Perpendicularity between speed of light and time in the Kaluza-Klein theory

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Abstract
Scenario of the hypothetical contact point between the fourth and fifth dimensions is analyzed from the Kaluza-Klein theory, where we found that in such a point, speed of light must be perpendicular to the time.

Keywords: Kaluza-Klein theory, Fifth-dimension.

Resumen
El escenario del hipotético punto de contacto entre la cuarta y la quinta dimensiones se analiza desde la teoría de Kaluza-Klein, donde encontramos que en tal punto, la velocidad de la luz debería ser perpendicular al tiempo.

Palabras clave: Teoría de Kaluza-Klein, Quinta dimensión.

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I. INTRODUCTION

As known, mathematician Th. Kaluza in 1921 [1] proposed a model that seeks to unify the two fundamental forces, gravitation described by the Einstein's general relativity [2] and electromagnetism described by Maxwell [3], deriving an expression for the geodesics in the fifth dimension (5D) space-time. The resulting equations can be separated into further sets of equations, where one of them is equivalent to Einstein field equations. In 1926, Oskar Klein proposed that the fourth spatial dimension is curled up in a circle of very small radius [4, 5], so that a hypothetical charged particle moving a short distance along that axis would return to where it began. The distance a charged particle can travel before reaching its initial position is said to be the size of the dimension. That spatial extra dimension is considered in the scale of very small particles, giving rise to the so-called Kaluza–Klein theory.

In this paper, scenario of the hypothetical contact point between the fourth and fifth dimensions is analyzed from the Kaluza-Klein theory, where we found that in such a point, speed of the light must be perpendicular to the time. Perpendicularity condition is also applied to derive an analogous expression to the Kaluza expression for the geodesics in the fifth dimension space-time, but in terms of speed of light and time.

II. PERPENDICULARITY BETWEEN SPEED OF LIGHT AND TIME IN THE CONTACT POINT OF 4D AND 5D

As background, space-time model proposed by Minkowski [6] in 4D reference can be written as the square of segment of line given by

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + c^2t^2, \tag{1}$$

where $dx_4{}^2 = c^2t^2$, t is the time and c is the speed of light that is included to make the units consistent throughout. This expression can be extended to a 5D frame of reference, giving

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + c^2t^2 + dx_5^2. \tag{2}$$

In the Th. Kaluza work [1], it is considered that multiplying the geometry of the gravity tensor of Einstein ($g_{\mu\nu}$) given in 4D, by the (antisimetric) electromagnetic tensor of Maxwell ($A_{\mu\nu}$), where μ, ν run 1, 2, 3 4, it is defined the product ($g_{\mu\nu}$) ($A_{\mu\nu}$), which is a matrix in 5D, given by

$$(\hat{g}) = (g_{\mu\nu})(A_{\mu\nu}) = ct, \tag{3}$$

where all hatted quantities are five-dimensional, and ct = 1 in $\hat{g}_{55}$. From the matrix in 5D, in the Kroneker delta value when $\hat{g}_{55}=1$ indicates that could exist an event or particle which occupies a position in the contact point P between 4D and 5D, which is when $dx_4 = dx_5$. Then, we can consider that when $dx_5 = 1$, yields

$$dx_4 - dx_5 = 0, \tag{4}$$

when 4D and 5D are excluding dimensions. Then, from the 4D reference, hence

$$dx_4^2 = c^2 t^2 = 1, \tag{5}$$

thus, it can be simplified as

$$c = \pm \frac{1}{t}. \tag{6}$$

Parameter x_4 determines the motion of a hypothetical charged particle in 4D (like a photon via ct). Sign of expression (6) indicates the direction of the vector velocity, where c and t are geometrically perpendiculars, since their respective slopes are perpendiculars, having from the matrix that g^{ij} is inverse with respect to g_{ij}. Thus, from expressions (6), it is possible to consider that when $\hat{g}_{55}=1$, being the contact point between 4D and 5D, charged particle path (given by the speed of the light) must be geometrically perpendicular to the time.

III. KALUZA-KLEIN THEORY FOR THE CONTACT POINT BETWEEN 4D AND 5D

In the Th. Kaluza paper, expression for the geodesic in 5D space-time is defined as

$$ds^2 = (dx_5 + \beta\varphi dx^i)^2 + g_{ik} dx^i dx^k, \tag{7}$$

where i, k run 1, 2, 3, 4, and

$$\beta = \sqrt{2k}, \tag{8}$$

where k the so-called Einstein gravitational constant given by

$$k = \frac{8\pi G}{c^2}, \tag{9}$$

where G is the Newtonian constant of gravity. Developing the square power of first term from expression (7), hence

$$ds^2 = dx_5^2 + 2dx_5 \beta\varphi_i dx^i + (\beta\varphi_i dx^i)^2 + g_{ik} dx^i dx^k, \tag{10}$$

where we can separate in groups according to the corresponding dimensions, hence

$$\begin{aligned} &5\text{D} \Rightarrow dx_5^2, \\ &4\text{D},5\text{D} \Rightarrow 2dx_5 \beta\phi_i dx^i, \\ &4\text{D} \Rightarrow (\beta\phi_i dx^i)^2 + g_{ik} dx^i dx^k. \end{aligned} \tag{11}$$

In this way, we can find out that first term of expression (11) is defined in 5D, second term is a point shared in 4D and 5D, being a contact point between both, x_4 and x_5 references. Third term is defined in 4D.

We can try to derive an equivalent expression for (10), but in terms of speed of light and time by developing equivalence for the second term of (11) through to apply some equivalences and the perpendicularity condition given by (6). Considering O. Klein expression for the momentum that some hypothetical charged particle q should go in the fifth dimension space-time, given by

$$p_5 = m\dot{x}_5 = \frac{q}{c\beta} = \frac{q}{c\sqrt{\frac{16\pi G}{c^2}}}. \tag{12}$$

Square power of expression (12) has some equivalent forms, such as

$$m^2 \dot{x}_5^2 = m^2 \dot{x}_5 \dot{x}_5 = m^2 \dot{x}_5 v_5 = \frac{q^2}{\left(\frac{16\pi G}{c^2}\right)c^2}, \tag{13}$$

where v_5 is the velocity of a hypothetical charged particle in the contact point between 4D and 5D.

Considering the equivalence for the electric field, given by $\mathbf{E} = c\times\mathbf{B}$, where $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field, and replacing in expression (13), yields

$$m^2 \dot{x}_5 v_5 = \frac{q^2 \mathbf{B}^2}{16\pi G \mathbf{B}^2}, \tag{14}$$

Having a hypothetical charged particle in the contact point between 4D and 5D that behaves like a fluid in the medium, then we could consider the velocity in which a spherical droplet will sink in a medium of density ρ [7]. Stokes velocity for a bubble of gas, it is given by

$$v = \tfrac{1}{3} R^2 \rho \frac{g}{\eta}, \tag{15}$$

where R is the radius of sphere, ρ is the density of the medium, g is the gravitational acceleration and η is the dynamics viscosity. Considering that rate between gravitational acceleration and density of the medium is the unit, then we can reduced expression (15) as $v = {}^1/_3 R^2\rho$. Thus, replacing it in expression (14), yields

$$m^2\dot{x}_5(\tfrac{1}{3}R^2\rho)=\frac{q^2\mathbf{B}^2}{16\pi G\mathbf{B}^2}. \quad (16)$$

Dividing expression (16) by R and reordering, hence

$$\frac{2m^2\dot{x}_5}{R}=\frac{q^2\mathbf{B}^2}{2(\frac{4}{3}\pi R^3\rho)G\mathbf{B}^2}=\frac{q^2\mathbf{B}^2}{2GM\mathbf{B}^2}, \quad (17)$$

where M is the mass of a massive body. Reordering terms according to the perpendicularity condition given by (5), yields

$$2\dot{x}_5\frac{2GM}{R}\frac{m^2\mathbf{B}^2}{q^2\mathbf{B}^2}=1, \quad (18)$$

where we can find a correspondence with two known equivalences; one of them is the Schwarzschild radius [8] defined as

$$c^2=\frac{2GM}{R}, \quad (19)$$

On the other hand, typical motion of charged particles is in circular or helicoidal path around the lines of force from the magnetic field. This motion is named Larmor rotation (spin, cyclotron motion). Frequency of motion is named cyclotron frequency given by $\omega = qB/m$ (which is inverse to the time), and the Larmor radius (or gyroradius) of the orbit [9] is given by

$$r=\frac{mv}{q\mathbf{B}}=\frac{m(r/t)}{q\mathbf{B}}\therefore t=\frac{m}{q\mathbf{B}}, \quad (20)$$

where r is the Larmor radius and v the velocity of the hypothetical charged particle. Thus, replacing (19) and (20) in expression (18), yields

$$2\dot{x}_5\mathbf{B}^2c^2t^2=1, \quad (21)$$

which correspond to the second term of expression (10). It is an approximated equivalent expression for that condition of a hypothetical charged particle located in the contact point between 4D and 5D, but related by the speed of light and time. This equivalence can be written as

$$ds^2=dx_5^2+2\dot{x}_5c^2t^2\mathbf{B}^2+\left(c^2t^2\mathbf{B}^2\right)^2+c^2t^2$$
$$+dx_3^2+dx_2^2+dx_1^2, \quad (22)$$

which can be reduced to the form of expression (7), hence

$$ds^2=\left(dx_5+c^2t^2\mathbf{B}^2\right)^2+c^2t^2+dx_3^2+dx_2^2+dx_1^2, \quad (23)$$

that is an analogous expression to the Kaluza expression (7) for the geodesics in the fifth dimension space-time, but in terms of speed of light and time.

IV. CONCLUSIONS

Kaluza-Klein theory can be analyzed to derive geometry in a high dimensional references as the geometrical reference between speed of light and time in the hypothetical contact point between 4D and 5D, resulting that a hypothetical charged particle should be in perpendicular position with respect the time. According to the perpendicularity condition an analogous expression of the Kaluza expression for the geodesics in the fifth dimension space-time can be derived by applying some equivalent expressions and the condition of perpendicularity.

Regarding to the education, classical Kaluza-Klein theory is revisited describing the main concepts of this theory defined in the five-dimension space-time, where it is showed the possibility to apply some of the known equivalences to consider another possible properties from the classical theories.

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