Introduction of Atwood's machines as Series and Parallel networks

Jafari Matehkolaee, Mehdi
Islamic Azad University of Sari, Iran, Mazandaran, Sari.

E-mail: mehdisaraviaria@yahoo.com

(Received 9 May 2011; accepted 30 June 2011)

Abstract

This paper shows that Atwood's machines are mechanical systems which can be considered in combination of series and parallel. By the concept of equivalent mass for series and parallel, it will be easy to analyze many problems in relation with networks of Atwood's machines. We defined specific relation separately for series and parallel network. These relations are not like relations of resistors and capacitors and springs networks.

Keywords: Atwood's machine, series, paralell, equivalent mass.

Resumen

El artículo muestra que las máquinas de Atwood son sistemas mecánicos los cuales pueden ser considerados en la combinación de series y paralelo. Por el concepto de masa equivalente para series y paralelo, serán fáciles de analizar muchos problemas en relación con las máquinas de trabajos de red de Atwood. Hemos definido la relación específica por separado para la red de trabajo en series y paralelo. Estas relaciones no son como las relaciones de resistencias, condensadores y las redes de trabajo de resortes.

Palabras clave: Máquina de Atwood, en serie, en paralelo, masa equivalente.

PACS: 45.20.Da-,45.20.da,45.30.+s **ISSN1870-9095**

I. INTRODUCTION

The concepts of series and parallel are very familiar, because we encounter them in various parts of physics. For example resistors and capacitors networks in electricity and springs networks in mechanics. In primary mechanics we sometimes consider the connection of mass and spring as series or parallel [1]. In spring networks our criterion for distinguishing between series and parallel were displacement of springs from origin. Indeed considering equivalent spring constants reduces difficulty in many problems. We tried to define the new conception of series and parallel in Atwood's machines. Our criterion for series and parallel combinations of Atwood's machines are tension of strings of the system. In this paper we assume that mass of all pulleys and strings can be ignored. Also the friction between of pulleys and strings are ignorable. At the first step with an example we try to show that analysis of Atwood's machines depends on the manner of connection of masses to strings and pulleys. In the Fig. 1, with change of the masses (m_1, m_2), whether increase or decrease, the tension of string will be consistently equal to zero and acceleration of m_1 and m_2 will be equal to g (g~9/8m/s^2).

This example shows that, arrangement of strings, masses and pulleys performs important role in determination of acceleration of system. Therefore we can introduce series and parallel networks with several examples.

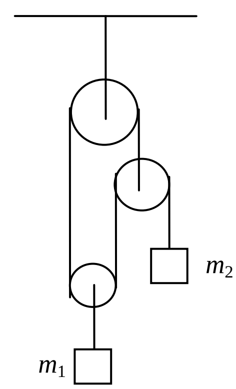

FIGURE 1. The system Atwood's machines that tension of string is consistently equal to zero.

II. SERIES NETWORK

In the Fig. 2, a pulley and two masses (m_1, m_2) are hung from a string with tension of T.

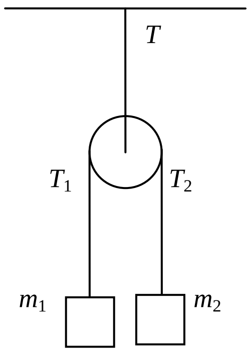

FIGURE 2. A simple Atwood's machine, that it consisting of two suspended masses connected by a string that passes over a pulley.

In this figure we can write:

$$T = T_1 + T_2\,, \tag{1}$$

T_1 and T_2 are tension of strings hung from pulley, where

$$T_1 = T_2 = \frac{2m_1m_2}{m_1+m_2}g, \qquad \text{and}$$

$$T = \frac{4m_1m_2}{m_1+m_2}g. \tag{2}$$

According to our primary assumption we ignore the mass of strings and pulleys. So we introduce the equivalent mass for the system is

$$\frac{4m_1m_2}{m_1+m_2}, \qquad \text{or}$$

$$m_t = \frac{4}{\dfrac{1}{m_1}+\dfrac{1}{m_2}}\quad . \tag{3}$$

By equivalent mass we mean the mass of a single body that can be substituted for the combination with no change in the operation of the rest of the system. Also by considering equivalent mass it can be equivalent system for Atwood's machines that makes less number of equations. The equivalent mass can be calculated in another way.

In Fig. 3, Fig. (3-b) shows the equivalent mass for system shown in Fig. (3-a). We suppose that the two systems have equal downward acceleration (a').

Evidently by Newton's second law for m_1 and m_2 and m we have:

$$mg - T = ma',$$

$$m_1g - T/2 = m_1(a'-a), \tag{4}$$

$$m_2g - T/2 = m_2(a'+a).$$

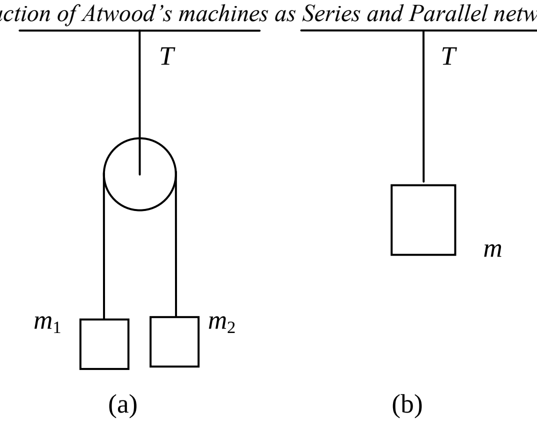

FIGURE 3. An Atwood's machine (a) and equivalent system for it (b).

In equations shown above, (a) represents acceleration of m_1 and m_2. By solving the equations we conclude

$$m = \frac{4}{\dfrac{1}{m_1}+\dfrac{1}{m_2}}.$$

Which is similar to relation (3).

In this relation $m \le m_1 + m_2$ and equality ($m = m_1 + m_2$) is for while masses are equal. In the Atwood's machine in series network primary tension of string , T, can be divided to smaller values ,T_1, T_2,…, T_n (for a network with n number of masses) , so that

$$T = T_1 + T_2 + \ldots + T_n\,. \tag{5}$$

If we hanging from a string of Atwood's machine, the another Atwood's machines in succession, then we have series network.

A. Example (1)

In the following figure m_1 = 4kg, m_2 = 3kg and m_3 = 1kg, suddenly we release the masses.

Calculate acceleration of m_1?

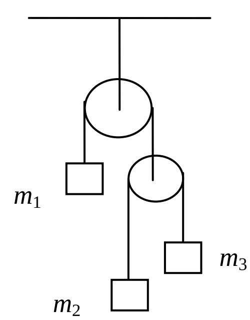

FIGURE 4. An Atwood's machine in series.

At first we should calculate the equivalent mass for m_2 and m_3

$$m_{2,3} = \frac{4}{\frac{1}{m_2} + \frac{1}{m_3}} = 3kg.$$

Now, we have an Atwood's machine with two masses which are $m_1 = 4$kg and $m_{2,3} = 3$kg so acceleration of m_1 and $m_{2,3}$ are equal to $g/7$ and $-g/7$ respectively. If we wanted to account acceleration m_2 or m_3 separately, then we use with equations that similar to second and third equations of relation (4) so acceleration of m_2 and m_3 are equal to $4g/7$ and $-4g/7$ respectively. maybe you feel since this example is solved in Fowles [2], using a standard Lagrangian approach, its not need to equivalent mass concept , but more increase the numbers of pulleys and masses more increase the number of equations and will be more difficult.

B. Example (2)

Consider an infinite Atwood's machine like following figure. Each of the masses is equal to m. suddenly we release the masses. Calculate acceleration of above mass? [3]

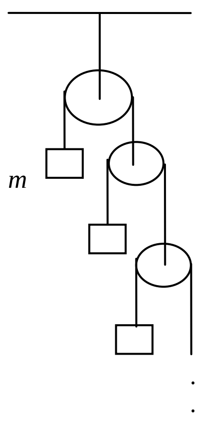

FIGURE 5. An infinite Atwood's machine.

By the conception of the equivalent mass we solve this problem. We suppose above system made of n number of masses (m) that are combine in series. If we consider x as mass of body hung from nth pulley, we can consider $f(x)$ as equivalent mass for masses of hanging from nth and (n-1) the pulley then according to relation (3):

$$f(x) = \frac{4}{\frac{1}{m} + \frac{1}{x}} = \frac{4x}{1 + \frac{x}{m}}.$$

Evidently the next pulley connects to x, so:

$$f(f(x)) = \frac{16x}{1 + \frac{5x}{m}}.$$

And we have for n masses

$$\left(\underbrace{f\,o\,f \cdots f}_{order\ n}\right)(x) = \frac{4^n x}{1 + \frac{(4^n - 1)x}{3m}}.$$

For the infinite Atwood's machine $n \to \infty$

So, equivalent mass is equal to $3m$. Then the acceleration of m is $g/2$. See to Fig. 6.

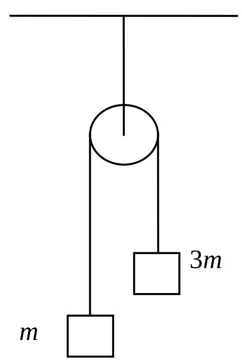

FIGURE 6. Equivalent system for Fig. 5, the acceleration of m is $g/2$.

III. PARALLEL NETWORK

In this combination tension of string is equal everywhere. This means that relation (5) can be written as

$$T_1 = T_2 = \ldots\ldots . T_n .$$

For a system made of n number of masses m_1, m_2,….m_n that are combine in parallel, the equivalent mass is

$$m_t = \frac{n^2}{\frac{1}{m_1} + \frac{1}{m_2} + \ldots . \frac{1}{m_n}}. \qquad (6)$$

And while the masses are equal:

$$m_1 = m_2 = \ldots = m_n = m,$$

So

$$m_t = nm,$$

by the equivalent mass in parallel network, will be easy accounting acceleration of masses and tension of strings. One system which can be considered for parallel network is

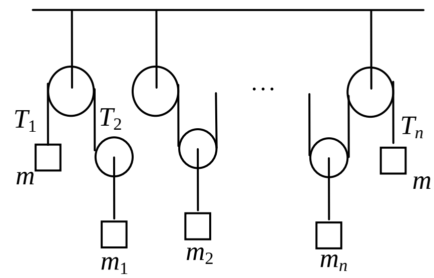

FIGURE 7. A parallel network for m_1, m_2… m_n and T_1=T_2=…=T_n.

The simplest figure for the system shown above is

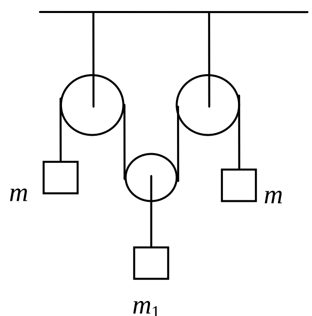

FIGURE 8. The simplest combination for parallel network.

Since length of string is constant, it can be easily concluded that acceleration of side masses are equal and opposite of the acceleration of middle mass. If we consider side masses as a single mass, 2*m*, equivalent arrangement for Fig. 8 is

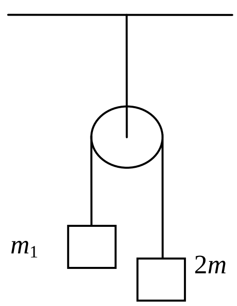

FIGURE 9. Equivalent arrangement, for Fig. 8.

Acceleration of m is equal to $g/3$ and acceleration of m_1 is equal to $-g/3$.

Now, we consider the next combination. Look at the Fig. 10:

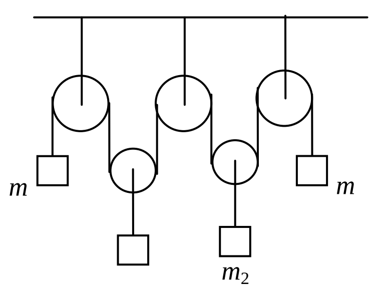

FIGURE 10. the parallel system with two middle masses.

For analyze Fig. 10 we suppose that the string is fixed and steady. In fact, we suppose that the acceleration of side masses is equal to zero.

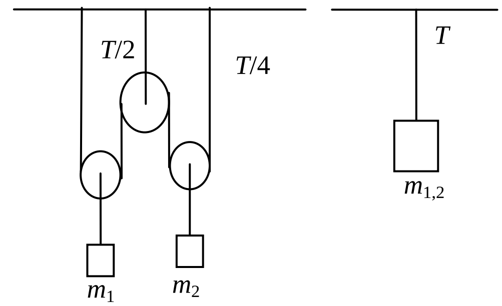

FIGURE 11. a) A system including the m_1 and m_2. b) An equivalent system for (a).

By writing Newton's second law for m_1 and m_2, tension of string is calculated

$$T=\frac{4m_1m_2}{m_1+m_2}g.$$

Therefore equivalent mass, $m_{1,2}$, is equal to

$$\frac{4}{\frac{1}{m_1}+\frac{1}{m_2}}.$$

Note that inserting n = 2 in Eq. (6) have the same result.

Now, we can imagine equivalent arrangement for Fig. 10.

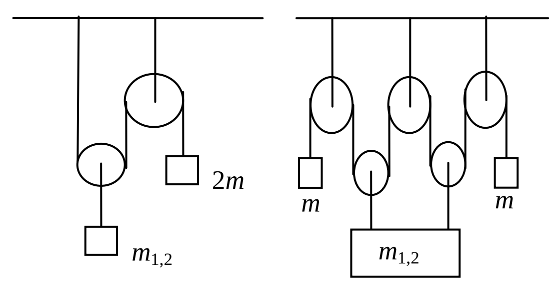

FIGURE 12. Two equivalent systems for Fig. 10.

Acceleration of each mass can be calculated

$$a=\frac{2m(m_1+m_2)-2m_1m_2}{2m(m_1+m_2)+m_1m_2}g,$$

$$a_1=\frac{2m(2m_2-m_1)-m_1m_2}{2m(m_1+m_2)+m_1m_2}g,$$

$$a_2=\frac{2m(2m_1+m_2)-m_1m_2}{2m(m_1+m_2)+m_1m_2}g.$$

On condition in which m_1 and m_2 and m are equal acceleration of side masses is equal to $2g/5$ and acceleration of m_1 and m_2 is equal to $g/5$ and $-g/5$ respectively.

Next parallel system includes three intermediate masses, m_1 and m_2 and m_3, and two side masses.

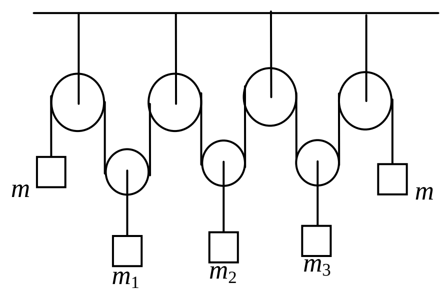

FIGURE 13. The parallel system with three intermediate masses.

Again we suppose that the string is fixed and steady. Therefore $a_1 + a_2 + a_3 = 0$ that a_1 , a_2 , a_3 are accelerations of m_1 , m_2 , m_3 respectively.

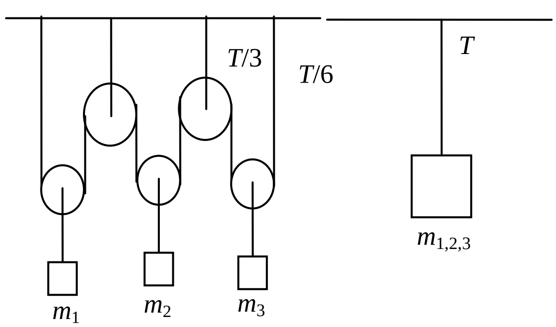

FIGURE 14. a) A system including m_1, m_2, m_3 b) An equivalent system for (a).

We assume that equivalent mass is $m_{1,2,3}$. By writing Newton's second law tension of string is calculated as follows

$$T = \frac{9g}{\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}}.$$

So equivalent mass, $m_{1,2,3}$ is equal to

$$\frac{9}{\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}}.$$

Note that inserting $n = 3$ in Eq. (6) has the same result.

Now we can imagine equivalent combination for Fig. 13 as following

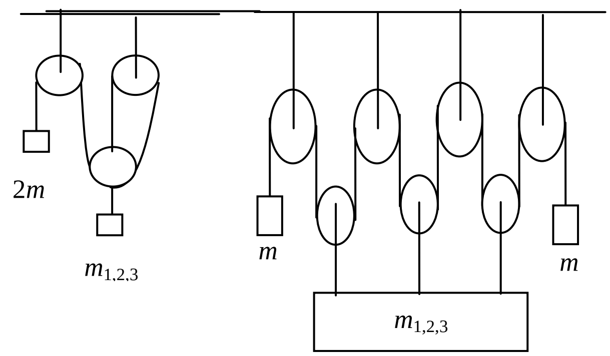

FIGURE 15. Two equivalent systems for Fig. 13.

If we assume that $m_1 = m_2 = m_3 = m$. Then acceleration of side masses will be equal to $3g/7$ and acceleration of intermediate masses will be equal to $-g/7$ may be another example makes bored you then next example without further explanation. In this combination we imagine a system with four intermediate masses and two side masses.

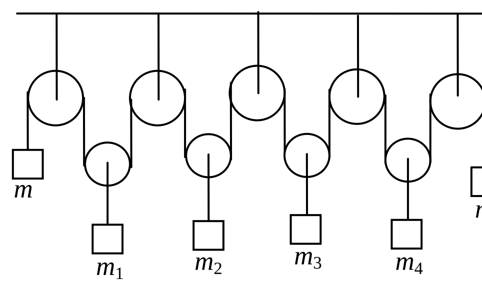

FIGURE 16. A system with four intermediate and two side masses.

The equivalent mass for m_1, m_2 , m_3 , m_4 is

$$m_{1,2,3,4} = \frac{16}{\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3} + \frac{1}{m_4}}.$$

That by putting $n = 4$ in Eq. (6) the same result will be obtained. The equivalent combination for Fig. 16 is

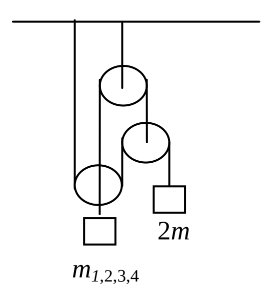

FIGURE 17. An equivalent system for Fig. 15.

By increasing the number of intermediate pulleys acceleration of side masses increases slightly and acceleration of intermediate masses tends to zero.

If we had infinite Atwood's machine in which all the masses are equal and combined in parallel form, acceleration of side masses will be as follows

$$a = \frac{n}{2n+1} g.$$

That if $n \to \infty$ then acceleration will tend to $g/2$.

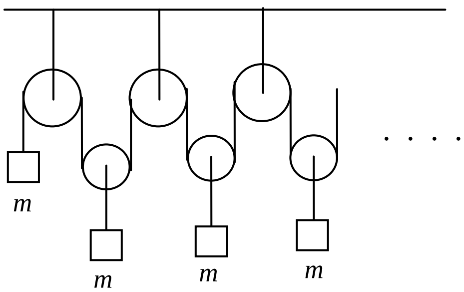

FIGURE 18. An in finite Atwood's machine.

IV. CONCLUSION

Atwood's machine in series and parallel combination can be considered as a basic field of mechanics. Because considering this matter deepens attitude to problems related to this systems**.** Also the concept of the equivalent mass and the acceleration of each of the masses could be derived with the method presented in the paper , and directly compared to the more standarad approach.

REFERENCES

[1] Resnik, R., Halliday, D. & Krane, K. S., *Physics (volume 1)*, 4th Ed. (Wiley, New York, 1992).

[2] Fowles, G. R. and Cassiday, G. L., *Analytical Mechanics*, 6th Ed. (Harcourt Brace College Publishers, USA, 1999).

[3] Maureen, D., Shpyrkv, O., *Competitions Physics Boston University America.4th ed.*, (Boston university, USA, 2004).