Investigation of Atwood's machines according to inertia and non inertia frames

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Abstract

In this paper, an extended review presented of Atwood's machines in inertia and non inertia frames. At each examples observer situation defined and analysed. The results are compared with the famous method Lagrange in analytical mechanics. It seemed example survey and comparing them with each other can make a simpler and also profound view of these systems.

Keywords: Atwood's machines – inertia observer- non inertia observable.

Resumen

En este trabajo, se presenta una revisión ampliada de las máquinas de Atwood en inercia y los marcos de no inercia. En cada situación son observados ejemplos definidos y analizados. Los resultados son comparados con el famoso método de Lagrange en la mecánica analítica. Esto es parecido a un estudio de ejemplo y comparándolos con los otros se puede hacer una vista simple y profunda de estos sistemas.

Palabras clave: Maquinas de Atwood– observador de inercia - no inercia observable.

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I. INTRODUCTION

The simple Atwood's machine is a dynamical system which consist on pulley, string and to definite masses body which are hanged down the strings. Basically the mass of pulley and string don't considering. Although there are various mechanics books in which this system mentioned them for example see to [1] or [2] (of course in these books this system analysed to different methods). Also the network of Atwood's machines offered in [3]. However mostly their analysis seems to be complicated comparing to another dynamical systems. More ever by to adding pulleys and bodies make hard to analysis. We believe, presenting a completed precise review of Atwood's machines along with determination of a observer who analysis dynamical of every body of such system separately, is highly effective.

Mentioning prominence of role of observer at analysis of these system it's important to mention the concept of virtual force at first. In order to achieve this goal consider these example. Suppose that a body with mass of m at rest on the elevator floor. Observer O_1, is out side the elevator. We consider this observer as inertia observer. According to the Newton's second law O_1 observer report the situation of body on mention:

$$N = mg. \tag{1}$$

Observer, O_2, also, report like mentioned equation. We consider this observer as non-inertia observer.

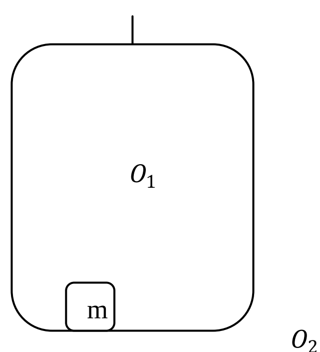

FIGURE 1. Observer O_1 is inside elevator and observer O_2 is outside elevator. The body is at rest on the floor of the elevator.

Now suppose elevator is going up with constant acceleration of a. This time observer O_2, report the result as mention:

$$N - mg = ma. \qquad (2)$$

Observer, O_1, sees the body at rest still, but he knows that the elevator is moving upward and considering that observer's knowledge is effective on measurement result so inevitably we relate ma, virtual force downward to the body:

$$N - mg - ma = 0. \qquad (3)$$

Now, we hang down the elevator roof, a simple Atwood's machine, which consist of pulley and strings and two bodies with masses of m_1 and m_2. We ignore the mass of pulley and strings. Again we suppose the elevator is going up by acceleration of a.

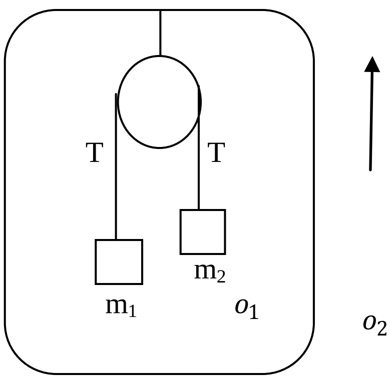

FIGURE 2. The elevator consist of the simple Atwood's machine is going up with acceleration of *a*.

The Newton's equations in o_1's opinion is as mention:

$$T - m_2 g - m_2 a' - m_2 a = 0, \qquad (4)$$

$$T - m_1 g + m_1 a' - m_1 a = 0, \qquad (5)$$

the a' is the acceleration that observer o_1, relate to each one of m_1 and m_2 equally and opposite.

Observer, o_2, report the above equations differently:

$$T - m_2 g = m_2(a_2' + a), \qquad (6)$$

$$T - m_1 g = m_1(a - a_1'). \qquad (7)$$

In which a_1' and a_2' are acceleration that relate to m_1 and m_2 respectively. For determining of acceleration of each one bodies we assume that the length of string constant. This measurement is as mention, in observer, o_1s' view. At first he asumes that the string length L, therfore: (he doesn't consider the constant amount of string roles over pulley.)

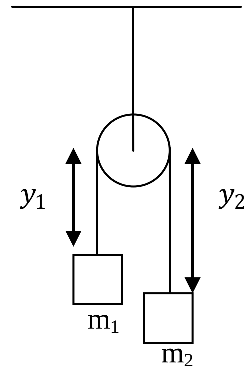

FIGURE 3. In this figure the string length is *L*, so that it is constant.

$$L = y_1 + y_2. \qquad (8)$$

By two time derivation from above equation we have:

$$0 = a_1' + a_2', \quad \text{or}$$
$$a_1' = -a_2'.$$

$$\left[\frac{(m_1 - m_2)}{m_1 + m_2}\right](g + a). \qquad (9)$$

But observer, o_2's result is different:

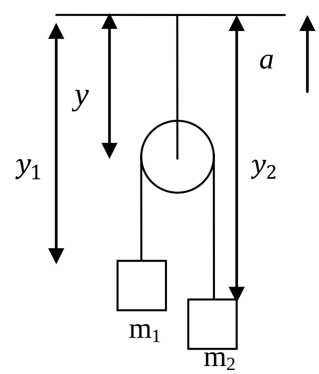

FIGURE 4. This figure shows that the new arrangement of distances.

$$(y_1 - y) + (y_2 - y) = L. \qquad (10)$$

Again by two time derivation from above equation:

$$a_1' + a_2' = 2a. \qquad (11)$$

So that a_1' and a_2' are acceleration of m_1 and m_2 respectively.

From Eqs. (6), (7), (11), we can obtain:

$$a_1' = \frac{[m_2(2a+g)-m_1 g]}{m_1+m_2}, \quad (12)$$

$$a_2' = \frac{[m_1(2a+g)-m_2 g]}{m_1+m_2}. \quad (13)$$

Each of above equation assuming $a = 0$ lead to familiar of

$$\left[\frac{(m_1-m_2)}{m_1+m_2}\right](g). \quad (14)$$

Next example is selected from analytical mechanics [2]. In this book Atwood's machine analysis according to the Lagrange equation. Prominent issue meanwhile, is result out put of above method are compatible with out put result of each frames (inertia or non- inertia).

We put the observer related to non- inertia frame, o_1, on the 2nd pulley and observer, o_2, is on the ground as mentioned. Obsever, o_1's report is as mention.

Each strings length rolled over 1th and 2nd pulley is L and L' respectively. Using [3] equivalent mass $m_{2,}$ m_3 *i.e.* $m_{2,3}$ is:

$$m_{2,3} = \frac{4}{\frac{1}{m_2}+\frac{1}{m_3}}. \quad (15)$$

Assuming $m_{1>}$ $m_{2,3}$ each one's acceleration is equal and opposite as mention below:

$$\frac{\left[m_1-\frac{4m_2m_3}{m_2+m_3}\right]}{\left[m_1+\frac{4m_2m_3}{m_2+m_3}\right]}(g). \quad (16)$$

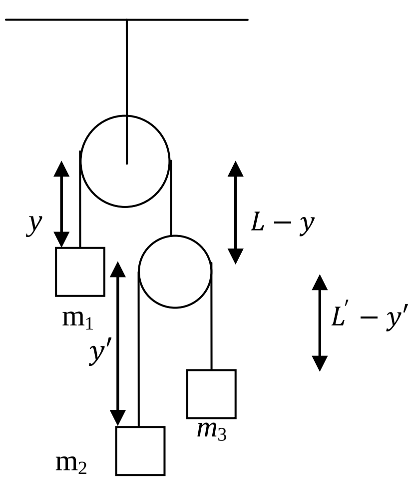

FIGURE 5. The figure aptitude from analytical mechanics as [2].

And each one of m_2 and m_3 masses acceleration are equally and opposite using Eq. (8) as mention below:

$$\frac{\left[m_1-\frac{4m_2m_3}{m_2+m_3}\right]}{\left[m_1+\frac{4m_2m_3}{m_2+m_3}\right]}(g+a).$$

But the observer o_2's report is completely different, see to Fig. (6):

$$m_1 g - T = m_1 a_1, \quad (17)$$

$$m_2 g - \frac{T}{2} = m_2 a_2, \quad (18)$$

$$m_3 g - \frac{T}{2} = m_3 a_3. \quad (19)$$

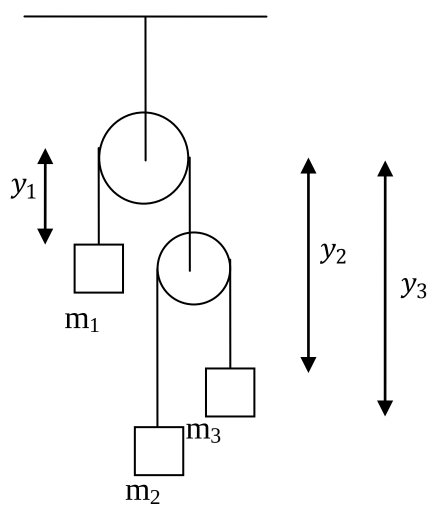

FIGURE 6. The analysis different from non-inertia view.

We consider distance between two pulleys equal h. So we have:

$$y_1 + h = constant, \qquad \text{and}$$

$$(y_2 - h) + (y_3 - h) = constant.$$

Then:

$$y_1 + \left[\frac{y_2+y_3}{2}\right] = constant.$$

Or

$$2a_1 + a_2 + a_3 = 0. \quad (20)$$

Using Eqs. of (17), (18), (19) and (20) we can obtain value of a_1, a_2, a_3.

It's clear that equation obtained by Lagrange method is compatible with the report of the observer, o_1. For example we consider m_1=4kg, m_2=3kg and m_3 =1kg then $m_{2,3}$ = 3kg and acceleration of m_1 and $m_{2,3}$ are equal to g/7 and $-g$/7 respectively. So acceleration of m_2 and m_3 are equal to 4g/7 and -4g/7 respectively (g is the gravity acceleration). By subtitute the value m_1, m_2, m_3 in Lagrange equations, it is clearly this results compatible with Lagrange method. But these results based on the (17), (18), (19) and (20) is completely different.

II. CONCLUSIONS

We show that analysis of Atwood's machines exactly so that later maybe make simpler solution of them. The result of dynamically analysis of Atwood's machines depend on the position of inertia or non-inertia observer. The result of Lagrange method is compatible with the result of non-inertia.

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