# Wind speed distribution and performance of some selected wind turbines in Jos, Nigeria

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## Abstract

The wind speed data of Jos have been shown to be either gamma or normally distributed. The wind is available and could be used for both residential and large scale power generation. At respective hub heights of 15m and 100m, at least 23% and 40% of Iskra AT5-1 and Micon NM 82 wind turbines installed capacities can averagely be generated respectively. A Micon NM 82 installed at a height of 100m in Jos can service about 210 homes each consuming at most 3kW of electrical power. The site is therefore good and the performance of the best two wind turbines attractive for investors.

**Keywords:** Wind speed distribution, Turbine performance, Average power output.

## Resumen

Los datos de velocidad del viento de Jos han demostrado ser gamma o una distribución normal. El viento está disponible y puede ser utilizado tanto para la generación de energía residencial y de gran escala. En alturas de hub respectivas de 15m y 100m, por lo menos 23% y 40% de Iskra AT5-1 y Micon NM 82 capacidades de turbinas de viento instaladas que ser pueden generadas respectivamente. Un Micon NM 82 instalado a una altura de 100m en Jos puede abastecer a unas 210 viviendas cada una consume en su mayoría 3kW de energía eléctrica. El sitio es bueno y por lo tanto el rendimiento de las dos mejores turbinas de viento es atractivo para los inversores.

**Palabras clave:** Distribución de la velocidad de viento, Rendimiento de Turbina, Potencia media de salida.

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## I. INTRODUCTION

Investments in wind energy require detail statistical assessment of sites' wind distributions and availability. It also requires selection and installation of suitable wind turbines for wind power extraction at the sites.

In Nigeria, most of these statistical assessments starts by assuming a Weibull distributed wind speeds [1, 2]. These assumptions might lead to errors of over or under estimation of potentials of candidate sites especially when the distribution of the wind speeds of the sites are not really Weibull.

In this work, four distribution functions were fitted unto constructed histogram of the wind speed data of Jos and the best fitted distribution function used for its detail energy potential assessment.

## II. MATERIALS AND METHODS

### A. Data

Ten years (1978-1987) monthly average wind speed data (in ms$^{-1}$), measured at a height of 3m, in Jos (Latitude: 09.867, Longitude: 08.900 and Altitude: 1286m) were used for this study. They were obtained from the Nigerian Meteorological Agency, Oshodi, Lagos and are re-structured into a histogram of class interval of 1 in Fig. 1.

### B. Distribution Functions

Four different distribution functions: normal, $n\ (v)$, gamma, $g\ (v)$, Weibull, $h\ (v)$ and Rayleigh, $R\ (v)$ were fitted and their probability density function (pdf) expressions for the $i^{th}$ wind speed, $v_i$ are given [3] as

$$n\ (v) = \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{1}{2}\left(\frac{v_i-\mu}{\sigma}\right)^2\right], 0 \leq v \leq \infty, \quad (1)$$

$$g\ (v) = \frac{v_i^{\alpha}}{\beta^{\alpha}\Gamma(\alpha)} exp\left[-\frac{v_i}{\beta}\right], \alpha, \beta, v > 0, \quad (2)$$

$$h\ (v) = \frac{k}{c}\left(\frac{v_i}{c}\right)^{k-1} exp\left[-\left(\frac{v_i}{c}\right)^{k}\right], v \geq 0, k, c > 0, \quad (3)$$

$$R\ (v) = \frac{2}{c^2} v_i exp\left[-\left(\frac{v_i}{c}\right)^2\right], v_i \geq 0, c > 0, \quad (4)$$

where $k, c, \sigma, \mu, \alpha$ and $\beta$ are the parameters of the distribution functions.

These parameters were determined statistically [3] at various heights from predicted wind speeds $v_h$ at some selected wind turbines hub heights, $h$ using the Hellmann power law [4].

$$v_h = v_g \left(\frac{h}{h_g}\right)^{\gamma} , \tag{5}$$

where $v_g$ is the reference wind speed measured at the reference height, $h_g$(= 3m) from the ground surface and $\gamma$ is the ground surface friction coefficient (taken as 0.2 for Jos). These distribution functions were validated with some statistical goodness-of-fit tests: Chi-square, Kolmogorov-Smirnov (KS) and Anderson-Darling (AD) tests. The details of these statistics could be found elsewhere [3, 5, 6].

## C. Wind Power Densities

The extractible root-mean-cube wind power, $P_{rmc}$ at a rotor efficiency, $C_P$ = 50% is given [4] as

$$P_{rmc} = \frac{1}{4}\rho A v_{rmc}^3 , \tag{6}$$

and the root-mean-cube wind speed, $v_{rmc}$ for the best fitted distribution function, $f(v)$ is given as

$$v_{rmc} = \left(\int_0^{\infty} v_i^3 f(v) dv\right)^{\frac{1}{3}} . \tag{7}$$

Hence the root-mean-cube extractible power density, $P_s$ is then

$$P_s = \frac{P_{rmc}}{A} = \frac{1}{4}\rho v_{rmc}^3 , \tag{8}$$

where $v_{rmc}^3$ is derived from fitted gamma distribution function as

$$v_{rmc}^3 = \beta^3 \alpha(\alpha+2)(\alpha+1). \tag{9}$$

And the corresponding mean wind speed, $\bar{v}$ is given by

$$\bar{v} = \int_0^{\infty} v f(v) dv = \alpha\beta^2 . \tag{10}$$

## D. Turbine Electrical Power Output

The extractible wind power given by equation (8) does not automatically translate into electrical power output of turbines due to transmission and generator losses. The electrical output powers depend upon turbines characteristics and their performance at each wind speed. The annual average electrical power output, $P_a$ of a turbine between its cut-in, $v_{in}$ and cut-out, $v_{out}$ wind speeds is given [7] as

$$P_a = \int_{v_{in}}^{v_{out}} P(v) f(v) dv , \tag{11}$$

where $P(v)$ is the power delivered by a turbine at each wind speed and fitted in this work by a third degree polynomial as

$$P(v) = a_0 + a_1 v + a_2 v^2 + a_3 v^3 , \tag{12}$$

where $a_0$, $a_1$, $a_2$ and $a_3$ are coefficients determined from fitted polynomial unto the performance data [8, 9] of the selected wind turbines. The list of the selected turbines and their characteristics are given in Table I.

In Jos, since the maximum recorded wind speed, $v_{max}$ excluding gust, is less than most turbine's rated wind speed, $v_r$ (12-15ms$^{-1}$), Eq. (11) was simply evaluated.

**TABLE I.** Characteristics of some selected wind turbines.

| Turbine | $v_{in}$ (ms$^{-1}$) | $v_r$ (ms$^{-1}$) | Hub, $h_t$ (m) |
|---|---|---|---|
| Small- turbines | | | |
| Proven WT 6000 | 2.5 | 12 | 9-15 |
| BWC Excel-S | 3.5 | 13.8 | 18-30 |
| Jacobs WTIC-29 | 3.0 | 14 | 36.6 |
| Whisper H 175 | 3.0 | 12 | 15 |
| AOC 15/50 | 4.6 | 11.3 | 30 |
| Iskra AT5-1 | 3.5 | 11 | 12-30 |
| Nordtank 65 kW | 4.0 | 12 | 23 |
| Medium size turbines | | | |
| Suzlon S. 64/950 | 3.5 | 13 | 64.5 |
| Suzlon S 64/1000 | 3.5 | 13 | 64.5 |
| Bonus 300 Mk II | 4.6 | 15.5 | 30 |
| Nordtank 150 kW | 4.0 | 12 | 24.5 |
| Turbowinds T600 | 3.5 | 12.5 | 50-60 |
| Micon 108kW | 4.0 | 15 | 23 |
| Large turbines | | | |
| Suzlon S 62/1000 | 3.5 | 14 | 64.5 |
| Suzlon S 88/2100 | 4.6 | 14 | 80 |
| GE 1.5 SL 77 | 3.5 | 12 | 85/100 |
| Nordex S 77 | 4.6 | 14 | 80 |
| Micon NM 82 | 4.0 | 14 | 70-100 |
| Nordex N 90 | 4.6 | 15 | 80/100 |

(Sources: INL, Sayler, Manufacturers websites [8, 9]).

between $v_{in}$ and $v_r$ such that

$$P(v) = \begin{cases} 0, & v_i < v_{in}, \\ P_r, & v_{in} < v_i \leq v_{max} , v_r . \end{cases} \tag{13}$$

Using the values of the coefficients for each of the selected wind turbines and the corresponding values of $\beta$ at the fixed shape parameter, $\alpha$ Eq. (11) was then solved numerically at various heights for the yearly average electrical power output for each of the selected wind turbines.

## E. Turbine Capacity Factor

Wind turbines performance are measures of how much of their rated powers are relatively generated at a site per annum. The capacity factor, CF (%) is the figure of merit that determines these measurements [4] and is simplified as

$$CF = \frac{P_a}{P_r} . \tag{14}$$

## III. RESULTS AND DISCUSSION

The fitted distribution functions unto the constructed histogram of the measured or observed wind speed data of Jos is given in Fig. 1. Table II gives corresponding values of their relative goodness-of-fit test results.

It could be visually seen that gamma and normal distribution functions are better fitted to the constructed histogram than Rayleigh and Weibull distribution functions.

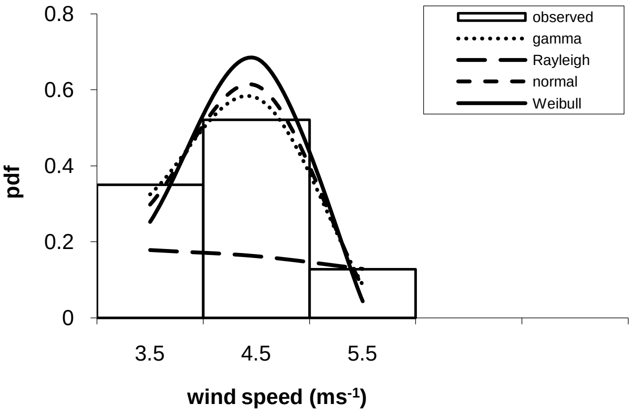

**FIGURE 1.** Fitted distribution functions.

These visual observations are equally validated by the goodness-of-fit test results given in Table II.

Despite the fact that the specific AD test is unavailable for gamma distribution function, the two distribution functions (gamma and normal) are still together the best fitted distribution functions. Therefore, either of the two could be used in Jos. In this work, the gamma distribution function is used.

Table III gives the predicted minimum, maximum, mean, mode and root-mean-cube wind speeds of Jos and their corresponding extractible wind power densities at various heights.

**TABLE II.** Statistical goodness-of-fit tests for fitted distribution functions.

| pdf | K-S test | | Anderson-Darling test | | |
|---|---|---|---|---|---|
| | Test value | Decision Rule | Test value | CV | Decision rule |
| $h$ (v) | 0.115 | reject | $2.7\text{x}10^{-5}$ | 0.050 | reject |
| $g$ ($v$) | 0.056 | accept | - | - | - |
| $R$ ($v$) | 0.342 | reject | $2.9\text{x}10^{-49}$ | 0.050 | reject |
| $n$ ($v$) | 0.075 | accept | 0.5811 | 0.747 | accept |
| CV | 0.1257 | | | | |

It is seen that the average minimum wind speed is greater than $4\text{ms}^{-1}$ hence a suitable cut-in for most modern wind turbines. With these range of power densities at the various heights, the wind in Jos could be considered available for most categories of wind turbines. Also in Table III are the values of the $\beta$ parameter of the gamma distribution function at different heights. The values of the modal wind speeds given in Table III give estimates of the speeds at which the wind blows most of the time and at the various heights in Jos.

The goodness of their fits is indicated by the values of the coefficient of determination, $R^2$ which were obtained to be greater than 0.9 for all the fitted turbines performance data.

The predicted annual average electrical power output of the selected wind turbines are presented alongside their corresponding capacity factors in Table IV. It could be seen that Iskra AT5-1 wind turbine at a hub height of 15m can generate, relative to its installed capacity, more electrical energy than Proven WT 6000 and Whispers H175 at the same hub height.

The two other turbines are themselves not too poor. All the three turbines could be used for residential purposes especially where 1kW of electrical power consumption is required.

**TABLE III.** The predicted mean, mode and root-mean-cube wind speed (in $\text{ms}^{-1}$) and extractible wind power densities (in $\text{Wm}^{-2}$) at various heights at a $C_p = 0.5$ and $\rho$ at corresponding heights.

| h (m) | $\beta$ | $v_{min.}$ | $v_{max.}$ | $v_{mean}$ | $v_{mode}$ | $v_{rmc}$ | $P_{mean}$ | $P_{mode}$ | $P_{rmc}$ |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 0.1179 | 4.26 | 8.02 | 5.88 | 6.07 | 6.00 | 61.16 | 67.36 | 64.89 |
| 20 | 0.1248 | 4.51 | 8.50 | 6.23 | 6.43 | 6.35 | 72.61 | 79.97 | 77.04 |
| 25 | 0.1305 | 4.72 | 8.88 | 6.51 | 6.72 | 6.64 | 82.93 | 91.34 | 87.99 |
| 30 | 0.1354 | 4.89 | 9.21 | 6.75 | 6.86 | 6.89 | 92.70 | 97.30 | 98.35 |
| 40 | 0.1434 | 5.18 | 9.76 | 7.15 | 7.27 | 7.30 | 110.06 | 115.52 | 116.77 |
| 50 | 0.1500 | 5.42 | 10.20 | 7.48 | 7.60 | 7.63 | 125.71 | 131.94 | 133.37 |
| 60 | 0.1555 | 5.62 | 10.58 | 7.76 | 7.88 | 7.91 | 140.10 | 147.05 | 148.64 |
| 70 | 0.1604 | 5.79 | 10.91 | 8.00 | 8.13 | 8.18 | 153.53 | 161.14 | 162.89 |
| 80 | 0.1647 | 5.95 | 11.21 | 8.22 | 8.35 | 8.38 | 166.17 | 174.41 | 176.30 |
| 100 | 0.1723 | 6.22 | 11.72 | 8.59 | 8.87 | 8.76 | 190.35 | 209.63 | 201.95 |

The medium size turbines category, Suzlon S.64/950, Suzlon S.64/1000 and Turbowinds have shown better performance relative to their individual installed capacities. Therefore could be used for medium scale power generation in Jos especially with Suzlon S.64/950 because of its higher capacity factor than the others For the larger size, G.E. SL 77 and Micon NM 82 have shown better relative power generation than the others. At least 36% of their installed capacities can be generated hence excellent turbines for large-scale power generation in Jos.

**TABLE IV** Average electrical power output, $P_a$, (in kW) and capacity factor, CF (in %) of some selected wind turbines at various heights

Small turbines

| Turbine / h (m) | Jacobs WTIC-29 (17.5 kW) | | Whisper H175 (3.5 kW) | | Proven WT6000 (6 kW) | | AOC 15/50 (50 kW) | | Iskra AT5-1 (5 kW) | | Nordtank (65 kW) | | BWC Excel-S (10 kW) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF |
| 15 | - | - | 0.68 | 19.43 | 1.31 | 21.83 | - | - | 1.19 | 23.8 | - | - | - | - |
| 20 | 2.87 | 16.40 | 0.82 | 23.43 | 1.60 | 26.67 | 12.03 | 24.06 | 1.43 | 28.6 | 10.71 | 16.48 | 2.07 | 20.7 |
| 25 | 3.39 | 19.37 | 0.94 | 26.86 | 1.84 | 30.67 | 13.98 | 27.96 | 1.65 | 33.0 | 12.30 | 18.92 | 2.38 | 23.8 |
| 30 | 3.85 | 22.00 | 1.05 | 30.00 | 2.06 | 34.33 | 15.71 | 31.42 | 1.83 | 36.6 | 13.78 | 21.20 | 2.66 | 26.6 |

Medium size turbines

| Turbine / h (m) | Suzlon S.64 (950 kW) | | Nordtank (150 kW) | | Bonus mkll (300 kW) | | Suzlon S.64 (1000 kW) | | Turbowinds (600 kW) | | Micon (108 kW) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF |
| 30 | - | - | 44.16 | 29.44 | 48.97 | 16.33 | - | - | 102.92 | 17.15 | 22.78 | 21.09 |
| 40 | 212.79 | 22.40 | 52.28 | 34.85 | 60.18 | 20.06 | 212.36 | 21.24 | 126.85 | 21.14 | 27.23 | 25.21 |
| 50 | 248.26 | 26.13 | 59.23 | 39.49 | 70.06 | 23.35 | 247.16 | 24.72 | 148.56 | 24.76 | - | - |
| 60 | 279.72 | 29.44 | - | - | 78.67 | 26.22 | 278.13 | 27.81 | 167.93 | 27.99 | - | - |
| 70 | 308.79 | 32.50 | - | - | - | - | 306.91 | 30.69 | - | - | - | - |

Larger size turbines

| Turbine / h (m) | Suzlon S.62 (1000 kW) | | Suzlon S.88 (2100 kW) | | G.E. SL 77 (1500 kW) | | Nordex S77 (1500 kW) | | Micon NM82 (1500 kW) | | Nordex N90 (2300 kW) | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF | $P_a$ | CF |
| 70 | 262.97 | 26.30 | 544.32 | 25.92 | 440.12 | 29.34 | 384.53 | 25.64 | 512.89 | 34.19 | 573.30 | 24.93 |
| 80 | 286.96 | 28.70 | 595.02 | 28.33 | 478.69 | 31.91 | 423.76 | 28.25 | 554.79 | 36.99 | 626.76 | 27.25 |
| 100 | 330.92 | 33.09 | 687.71 | 32.75 | 548.46 | 36.56 | 496.12 | 33.07 | 630.16 | 42.01 | 725.33 | 31.54 |

## IV. CONCLUSION

The wind speed distribution of Jos was found to be either gamma or normally distributed. It is found that with a minimum yearly average wind speed of > $4 ms^{-1}$, the wind could therefore be considered available for most categories of wind turbines. At respective heights of 15m and 100m, residential and large-scale wind power generation is very satisfactory and equally attractive for investment.

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