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Comparison between gravitational and inertial mass: Two experiments with Real Time Laboratory

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Abstract

Two simple experiments performed with RTL (Real Time Laboratory) offer an opportunity to make clear for the students the difference between weight and mass, and between inertial and gravitational mass. The role of the hydrodynamic mass is also investigated, and various effects on the motion of different bodies due to the presence of air are discussed.

Keywords: Inertial mass, gravitational mass, hydrodynamic mass, hydrostatic force, Microcomputer Based Laboratory.

Resumen

Dos experimentos sencillos realizados con RTL (Laboratorio de Tiempo Real) ofrecen una oportunidad para dejar en claro a los estudiantes la diferencia entre peso y masa, y entre masa gravitacional e inercia. Se investigó el papel de la masa hidrodinámica también, y también se discutieron diversos efectos en el movimiento de diferentes cuerpos debido a la presencia de aire.

Palabras clave: Masa inercial, masa gravitacional, masa hidrodinámica, fuerza hidrostática, Laboratorio Basado en Microcomputadoras.

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I. INTRODUCTION

"What is heavier: 1 kg of iron or 1 kg of Styrofoam?" Most students at elementary level learn that the correct answer is "*Neither one: They have the same weight*". This may be satisfactory at elementary level, if the question is addressed to point out the different concepts of density and weight, with students whose experience suggests that iron is always heavier than Styrofoam. But the physics teacher knows that weight is not the same.

Confusing *weight* with *mass* is a common mistake among students, and it might be due to various reasons. In the Practical Unit System, the same number indicates both the gravitational *force* acting on a body and its *mass*. As a consequence the weighing balance is often assumed to be a device that directly measures the mass of a body instead of the force acting on it. Moreover the upward hydrostatic force due to air (also named Archimedes's force or hydrostatic force) acting on the objects usually weighed in the laboratory, or in everyday life situations, is normally negligible with respect to the force acting on the same objects due to the Earth gravitational force.

The tricky question mentioned above may be reformulated more clearly as follows: "If two blocks, one made of iron and one of Styrofoam, have the same mass, would they show the same weight, measured by a weighting balance in air?

Here the answer is "no: iron weighs more than Styrofoam because its density is higher and therefore the air hydrostatic force is smaller than in the case of Styrofoam".

The weighing balance does measure the vectorial sum of two *forces* (hydrostatic and gravitational), and it may be calibrated in *mass* unit only if the hydrostatic term of the measured force is negligible.

Being the usual balance a device unable to produce correct mass measurements, how can we measure the mass of a body?

Nowadays the real-time data acquisition systems allow us to use fast and simple apparatuses to perform this task. We describe here two experiments aimed at pointing out the *difference between mass and weight,* by using only motion and force sensors and few other simple objects.

The investigation requires to measure mass and weight of the same object separately. For this purpose the different concepts of *inertial* and *gravitational* mass will also be considered.

II. A FIRST EXPERIMENT: BALLOON BOUNCES

For this experiment we use a gimmick large balloon[1] (air-filled with a bicycle pump to a diameter of about 40cm), a sonar[2] and a force sensor connected through a CBL™ interface to a Texas TI-89 graphic calculator[3].

The first step is to hang the balloon to the hand-held force sensor, by means of a wire loop (hold in place by sticky tape). The measured force F is the balloon "weight", the force sensor acting as a balance which gives the total force acting on it.

Assuming that only gravitational force is acting, we would obtain for the balloon mass the value:

$$m=F/g. \qquad (1)$$

In our case[4] the measured "weight" F=8.80N corresponds to a mass $m= F/g=$ 0.898kg.

In the second step we compare this value with the one obtained dynamically as

$$m=F/a, \qquad (2)$$

where F is the applied force and a the resulting acceleration.

We expect these two values of m to be identical, even if they refer to different properties of the same body.

The first value corresponds to the gravitational mass m_g, defined by the gravitation law $F= m_g(\gamma M/r^2)$, on the Earth usually simplified into $F=m_g g$, where $g=(\gamma M/r^2)$ is the gravity acceleration, depending on the Earth mass M and on the distance r of the body from the Earth center of mass (neglecting the effect of Earth rotation). The second value corresponds to the inertial mass m_i, defined by Newton's Law $F=m_i a$.

The inertial mass is obtained by recording the balloon movement during some bounces, and calculating its acceleration a during the upward and downward motion.

The sonar is held still at about 2 meters above the ground, facing the balloon bouncing underneath. Three plots of distance, velocity and acceleration versus time are automatically produced by the system and made available on the screen of the graphic calculator[5], like those shown in Fig. 1.

From any of the plots of Fig. 1 we may obtain an evaluation of the balloon acceleration a during free fall (both during the rise and the fall of the balloon), by quadratic fitting in the first plot, by linear regression in the second one and by simple average in the third one.

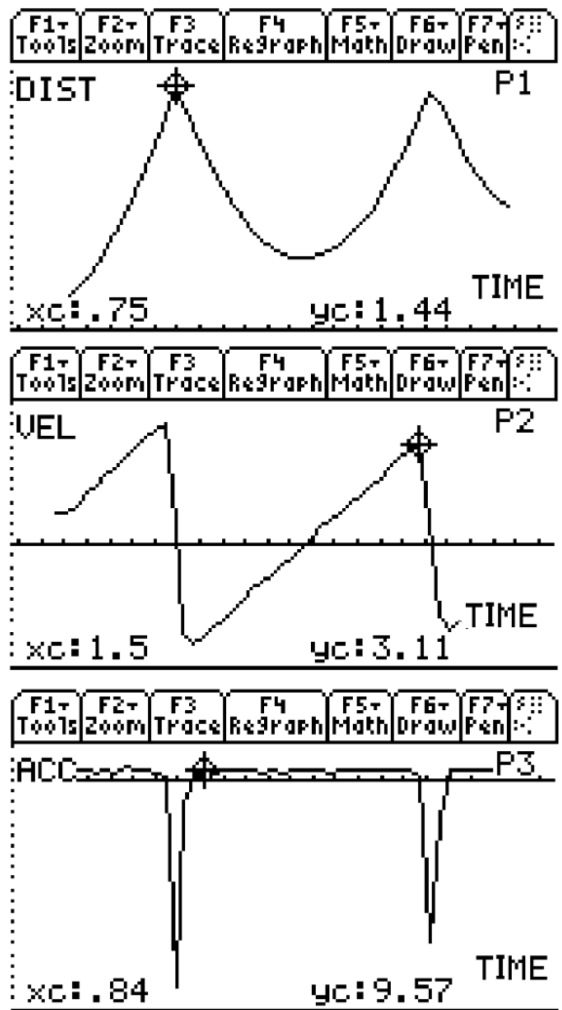

FIGURE 1. Balloon position, velocity and acceleration as functions of time.

We note that we cannot detect sensible effects of the air friction on the balloon acceleration.

A friction force in fact acts in the direction opposite to the velocity, and it is expected to add-up to the gravitational force during upward motion and to be subtracted from gravitational force during downward motion. As a consequence the presence of air friction should produce different slopes in the velocity versus time plot.

[1] Available in sports shops at a cost of about 15$.
[2] Here we use the tern *sonar* as a short name for the Motion Detector (see http://www.vernier.com), a sensor based on the sonar technology to measure in real time the position of an object by using the time of flight of an acoustic wave pulse reflected by the object (the "eco" effect).
[3] CBL (Computer Based Laboratory) is produced by Texas Instruments (see: http://www.education.ti.com)
[4] In Padova (Italy), the gravity acceleration is g=9.81m/s^2. Small changes with altitude or latitude (of the order of few parts per thousand) may be found in some textbooks, *e.g.* Tipler Paul A., *Physics*, (Worth Pub., New York, 1965).
[5] The Texas Instruments graphic calculators usually show graphs without numerical labels on the axes, because they are "live" graphs, where a marker moving along the curves shows the coordinate values, and this allows to obtain the plot scale.

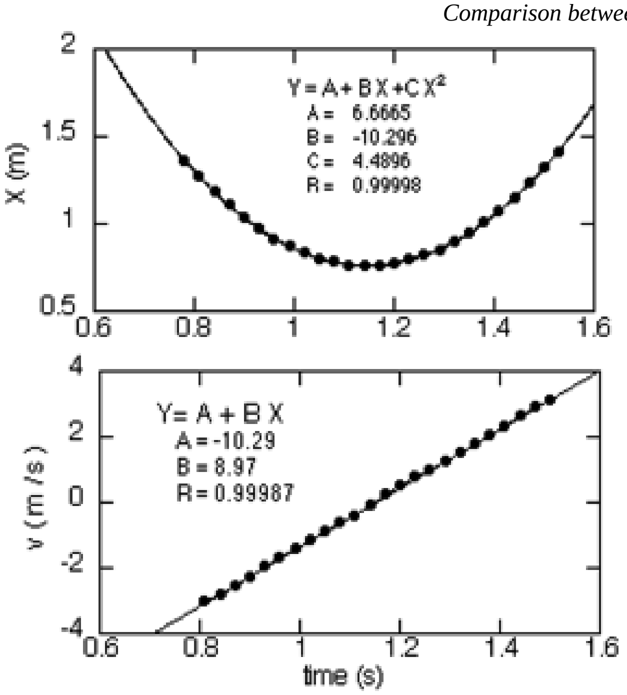

FIGURE 2. Quadratic and linear fits to calculate the acceleration.

On the contrary our experimental results (in Fig. 2) show equal slopes during the upward and the downward motion. (Actually a slight difference may be found by using separate fits for upward and downward motion, and these details are discussed in the Appendix).

Being the measured value of the acceleration $a \approx 9\text{m/s}^2$ (that we just proved not to be affected by systematic error due to friction) well below the gravity acceleration g, the calculated inertial mass results larger than the one statically measured: $m_i = F/a$=0.980kg.

The difference from the value measured as $m_g = F/g$= 0.898kg, is about 8%, well above the experimental uncertainty. A systematic error in these measurements is introduced by assuming a given value (344m/s) for the sound speed in air (used to convert the sonar output into distance measurements) corresponding to sound speed at about 22°C. Due to the temperature dependence of this parameter a maximum error of about ±1% is introduced when working at room temperature of 18°C or 28°C, respectively.

We may try to explain this disagreement by taking into account the hydrostatic force due to the air, acting on the balloon.

III. A FIRST CORRECTION: THE HYDROSTATIC FORCE

When we include in our analysis the hydrostatic force directed upward $F_A = \rho g V$, the total force measured by the sensor becomes $F = m_g g - \rho g V$, and therefore the calculated gravitational mass changes into:

$$m_g = (F + \rho\, g\, V)\,/g. \qquad (3)$$

To calculate the hydrostatic force $F_A = \rho g V$ we must know the balloon volume V, and the air density ρ. The volume $V=(4/3)\pi R^3$ may be obtained by measuring the length $2\pi R$ of a thin wire wound around the balloon. From the measured radius R= 21.5cm, we obtain V= 41600cm^3.

By assuming for the air density the value $\rho=1.2\text{kg/m}^3$ quoted in many textbooks (dry air at 0°C and 100kPa), the hydrostatic force results F_A = 0.49N, which is about 5.6% of the measured weight.

Therefore the calculated gravitational mass becomes m_g = (0.947±0.015) kg. This value is still smaller than the inertial mass obtained by dynamic measurement $m_i = F/a$ = (0.980±0.020) kg.

The uncertainties on m_i and m_g are evaluated assuming 1% of uncertainty on the force F, 2% on the radius R, and 3% on the air density ρ.

Our analysis could stop here, because the error bars make the two values compatible.

But repeated measurements prove that the m_i value is always in excess with respect to the m_g value.

We could guess that the assumed value for the air density is too small, and this would underestimate the correction introduced in calculating the m_g value. But considering that we work at room temperature (not at 0°C), and that the humid air typical of Padova has a *lower* density with respect to dry air, we must conclude that the adopted value for ρ is in excess, not in defect. Therefore we must find a different reason for the observed systematic larger values for m_i.

IV. A SECOND CORRECTION TO THE MODEL: THE HYDRODYNAMIC MASS

The neglected effect must in some way increase the effective value of the inertial mass m_i with respect to the value of the gravitational mass m_g.

Such effect is indeed the hydrodynamic mass, *i.e.* the increase of the inertia of an object moving in a fluid, due to the fact that also some fluid mass must be displaced.

A complete calculation of the hydrodynamic mass may be found in the literature [1]. For a sphere it is (1/2)ρV, *i.e.* one half of the mass of the displaced air.

Accounting for this effect and using Eq. (3) for m_g, the motion equation becomes:

$$a = \frac{F}{m_i} = \frac{F}{m_g + (1/2)\,\rho V} = \frac{F}{F/g + (3/2)\,\rho V}, \qquad (4)$$

which predicts, for our balloon, a=9.05m/s^2, in good agreement with the experimental result.

In Eq. (4) the quantity F/g is the “mass measured by a weighting balance”, and we may therefore conclude that the effective inertia in the observed motion is equivalent to that of a balloon whose mass is increased, with respect to the value measured by its weight, by the amount (3/2)ρV.

In our system the inertial mass involved in the motion is not only the balloon mass. We must consider not only the balloon but the *system as a whole*, in a way similar to the case of the horizontal Atwood machine (where the accelerated mass is the sum of the cart mass and the mass of the pulling weight), or to the case of rolling bodies on the incline (where the driving force is produced by gravity but the inertia is affected by the mass distribution around the rotating axis…).

V. A SECOND EXPERIMENT: BALLOON OSCILLATIONS

We use the same RTL system, a spring (made of a rubber band, with an elastic constant k=27N/m and mass m_s=0.005kg.), and a rigid stand to hold the force sensor as shown in Fig. 3.

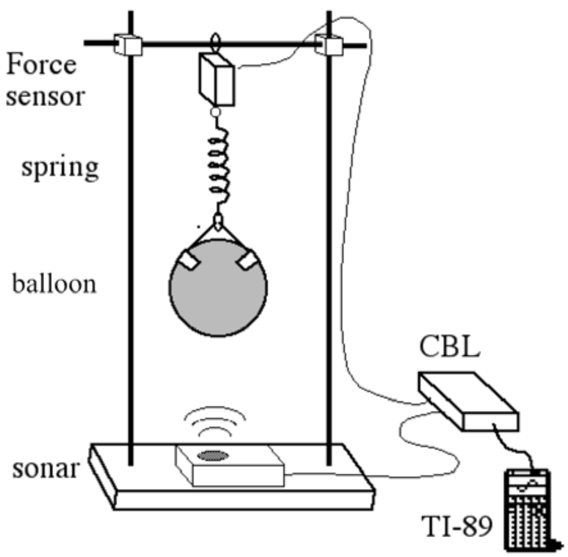

FIGURE 3. Setup to investigate balloon oscillations.

The mass-spring oscillator is an apparatus well known to physics teachers, but here, unlike the more usual teaching situations, it is essentially used to measure the inertial mass of a body, in a way similar to that used for the *inertial* balance in the PSSC textbook [2].

The mass-spring system oscillates with an angular frequency ω given by $\omega^2 = k/m_i$, where k is the spring elastic constant and m_i the inertial mass. In the absence of a force sensor the system inertial mass might be obtained by measuring separately the frequency ω from the slope of the acceleration versus distance plot, and the elastic constant k from the slope of the force versus distance plot. The inertial mass is then calculated as: $m_i = k/\omega^2$.

Using the force sensor the inertial mass is directly obtained from the slope of the force versus acceleration plot, exploiting the Newton equation $F = m_i\, a$.

An example of the plots obtained with our balloon is reported in Fig. 4.

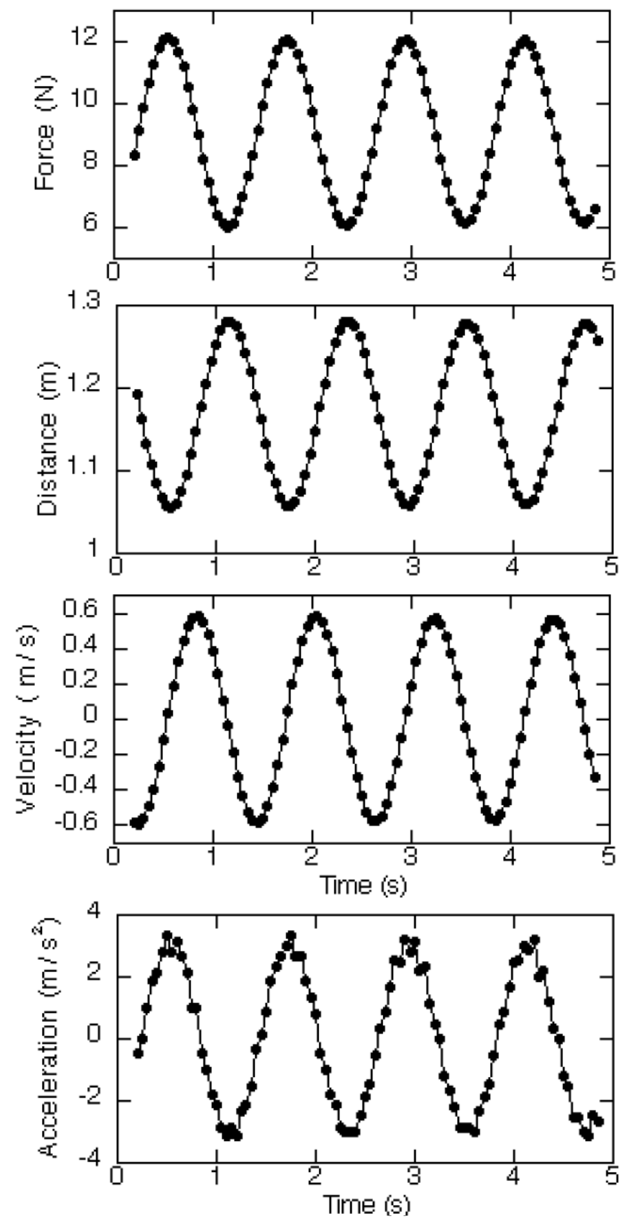

FIGURE 4. Force, distance, velocity and acceleration versus time.

From the slope of the force versus acceleration plot, shown in Fig. 5, we obtain m_i = 0.974kg a value 8% greater than the value F/g obtained from the balloon “weight”.

As in the first experiment, the hydrostatic effects by alone (the correction ρV = 0.05kg) cannot account for such discrepancy. To achieve a value closer to the measured value of the total inertial mass m_i we need to add also the contribution $\rho V/2$ of the hydrodynamic mass.

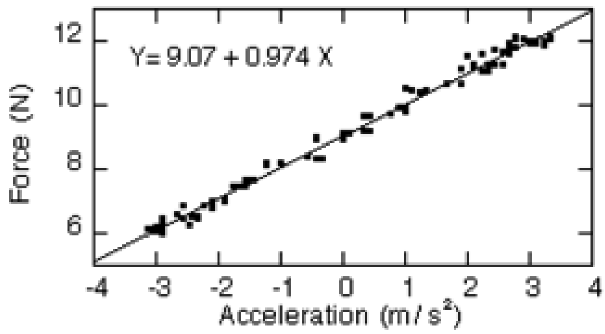

FIGURE 5. Force versus acceleration plot.

One might argue that there is still a missing term in our model for the mass-spring oscillator: the effective inertia of the oscillating spring. If the spring has a mass m_s it gives [3] a contribution $m_s/3$ to m_i, which may be made negligible by using a light spring (this is the reason why we use a rubber band instead of a metallic coil spring).

Resuming: The force sensor (dynamometer, or "weighing balance") measures the value $F/g=m_g-\rho V$, *underestimating* the gravitational mass, while the "inertial balance" method measures the value $m_i+m_h = m_i+(1/2)\rho V$, *overestimating* the balloon inertial mass.

VI. CONCLUSIONS

We have shown how the use of RTL may help to investigate the motion in air taking into account effects that are important in the study of *real* motion but that are often neglected in the traditional curricula (the hydrostatic force due to air, the hydrodynamic mass, the conceptual distinction between gravitational and inertial masses, the effects of dissipative and conservative forces on the acceleration), and we proved how important it is to make a correct choice of the model used to analyze the experimental data.

We started our investigation by posing the question whether a balance can give a reliable measurement of the mass of a body, but an alternative path might be to compare the predicted acceleration g of a free falling body with the measured value a that turns out to be sensibly smaller.

In both cases the experimental results may be used to discuss the different roles played by friction, by hydrostatic and gravity forces and by hydrodynamic mass, in affecting the acceleration in different situations.

The friction force increases or decreases the acceleration, depending on the sign of the velocity of the moving object. It can be easily calculated from the measured accelerations during a bidirectional motion when other acting forces are kept constant.

The hydrostatic force always subtracts to the gravity force (being itself produced by the gravitational field). It may be thought as an "effective change of gravity", and it must be considered when calculating the mass of an object from its weight.

The hydrodynamic mass accounts for the momentum, which is *temporarily transferred* from the moving body to the fluid, and subsequently *given back* to the body. This effect depends essentially on the fluid *density*, not on its *viscosity*. It is a dynamic effect, absent for a still body.

REFERENCES

[1] For example in Lamb H., *Hydrodynamics*, (Dover, N. Y., 1932), or in Landau, L. D., Lifshitz, E. M., *Fluid mechanics*, (Pergamon, N. Y., 1959).

[2] PSSC, *Teacher's Resource Book and Guide*, part III, chapt. 3 (Heath & Co, Boston, 1965).

[3] See for example Ruby, L., *Equivalent mass of a coil spring*, The Physics Teacher **38**, 140-141 (2000).

APPENDIX an evaluation of dissipative forces with different bouncing balls

An evaluation of the order of magnitude of the air friction on a falling sphere may be easily derived in two simple cases: either assuming *linear* or *turbulent* flow.

In the first case we are dealing with a force proportional to the air *viscosity*, in the second one with a force mainly proportional to the air *density*.

The transition between the two regimes is approximately marked by the value 1000 of the Reynolds number $Re=(\rho/\eta)rv$, where the air viscosity η at room temperature is about 1.8 10^{-5} in MKS units.

In our experiment the maximum velocity of the balloon falling from the height h=0.5m is $v=\sqrt{2gh}$ =3m/s, and a factor 10 less during the oscillations (Fig. 4).

At the highest speed $Re=10^4$, and in the turbulent regime the force may be written:

$$F_S= C\pi R^2 \rho v^2/2, \quad (5)$$

(C is an adimensional form factor, close to 0.4).

In this approximation, at the maximum speed, the drag force is about 0.3N, comparable to the hydrostatic thrust, but only 3% of the gravitational force.

Linear regressions of the velocity-time plots, close to the recoils, yield slightly different slopes for upward (a=9.32m/s^2) and downward (a=8.69m/s^2) motion, as shown in Fig. 6:

The relative change in the acceleration ($\Delta a/a = \pm 3\%$) with respect to the average value gives an experimental evaluation of the dissipating force, which is in good agreement ($\Delta F/F= F_S/mg =\Delta a/a$) with the model described in Eq. (5).

In the case of a ping-pong ball bounces the analysis should be different. Assuming the same value for h (with R=1.8cm, m= 0.002kg), at the maximum speed the motion is still in the linear regime, where the Stokes Law predicts for the drag force:

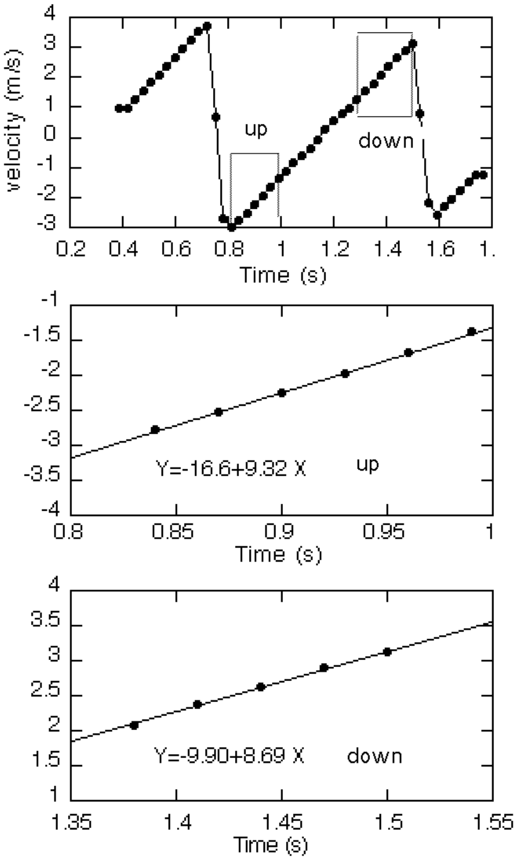

FIGURE 6. Linear regression in upward and downward motion.

$$F_S = 6\pi\, R\, \eta\, v, \tag{6}$$

The viscous force at maximum speed is about 2 10^{-5}N (0.1% of the gravitational force) while the hydrostatic force is about 3 10^{-4}N, so that the predicted acceleration (practically equal in upward and downward motion) is about 9.7m/s^2. The friction force cannot be easily detected, while the hydrostatic force still produces a sensible effect (about 1.5%) and the hydrodynamic mass is barely detectable (0.7% of the ball mass).

Using a rubber ball of similar size (m=0.030kg) all the mentioned effects become negligible (less than 1%) and the ball behaves as if it were falling in vacuum.