Thermodynamics of solar energy channels over the globe

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Abstract
The thermodynamics of the intercepted solar energy distributed over various channels reported by M. King Hubbert is discussed pedagogically; the role of green house gases in maintaining the temperature of the earth as well as the atmosphere making it suitable for life, the annual average rainfall of about one meter over the globe for supply of fresh water, the generation of wind power due to unequal heating of the earth via Curzon Ahlborn engine and finally the production of biomass through photosynthetic channel.

Keywords: Solar energy, earth, channels, thermodynamic calculations, Curzon-Ahlborn engine.

Resumen
La termodinámica de la energía solar interceptada distribuida sobre diversos canales reportados por M. King Hubbert, se discutió pedagógicamente; el papel de los gases con efecto invernadero en el mantenimiento de la temperatura de la Tierra, así como la atmósfera haciendo esto adecuado para la vida, la precipitación media anual de alrededor de un metro sobre el planeta para el suministro de agua potable, la generación de energía eólica debido al calentamiento de desigual la Tierra a través del motor de Curzon Ahlborn y, finalmente la producción de biomasa a través del canal fotosintético.

Palabras clave: Energía solar, Tierra, canales, cálculos termodinámicos, motor de Curzon-Ahlborn.

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I. INTRODUCTION

Every student of science realizes that the main natural source of energy on the earth[1], responsible for sustaining human as well biological life, is the sun. It produces direct heating of our planet to maintain its average temperature with the help of the atmosphere in such a way that the life can survive. It also recycles the sea/river/lake water by initial evaporation and subsequent average annual rainfall of around one meter over the globe. The unequal absorption of the heat at the equator, north and south poles results in the wind power over the globe. The visible range of the spectrum participates in the photosynthesis and provides biomass for the human beings and animals. A student of physics, however, by his very nature of being precise, would like to know the answer of the following questions.

1. What are the definite proportions through which the solar energy gets distributed among the above-mentioned channels?
2. If such proportions were known a priori what quantitative inferences can one draw regarding interesting physical/biological phenomena such as the planetary temperature, mass of the water evaporated and the subsequent rainfall, estimate of the wind energy generated and the amount of the consumable biological materials produced?
3. Can these numbers be correlated thermodynamically?

Answer to these questions would be of great benefit to physics students because they will have exposure to numbers related to solar radiation and also would realize how this knowledge can be fruitfully utilized to deal with some issues occurring in allied disciplines of geophysics, meteorology and agriculture. The aim of the present paper is to accomplish this task below one by one in three sections.

II. DISTRIBUTION OF THE INTERCEPTED SOLAR ENERGY

The half of the globe, on the average, intercepts[1] $(Q) 1.74 \times 10^{17} J$ of solar radiation per second. This energy is electromagnetic in nature which is characterized by wavelength λ, frequency ν and velocity c satisfying the relation

$$c = \nu\lambda, 0 \leq \lambda \leq \infty, \infty \geq \nu \geq 0, \tag{1}$$

The electromagnetic spectrum extends from below the radio frequencies at the long-wavelength end through gamma radiation at the short-wavelength end covering wavelengths from thousands of kilometers down to a fraction of the size of an atom. The Table Ia shows that the intercepted solar radiation lies between UV and infrared and Table Ib gives the distribution of this intercepted energy among various channels[1].

TABLE Ia. Distribution of the intercepted solar energy over the electromagnetic spectrum.

Name	Wavelengths	Contribution to the intercepted solar energy	Corresponding Percentage
Gamma-rays	$0-\ 10^{-14}m$	~ 0	~0
X-rays	$10^{-14}-10^{-10}m$	~ 0	~0
Ultra-violet	$10^{-10}-400\cdot 10^{-9}m$	$2.1x10^{16}$W	12.1%
Visible	$400\cdot 10^{-9}-700\cdot 10^{-9}m$	$6.3x10^{16}$W	36.7%
Infrared	$700\cdot 10^{-9}-10^{-3}m$	$8.8x10^{16}$W	51.2%
Microwave	$10^{-3}-0.1m$	$1.0x10^{9}$W	Negligible
Amateur	$0.1-10^{2}m$	$7.2x10^{9}$W	Negligible
Radio waves	$10^{2}-10^{4}m$	$3.8x10^{9}$W	Negligible
Long waves	$10^{4}-\infty m$	$4.7x10^{7}$W	Negligible

TABLE Ib. Distribution of the intercepted solar energy into various channels over the globe[1].

Channel	Electromagnetic Energy (W)
Solar radiation on the earth	$1.74x10^{17}$
Direct reflection	$5.2x10^{16}$
Direct conversion to heat	$8.2x10^{16}$
Evaporation & precipitation	$4.0x10^{16}$
Winds, waves, convection & currents	$3.7x10^{14}$
Photosynthesis	$4.0x10^{13}$

Before proceeding further the experimental facts about the earth and its atmosphere along with the assumptions made are worth mentioning. The earth is assumed to be an almost spherical body enveloped with the atmosphere rotating on its axis as well as around the sun. Various layers of the atmosphere are termed as troposphere, stratosphere, mesosphere, thermosphere and exosphere. About 80% mass of the atmosphere is confined in the troposphere and it extends up to an altitude of 10-15km above the surface of the earth and this plays an important role in deciding the climate over the globe. This height being too small in comparison to the radius of the earth it will be assumed that the value of acceleration due to gravity g does not change within the troposphere. The main constituents of the troposphere are nitrogen, oxygen, argon, water vapor and carbon dioxide and its density decreases with altitude almost exponentially. The presence of the atmosphere and oceans has made it suitable for the life.

II. THERMODYNAMIC CALCULATIONS

A. Temperature of the Earth & its Atmosphere

The atmosphere of the globe being transparent to the incident solar radiation, the intercepted solar energy $1.74\times 10^{17}W$ would yield the average temperature T of the whole earth through the expression

$$4\pi a^2\sigma T^4 = 1.74x10^{17}(1-A). \quad (2)$$

Where $\sigma = 5.67x10^{-8}W\ m^{-2}K^{-4}$ is the Stefan-Boltzmann constant, a is the radius of the earth and A is its albedo[2], *i.e.* the ratio of reflected to incident solar energy for the whole earth. The right hand side corresponds to the absorbed solar radiation while the left hand side represents the radiation emitted by the earth assuming it behaves as a blackbody at temperature T. The basic question posed here is that when the half of the globe intercepts the above mentioned solar energy then why the area of the whole globe is considered to find the average temperature. This is because the circulation of the atmosphere and the oceans redistribute the captured solar energy throughout the globe. Using the data[2,3].

$$a = 6.37x10^6m, A = 0.3, \quad (3)$$

gives the value of the average temperature of the earth as

$$T = 255K, \quad (4)$$

It is well known that the average temperature of the earth[2] is around 288K and hence the above calculated temperature gets modified due to the presence of the oceans as well as the atmosphere. The atmosphere not only redistributes the energy throughout the globe but it also works as a blanket[4] to keep it warm. The presence of the greenhouse gases carbon dioxide and water vapor in the troposphere capture the infrared radiation emitted by the earth which in turn emits radiation both upward as well as downward. Suppose the top most portion of the troposphere is at the temperature T_a. In the steady state condition the following expression will be satisfied

$$4\pi\sigma a^2 T_s^4 = 1.74 \cdot 10^{17}(1-A) + 4\pi\sigma a^2 T_a^4. \tag{5}$$

Where T_s is the steady state temperature of the earth and since the thickness of the troposphere is too small in comparison to the radius of the earth its thickness has been ignored. One can draw two very important conclusions from the above expression. Firstly the steady state temperature of the earth will always be greater than the temperature of troposphere at any altitude. Secondly the temperature of the troposphere will fall with the altitude. These two observations are very important in view of the Sandstrom's theorem[5] for the generation of the wind power and lifting of water vapor which finally precipitates. The knowledge of T_s at a point on the earth can be used to find the temperature T_a at the top of the troposphere corresponding to that location. For example the average temperature of the earth[2] is

$$T_s = 288K, \tag{6}$$

which corresponds to the average temperature of the air at the top of the troposphere as

$$T_a = 227K. \tag{7}$$

This gives us an approximate relation

$$T_s - T_a \sim 60K. \tag{8}$$

This is being satisfied at most of the locations on the globe such as at North Pole, South Pole or Mediterranean.

B. Evaporation & Precipitation

This channel is very important for the distribution of fresh water for life across earth's landscape by evaporating the saline water of the oceans. This is achieved by providing the latent heat to the water available at 288K in the oceans, lakes, rivers, etc. The value of the latent heat[6] at this temperature is $L = 588.9 kcal\ kg^{-1} = 2.46 \cdot 10^6 J\ kg^{-1}$. We will calculate the total amount of water being evaporated annually. This will become clear in the sequel why this is being done. The amount of water M_{water} evaporated in one year will be given by

$$\frac{Solar\ power\ going\ into\ evaporation\ \ Q_{EV} \cdot One\ year\ time\ \tau}{Latent\ \ heat\ L}\ kg\ y^{-1},$$

$$\tau = 3.15 \cdot 10^7 s, L = 2.46 \cdot 10^6 J\ kg^{-1}. \tag{9}$$

The amount of solar energy going into the channel evaporation has been quoted by M. King Hubbert as $Q_{EV} = 4.0 \cdot 10^{16} W$. Substituting these values in the above expression gives the evaporated amount of water annually as

$$M_{water} = 5.1 \cdot 10^{17} kg\ y^{-1}. \tag{10}$$

Since the density of water

$$D = 10^3 kg\ m^{-3}. \tag{11}$$

The volume of water being evaporated will be[7]

$$V_{water} = \frac{M_{water}}{D} = 5.1 \cdot 10^{14} m^3 y^{-1}. \tag{12}$$

The water evaporated from the globe finally precipitates and suppose if one collects this water for one year the width of this water $\delta\, m$ on the earth is well known annual rainfall. This can be determined by finding out the volume of this water shell V_{shell} covering the earth.

$$V_{shell} = Volume\ of\ the\ (earth + shell) - volume\ of\ the\ earth$$

$$= \frac{4\pi}{3}(a+\delta)^3 - \frac{4\pi}{3}a^3,$$

$$= \frac{4\pi}{3}a^3\left(1+\frac{3\delta}{a}\right) - \frac{4\pi}{3}a^3,$$

$$= 4\pi a^2 \cdot \delta\ m^3. \tag{13}$$

Equating the volume of evaporated water V_{water} and the volume of water shell gives

$$\delta = \frac{5.1 \cdot 10^{14}}{4\pi a^2} m. \tag{14}$$

Putting the value of the radius of the earth $a = 6.37 \cdot 10^6 m$ in the above expression gives the value of annual rainfall on the earth

$$\delta \cong 1m. \tag{15}$$

This value is consistent with the experimental records[8] over the years. This channel is called evaporation and precipitation because during evaporation the above mentioned solar energy is consumed and also the same amount of energy is released during condensation, may be at some other location in the atmosphere. In the first case it cools the earth while in the second process it provides heat to the earth. Now we take the case of solar energy going into wind power.

C. Wind Power

The generation of the wind power can be achieved through the temperature gradient present in the atmosphere of the earth. There are basically three temperature gradients available as follows.

1. The earth being a sphere the incident solar radiation strikes the portions around the equator at right angles while the parts away from it either towards North or South Pole the angle of incidence goes on decreasing.

The rays become almost tangential at extreme points which are north and south poles. Thus the portions around the equator will receive more heat and as one moves away the heating of the earth declines and at extreme north and south poles the amount of solar energy captured becomes practically zero. This phenomenon gives rise a temperature gradient between the equator and the North Pole as well as between the equator and the South Pole. The average annual temperature around the equator[9] is 30 degree Celsius whereas at the North Pole the average winter temperature[10] is around -34 degree Celsius and the average summer temperature is around 0 degree Celsius. The corresponding average temperature during winter at the South Pole is -58 degree Celsius while during summer[11] it is -25.9 degree Celsius.

2. The second temperature gradient develops because of day and night[12]. During the day the earth receives solar energy while it cools off during the night. The available data show that it is around 30 degree Celsius during the day and 23 degree Celsius in the night.

3. The third temperature gradient present in the troposphere is with respect to the altitude as discussed above.

All these three temperature gradients, in turn, cause pressure gradient which moves the air from high pressure zone to a lesser one. While the first two pressure gradients generate the wind power the third one lifts the evaporated water as well as hot air to higher altitudes for precipitation and recycling of the wind through Hadley cell. The power of this channel comes out to be $3.7 \cdot 10^{14} W$ and this is being answered in the Section IV where a thermodynamic attempt is made to correlate the numbers reported by M. King Hubbert[1].

D. Photosynthesis

This channel provides biomass in the form of food materials to the population residing on the globe. According to M. King Hubbert[1] the photosynthesis channel consumes $Q_{PS} = 4 \cdot 10^{13}$W of solar energy. On the annual basis this corresponds to the harvested solar energy by photosynthesis as

$$Q_{PS}^{A} = Q_{PS} \cdot \tau \, J y^{-1} = 4 \cdot 10^{13} \cdot 3.15 \cdot 10^{7} = 1.2 \cdot 10^{21} J y^{-1}. \quad (16)$$

It is known that for every one joule of energy 6.45×10^{-8}Kg of dry biomass[2] is produced through photosynthesis. Thus the total projected dry biomass being produced annually over the globe would be

$$M_{BIO}^{A} = 6.45 \cdot 10^{-8} \cdot 1.2 \cdot 10^{21} = 8 \cdot 10^{13} kg\, y^{-1}. \quad (17)$$

This is quite comparable with the achieved biomass production[13] of $2 \cdot 10^{14} kg\, y^{-1}$ in view of the fact that the actual biomass produced contains substantial amount of moisture in it.

III. THERMODYNAMIC EXPLAINATION

The channels of the solar energy distribution over the globe can also be examined thermodynamically. That is one can look for the heat and work parts in the utilization of the net intercepted solar energy and see if these numbers are consistent with an engine which operates on the globe. So let us first separate out the work and heat parts in the numbers listed in Table 1.

Net Incident Solar Energy $[Q_1] = 1.74 \cdot 10^{17} - 0.52 \cdot 10^{17}$

$$= 1.22 \cdot 10^{17} W. \quad (18)$$

This is the input energy to an engine which is of course not a Carnot engine because the output power of it is practically zero as it takes infinite time to operate it. The most realistic engine will be endoreversible Curzon-Ahlborn engine[14]. In the case of our globe this can be hypothetically operated in between the temperature gradient available in the troposphere. The Curzon-Ahlborn[14] efficiency of this engine will be

$$\eta = 1 - \sqrt{\frac{T_a}{T_s}}. \quad (19)$$

From the discussion above we have seen that the bottom of the troposphere is at the temperature $T_s = 288K$ [cf. (6)] whereas the top of the troposphere has the temperature $T_a = 227K$ [cf. (7)]. These values corresponds to the efficiency

$$\eta = 1 - \sqrt{\frac{T_a}{T_s}} = 1 - \sqrt{\frac{227}{288}} = 1 - 0.89 = 0.11. \quad (20)$$

If we employ the value of net input energy through (16) the work generated will be

$$P = Q_1 \cdot \eta = 1.22 \cdot 10^{17} \cdot 0.11 = 1.3 \cdot 10^{16} W. \quad (21)$$

This value will now be compared with the total work being produced from the various channels mentioned in the Table 1; the last three channels contain either partially or completely work parts. The channels wind, convection, waves and current as well as the photosynthesis correspond to work being produced whereas the channel evaporation, precipitation, etc consists of heat and work parts both. Here the latent heat is partially utilized in doing work W_{ST} against the surface tension and subsequently the work W_{EAP} in expanding this vapor against the external air pressure of one atmosphere. This exercise has been done by Agrawal and Menon[15]. They report that at 288K these are

$$W_{ST} = 56.6\, calorie\, g^{-1} = 2.38 \cdot 10^{5} J kg^{-1}, \quad (22a)$$

$$W_{EAP} = 31.8\, calorie\, g^{-1} = 1.34 \cdot 10^{5} J kg^{-1}. \quad (22b)$$

Next this vapor is lifted to higher altitude by the force of buoyancy where it condenses giving the rain. For simplicity let us assume that the vapor is lifted to the height of

$h = 10000m$ for precipitation then the work done W_{LV} in lifting one kilogram vapor will be

$$W_{LV} = mass\ of\ vapor \times acceleration\ due\ to\ gravity \times height$$

$$= 1kg \times 9.8ms^{-2} \times 10000m = 9.8 \times 10^4 J. \quad (22c)$$

Thus the total work being done in the process of evaporation and its lift will be sum of (22a), (22b) and (22c) which is

$$W_{EV} = W_{ST} + W_{EAP} + W_{LV} = 2.38 \times 10^5 + 1.34 \times 10^5 + 9.8 \times 10^4$$
$$= 4.7 \cdot 10^5 Jkg^{-1}. \quad (23)$$

Multiplying this with total mass of the water vapor M_{water} evaporating per second gives the work part in the evaporation and precipitation channel as

$$W_{EPC} = M_{water} \times W_{EV}. \quad (24)$$

Putting the value of the total mass of water vapor evaporating per second which is

$$M_{water} = \frac{Solar\ power\ going\ into\ evaporation}{Latent\ heat} = \frac{4 \times 10^{16}}{2.48 \times 10^6}$$
$$= 1.6 \cdot 10^{10} kg\ s^{-1}, \quad (25)$$

in the above expression gives the rate of work being done in the channel of evaporation and precipitation as

$$W_{EPC} = 1.6 \cdot 10^{10} \cdot 4.7 \cdot 10^5 = 7.5 \cdot 10^{15} W. \quad (26a)$$

The wind channel produces the total power

$$W_{WIND} = 3.7 \cdot 10^{14} W, \quad (26b)$$

and the photosynthesis channel produces biomass which is equivalent to the value of work as

$$W_{PC} = 4 \cdot 10^{13} W. \quad (26c)$$

The sum of (26a), (26b) and (26c) gives the value of total work W_{TOTAL} being produced by the net intercepted solar energy. This is as follows

$$W_{TOTAL} = W_{EPC} + W_{WIND} + W_{PC} = 8 \cdot 10^{15} W. \quad (27)$$

A comparison of the work part (21) derived by applying the endoreversible Curzon-Ahlborn[14] engine with the sum of work parts (27) of the channels evaporation, wind and photosynthesis is quite satisfactory in view of the pedagogic nature of this paper. This also demonstrates another success of Curzon Ahlborn[14] engine which is quite popular for the last several decades.

IV. CONCLUSIONS & DISCUSSION

M. King Hubbert[1] had reported the distribution of intercepted solar energy into various channels and while teaching this topic the author had faced lot of questions from the undergraduates. This article has attempted to provide their answers for the benefit of the students and helping them in drawing quantitative inferences regarding the interesting physical/biological phenomena due to the intercepted solar energy by the earth. These are as follows

- The steady state temperature T_s of the earth will always be greater than the temperature of troposphere at all altitude and the maximum temperatures difference $T_s - T_a \sim 60K$; T_a being the temperature of the air at the top of the troposphere.
- The solar energy going into the evaporation channel causes the annual global average rainfall of around one meter.
- The photosynthesis channel results in the annual production of around $8 \cdot 10^{13}$ kilogram of dry biomass which is quite comparable with the achieved biomass production of $2 \cdot 10^{14} kg\ y^{-1}$ in view of the fact that the actual biomass produced contains substantial amount of moisture in it.
- The thermo dynamical model presented here confirms that the net intercepted solar energy produces work through a hypothetical Curzon Ahlborn[14] engine and it is consistent with the produced work through the process of evaporation, wind and photosynthesis.
- The values of wind power obtained by M. King Hubbert[1], Gordon and Zarmi[12] as well as the one projected in the present paper are not consistent with each. This has been elaborated below.

It is clear from the above discussion that the major portion of the work being produced by the combination of oceans and atmosphere goes into the work of evaporation of water and a marginal part is used by photosynthesis. There is some uncertainty as far as the amount of global wind is concerned. According to the present model this could be the net available work which is the difference of the work derived from Curzon Ahlborn[14] engine and work done in the evaporation process ignoring the marginal photosynthesis channel. This is

$$W_{WIND}^{PROJECTED} = P - W_{EPC} = 1.3 \times 10^{16} - 7.5 \times 10^{15} = 5.5 \cdot 10^{15} \text{W}, \quad (28)$$

and could be wind generated over the globe. Dividing this by the total surface area of the earth gives

$$w_{wind}^{projected} = \frac{W_{WIND}^{PROJECTED}}{4\pi a^2} = \frac{5.5 \times 10^{15} \text{ W}}{5.1 \times 10^{14} \text{ m}^2} = 10 W m^{-2}, \quad (29)$$

where as the corresponding value by M. King Hubbert[1] is 0.7W/m^2 and Gordon and Zarmi[12] arrive at a value of 17W/m^2 through finite time thermodynamics. However, a nonendoreversible model given by Barranco-Jime and Angulo-Brown[16] predicts the wind power equal to 7W/m^2 for $T_s = 294.4K$ and $T_a = 239.7K$ which are quite comparable to the corresponding values [cf. (6), (7), and (29)] mentioned above.

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