rules for the classroom*, IEEE Trans. Educ. **43**, 262-265 (2000).
[9] Denardo, B., *Temperature of a lightbulb filament*, Phys. Teach **40**, 101-105 (2002).
[10] Menon, V. J. and Agrawal, D. C., *Lifetimes of incandescent bulbs*, Phys. Teach **41**, 100-101 (2003).
[11] Ray, B., *Don't zap that light bulb*, Phys. Teach **44**, 374-375 (2006).

[12] Gluck, P. and King, J., *Physics of incandescent lamp burnout*, Phys. Teach **46**, 29-35 (2008).

[13] Agrawal, D. C. and Menon, V. J., *Illuminating physics with gas-filled lamps:Exponent-rules*, Lat. Am. J. Phys. Educ. **3**, 33-37 (2009).

[14] De Izarra, C. and Gitton, M. J., *Calibration and temperature profile of a tungsten filament lamp*, Europ. J. Phys. **31**, 933-942 (2010).

[15] van Horn, D. D., *Fundamentals of incandescent lamp design*, GEL Lecture notes, (General Electric Company, Nela Park, OH) (1972).

[16] Vukcevich, M. R., *General equations for incandescent lamps*, GEL Report, (General Electric Company, Nela Park, OH) (1974).

[17] Vukcevich, M. R., *Radiation from coiled filaments*, GEL Report, (1977), IES Meetring, St. Louis (1984)

[18] Incandescent Lamps, Pamplet TP-110R2 (General Electric Company, Nela Park, OH 1984)

[19] Vukcevich, M. R., *The Science of Incandescence,* (Advanced Technology Department, GE Lighting, Nela Park, Cleveland, 1992), Chapter 9.

[20] Brett, J., Fontana, R. P., Walsh, P. J., Thouret, W. E., Gerber, M., Kaufman, R. and Thorington, L., *Filaments for incandescent lamps with energy conserving envelopes*, IEEE Trans Indus. Appl. IA-17, 210-216 (1981).

[21] Raczkevi, B. and Molnarka, G., *The calculation of the shadow factor of single coil filament*, Industry Application Society Annual Meeting 1988, Conference Record of the 1988 IEEE (2-7 October 1988), Pittsburgh, Pa, USA, volume 2, pp. 1842-1847.

[22] Howard, A. J. and Langmuir, I., *The characteristics of tungsten filaments as functions of temperature Part II*, Gen. Elect. Rev. **30**, 354-361 (1927).

[23] Agrawal, D. C. and Menon, V. J., *Incandescent Bulbs:Illuminating thermal expansion*, Quantum 35-36 (1998).

[24] Menon, V. J. and Agrawal, D. C., *A theory of filament lamp's failure statistics*, European Physical Journal Applied Physics **34**, 117-122 (2006).

[25] Menon, V. J. and Agrawal, D. C., *A theory for the mortality curve of filament lamps*, Jour. Mater. Engg. Perf. **16**, 1-6 (2007).

[26] Menon, V. J. and Agrawal, D. C., *A model for mass loss in burned out filaments of incandescent lamps*, Leukos **1**, 93-100 (2004).

[27] Agrawal, D. C. and Menon, V. J., *A relation between mass loss and life of incandescent filament lamps,* Light Resear Techn **42**, 95-101 (2010).

[28] Agrawal, D. C. and Menon, V. J., *Replacement rate of filament lamps in an organization:A rule of thumb*, Light Resear Techn **41**, 343-348 (2009).