The direction of the electric field of the uniform line of charge

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Abstract

This article presents that the direction of the electric field of the uniform line of charge can be shown as geometrical rule. We investigate this rule for finite line, semi-infinite line and infinite line. This subject shows compatibility of geometry and physics.

Keywords: geometry,direction,line of charge

Resumen

Este artículo presenta que la dirección del campo eléctrico de la línea uniforme de carga puede ser mostrada como regla geométrica. Investigamos esta regla para la línea finita, línea semi-infinita y línea infinita. Este tema muestra la compatibilidad de la geometría y la Física.

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The many parts of the physics and geometry are compatible. The concept of stable equilibrium in physics makes easy the solution some of the problems of geometry.

As example suppose we wish to find the point on the plane triangle ABC so that sum of the distances of point to triangle heads are the minimum value.

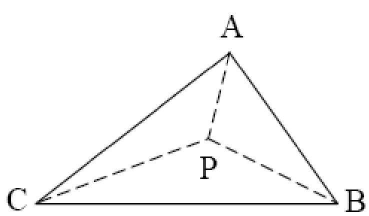

FIGURE 1. P is the point that, (PA+PB+PC) is the minimum value.

This point called to Fermat's point. That is from this point the three side of triangle is seen with equality angles. This point can be obtain by the another method that is based on stable equilibrium and the minimum potential energy. (We hang the same three strings with tension of T and the same three bodies with masses of m from the point of P. so that the strings lie on the PA, PB and PC and the system is in the stable equilibrium state. The point of P is equilibrium point due to potential energy is the minimum acquired that PA, PB and PC is the minimum value, because the length of the strings are constant).

Instead the determination of center of gravity and center of mass would facility by geometrical methods. The perception concepts such as flux and charge surface density will easier to solid angle and Gaussian curve. Also we show in this paper the direction of the electric field of line of charge by a theorem in geometry as a general rule.

At first we begin our discussion with a theorem.

Theorem:

The magnitude of the electric field at the point A due to the infinitesimal line of charge (side of BC in the ABC triangle in Fig. 2), is depends on the normal distance y and angle α. y is the normal distance from A to horizontal and α is the angle that side of BC can be seen from A.

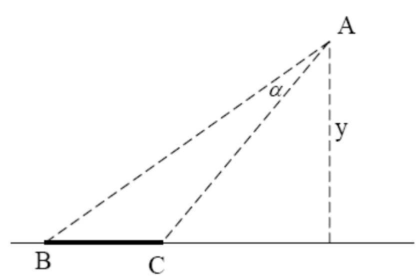

FIGURE 2. *y* is the normal distance from point A to horizontal line.

The magnitude of the electric field at point A proportional to $\frac{BC}{(AC)^2}$ or

$$E \sim \frac{BC}{\left(AC\right)^2}, \tag{1}$$

$$\frac{BC}{\left(AC\right)^2} = \frac{\frac{BC}{AC}}{AC} = \frac{\sin\alpha}{AC\sin\hat{B}}.$$

Since α is the small angle then

$$E \sim \frac{\sin\alpha}{AC\sin\hat{C}} = \frac{\sin\alpha}{y} \equiv f\left(\alpha, y\right), \tag{2}$$

now see to Fig. 3, this figure shows a uniform ring of charge. This rings of radius y carriers a uniform charge density. According to the figure the infinitesimal line of charge BC lies on the line so that this line is the tangent on the ring.

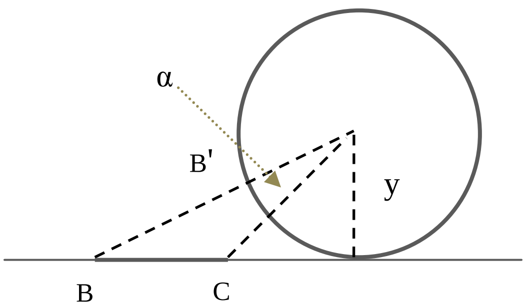

FIGURE 3. The ring of radius y carriers a uniform charge density.

According to the above theorem the magnitude of the electric field due to the arc B'C' and line of charge BC is similar.

Because both of them are depends on radius y and angle α. The magnitude of the electric field for them is [2]

$$E = \frac{\lambda}{2\pi\varepsilon_0 y}\sin\frac{\alpha}{2}. \tag{3}$$

This relation is compatible to relation (2). Clearly the direction of the electric field at the point A due to line of charge BC and the arc B'C' is similar too.

We consider a thin rod of length l that lies along the horizontal axis and carriers a uniformly distributed positive q, so that its linear charge density is λ. According to the principle of superposition [1] the rod made on by two rods length l' and $(l\text{-}l')$.

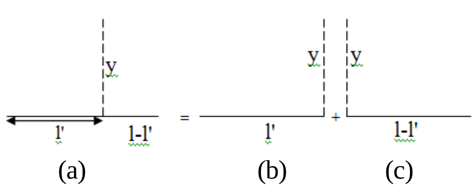

FIGURE 4. Fig. (a) to be made on Fig. (b) and Fig. (c).

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The electric field due to rod lengh l' at the point of P is

$$\vec{E}_b = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{\left(y-\sqrt{l'^2+y^2}\right)}{y\sqrt{l'^2+y^2}}\hat{i} + \frac{l'}{y\sqrt{l'^2+y^2}}\hat{j}\right]. \tag{4}$$

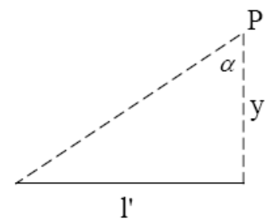

FIGURE (4-b). In this figure α is the angle that line of l' can be seen from the point of P. the point of P located in the normal distance y from the line of l'.

The magnitude of $\vec{E}_b$ is

$$E_b = \frac{\lambda}{2\pi\varepsilon_0 y}\sin\left(\frac{\alpha}{2}\right),$$

α in above relation is the angle that, all of the rod can be seen from the point P.

And the electric field to rod of length $(l-l')$ at the point P is

$$\vec{E}_c = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{\left(\sqrt{(l-l')^2+y^2}\right)-y}{y\sqrt{(l-l')^2+y^2}}\hat{i} + \frac{(l-l')}{y\sqrt{(l-l')^2+y^2}}\hat{j}\right] \tag{5}$$

finally the electric field due to rod of length l at the point P is $\vec{E} = \vec{E}_c + \vec{E}_b$.

If we suppose θ is the angle between along of the electric field vector ($\vec{E}$) due to rod of length l to horizontal then

$$\tan\theta = \frac{y(l-l')\sqrt{l'^2+y^2} + yl'\sqrt{(l-l')^2+y^2}}{\sqrt{l'^2+y^2}\left(\sqrt{(l-l')^2+y^2}-y\right) - y\sqrt{(l-l')^2+y^2}\left(\sqrt{l'^2+y^2}-y\right)},$$

so

$$\theta = \tan^{-1}\left(\frac{l\sqrt{l'^2+y^2}}{y\left(\sqrt{(l-l')^2+y^2}-\sqrt{l'^2+y^2}\right)} + \frac{l'}{y}\right), \tag{6}$$

by to limit of relation (6), $l' \to \frac{l}{2}$, then we conclude $\theta = \frac{\pi}{2}$.

This angle shows that the direction of the electric field due to rod of length *l* at the point P a normal distance *y* from the rod along its perpendicular bisector.

Now we interpret θ in relation (4). See to Fig. 5:

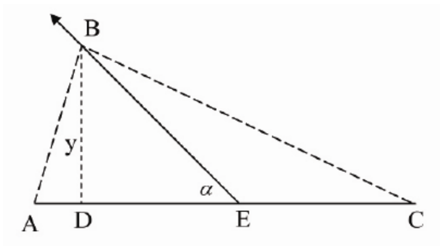

FIGURE 5. In this figure AD = *l'*, DC = (*l* - *l'*) and AC = *l* and BD = *y* is the normal distance.

Suppose the electric field vector due to rod length *l* lies on the BE. If we consider BE is the median triangle of ABC then AE = EC. That is the contribution of two part of rod is equal in the electric field. It is no logic.

So we assume that BE is the bisector of the triangle ABC. By a famous theorem geometry can be writing

$$\frac{BC}{BA} = \frac{CE}{AE}.$$

If we consider DE = *x* then

$$\frac{\sqrt{(l-l')^2 + y^2}}{\sqrt{l'^2 + y^2}} = \frac{l-(l'+x)}{l'+x}.$$

By the calculation above equality, the value of *x* is follow

$$x = \frac{l\sqrt{l'^2+y^2}}{\left(\sqrt{(l-l')^2} + \sqrt{l'^2+y^2}\right)} - l',$$

α can be obtain from the triangle BDE in Fig. 5:

$$\tan\alpha = \frac{y}{x} = \frac{1}{\dfrac{l\sqrt{l'^2+y^2}}{y\left(\sqrt{(l-l')^2} + \sqrt{l'^2+y^2}\right)} - \dfrac{l'}{y}}.$$

After working out calculation above equation gives us

$$\alpha = \tan^{-1}\left\{\frac{l\sqrt{l'^2+y^2}}{y\left(\sqrt{(l-l')^2+y^2} - \sqrt{l'^2+y^2}\right)} + \frac{l'}{y}\right\} \qquad (7)$$

relation (7) shows that the electric field vector due to rod each arbitrary point lies on the bisector of triangle is made by two end of rod. Because the relation of (7) is equal to relation (6).

By to limit of relation (6) or (7), $l' \to 0$, then we conclude

$$\delta = \tan^{-1}\left(\frac{l}{\sqrt{l^2+y^2} - y}\right) \qquad (8)$$

Eq. (8) gives us the direction of the electric field due to rod of length *l* at a normal distance *y* from the end of the finite rod. See to Fig. 6.

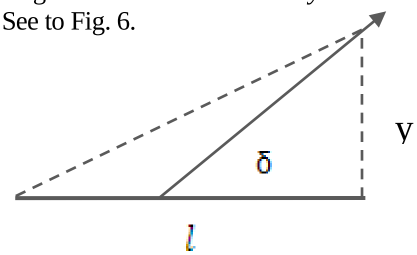

FIGURE 6. A rod of length *l* and along of the electric field vector due to rod.

By to limit of relation (8), $l \to \infty$, then we conclude $\delta = \frac{\pi}{4}$.

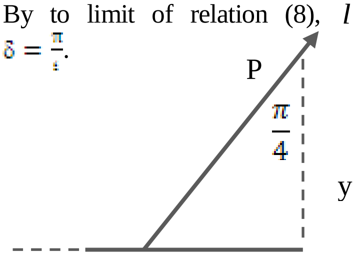

FIGURE 7. A semi-infinite rod, show that the electric field at the point P makes an angle $\frac{\pi}{4}$ with the rod.

For the another example see to Fig. 8.

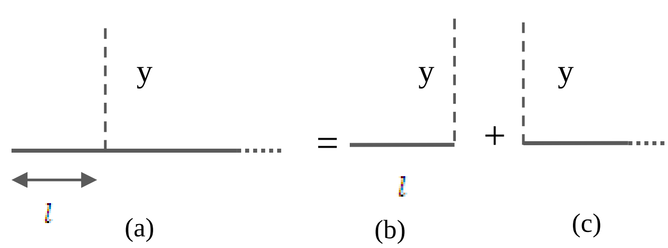

FIGURE 8. A semi-infinite rod made on by two Fig. (8-b) and (8-c).

Now according to rule, we draw the direction of electric fields due to rod of Fig. (8-b) and Fig. (8-c) on the semi- infinite rod Fig. (8-a).

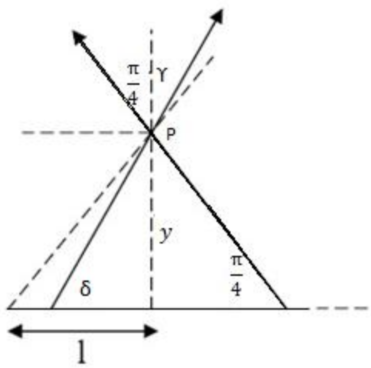

FIGURE 9. This figure shows the geometry for the calculation. □ is the relation (8).

We conclude from Fig. 9, the electric field due to a semi-infinite rod (Fig. (8-a) makes an angle $\beta = \frac{\pi}{4} + \gamma$ with the rod. Then $\tan\beta = \tan\left(\frac{3\pi}{4} - \theta\right)$

From the trigonometry relation

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}.$$

And from the relation (8) we can write

$$\beta = \tan^{-1}\left(\frac{\sqrt{l^2 + y^2} + l}{y}\right) \qquad (8)$$

By to limit $l \to 0$, above relation lead to $\beta = \frac{\pi}{4}$ finally see to Fig. 10.

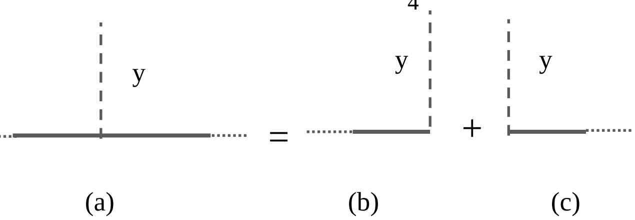

FIGURE 10. An infinite rod made on by two semi-infinite rod.

We draw the direction of electric field due to semi-infinite rod of Fig. (10-b) and (10-c) on the infinite rod (10-a).

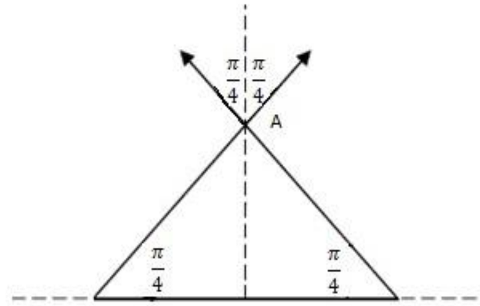

FIGURE 11. This figure shows that the electric field due to infinite rod makes an angle $\frac{\pi}{4}$ with the rod.

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CONCLUSION

We show that rely on geometry the direction of electric field due to line of charge can be shown as geometrical rule. By the principle of superposition and by drawing the electric field vectors we can get the direction of the electric field due to line of charge.

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