Towards a conceptual framework for identifying student difficulties with solving Real-World Problems in Physics

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Abstract
This paper develops a conceptual framework for identifying the challenges and obstacles university students encounter when solving real-world problems involving Physics. The framework is based on viewing problem solving as a modelling process. In order to solve a real-world problem, the problem solver has to go through the steps and do the tasks of such a process. The paper presents a theoretical analysis of what it takes to solve three real-world problems, demonstrating how the framework presented captures the essential aspects of solving them. Moreover, it is argued that three steps critical for real-world problem solving – initial analysis of the problem situation, choice of relevant physical theory (the so-called paradigmatic choice) and mathematization – are not covered by existing models of problem solving in Physics. Finally, the existing research on student difficulties with problem solving in Physics is placed within the framework.

Keywords: Problem solving, real-world problems, university level.

Resumen
Este artículo desarrolla un marco conceptual para identificar los retos y obstáculos que los estudiantes universitarios encuentran en la resolución de problemas reales relacionados con la física. El marco está basado en la visualización de la solución de problemas como un proceso de modelado. Con el fin de resolver un problema del mundo-real, el solucionador de problemas tiene que pasar por los pasos y hacer las tareas de dicho proceso. El documento presenta un análisis teórico de lo que se necesita para resolver los tres problemas del mundo-real, demostrando cómo el marco que se presenta recoge los aspectos esenciales de su solución. Por otra parte, se argumenta que los tres pasos críticos para la resolución de problemas del mundo-real - análisis inicial de la situación del problema, la elección de la teoría de los aspectos físicos (la elección paradigmática llamada) y la matematización - no están cubiertos por los modelos existentes de resolución de problemas en física. Por último, la investigación existente sobre dificultades de los alumnos con la resolución de problemas en física se coloca en el marco.

Palabras clave: Resolución de problemas, problemas del mundo-real, nivel universitario.

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I. INTRODUCTION

A major goal of Physics Education is to develop student competency in solving real-world problems using the concepts and theories of Physics. Since the 1960s, Physics educators and researchers have lamented that the type of problems so predominantly used in Physics education that they are called standard problems [1] are cleaned-up versions of real-world problems with much of the physical reasoning already done for the students due to the very formulation of the problems [2, 3, 4, 5]. Fig. 1 shows a typical example of a standard problem. In order to allow students to develop a broader problem-solving competency, other types of problems have been proposed [2, 3, 5, 6, 7]. As these problems supposedly simulate problems found in the real world, they are often called real-world problems.

The process of solving standard problems has been studied extensively, but only a few studies exist on the challenges and difficulties students encounter when solving real-world problems. In their study of high school students solving astronomy problems, Shin, Jonassen and McGee [8] concluded that solving both well- and ill-defined problems required domain-specific knowledge, but ill-defined problems in unfamiliar contexts also required planning and monitoring skills. Fortus [9] investigated the approaches of individuals with different physics backgrounds to both well-defined and real-world problems. He found that the skills needed to solve a well-

A pendulum consists of a small ball attached to one end of a light string of length L. The other end of the string is attached to hook fastened to the ceiling. A fixed peg is located vertically below the hook at a distance smaller than L. The ball is initially held at rest, with string taut and horizontal, and is then released. What must be the minimum distance between the hook and the peg so that the string is still taut when the ball reaches a point directly above the peg?

FIGURE 1. A standard problem: The pendulum problem.

defined problem in the domain of Newtonian mechanics were not sufficient to solve a real-world problem in the same domain that required making assumptions. Moreover, Fortus' results indicate that the step in the problem solving process causing most difficulties for the subjects studied was making reasonable assumptions concerning the problem.

Hence, real-world problems seem to present other difficulties for the problem solver than standard problems do. Unfortunately, we do not know much about what challenges and obstacles students encounter when solving real-world problems. The present paper is part of a general research program that aims at remedy this situation by identifying and characterizing these difficulties for problems posed by others and whose solutions require the use of Physics. This paper develops a conceptual framework that can be used in this endeavor.

A research framework is, 'a basic structure of the ideas (*i.e.* abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated' [10, p. 458]. It provides a structure that can be used for conceptualizing and designing empirical research studies, allows sense making of data, and enables us to go beyond common sense. Lester distinguishes between different kinds of research frameworks. A theoretical framework guides research activities with reference to formal theory, *i.e.* a theory that has been developed by using an established, coherent explanation of certain sorts of phenomena and relationships, *e.g.* Piaget's theory of cognitive development. In contrast, *conceptual frameworks*, such as the one presented here, are put together from a number of sources, including various theories and aspects of practitioner knowledge. A conceptual framework is, 'an argument that the concepts chosen for investigation, and any anticipated relationships among them will be appropriate and useful given the research under investigation' [10, p. 460] and its aim is to contribute to both fundamental understanding and the development of teaching.

In order to obtain such a framework for the present purpose, we need a detailed description of the problem solving process. For standard problems, several such models exist [11, 12, 13, 14, 15], but none of them focuses on the solving of real-world problems. Hence, the aim in this paper is to answer the following research questions:

What stages are involved in solving real-world Physics problems? What are the tasks at each stage?

A systematic and sufficiently detailed description of these stages and tasks would give us an account of the challenges that solving a real-world problem poses for students. Whether these challenges are in fact *obstacles* for the students is a question that will be studied empirically in subsequent papers.

The research question is investigated theoretically by solving the three real-world problems presented in Fig. 2 and by analyzing the stages and tasks involved in the solutions. In order to systematically identify the stages and tasks required in the problem solving process, the problem solving process is seen as a modelling process, where the solution to a problem is obtained via a mathematical model that is either constructed for the purpose or selected from the physicists' arsenal of models. This perspective allows us to draw on the research on modelling in Physics Education, in particular the work of Hestenes and Halloun. The justification of the framework is based on a theoretical argument that the model actually captures the essential steps in solving the three problems.

II. STANDARD AND REAL-WORLD PROBLEMS

Standard problems can be characterized as follows [1]: A situation is described for which certain information is provided, typically as numerical values for the variables of the situation. The job of the problem solver is to determine the value of one of the other variables of the situation.

The cannon problem
How does a cannon's firepower depend on the length of the cannon barrel?

The power plug problem
A power plug is connected to a water heater. Heat is generated in the power plug due to a loose connection. How much heat can possibly be generated in the power plug?

The water tap problem
How does the width of a column of water from a tap change down the column?

FIGURE 2. The real world problems considered in the paper. The cannon problem is taken from Højgaard Jensen (2003), the power plug problem from Højgaard Jensen (2004) and the water tap problem is an unpublished exam problem posed by Jens Højgaard Jensen.

Moreover, the problems are well-defined with the sought-after variable explicitly stated and only relevant variables appear in the problem statement.

To what extent are the so-called ‘real-world problems’ in the Physics Education literature (see, *e.g.* the problems in [4, 9]) related to problems found in real-world situations? On the one hand, the problems have some artificial features. For example, they are not set by the problem solvers themselves but by others, *e.g.* the teacher. This means that the problem framing stage of the problem solving process that some researchers emphasize as important in real-world problem solving (see, *e.g.* [18]) has already been done when the problem is presented to the solver. Consequently, ‘real-world tasks’ would perhaps be a more accurate name than ‘real world problems,’ but since the latter is used in the literature, it will also be used here. In addition, as the problems are set in a school context, they carry some connotations with them. Students can safely assume that the Physics of the problems has been covered in the classroom, that they can in fact be solved and that this can be done within a reasonable amount of time, etc. On the other hand, the problems also simulate aspects of problems in the real world. First, they refer to an authentic context: (A) The event described has taken place or has a fair chance of taking place; (B) the question posed in the task might actually be asked in the real-life event; (C) the data/information given in the problem is realistic in terms of the problem; and (D) the solution is consistent with what is regarded as an appropriate solution in the corresponding out-of-school situation.[1] Many standard problems do not fulfill points (A) and (B), *e.g.* who would like to know the answer to the problem in Fig. 1? Second, the steps required to solve the real-world problems correspond to those of solving a real-world task (except, of course, for the problem framing). More precisely, for a real-world problem the problem solver must make several decisions about (1) which specific variable(s) would be useful to answer the question; (2) which Physics concepts and principles could be applied to determine that variable; (3) what information would be needed; and (4) where or how that information could be obtained or estimated [2, 3, 20]. Moreover, the problem solver needs to make assumptions, approximations and idealizations of the problem situation [2, 9, 21, 22]. In contrast, standard problems do not require that the solver performs all or even most of these steps.

The three problems discussed in the present paper are formulated in everyday language; the situations described belong to the real world rather than an artificial physics world; their questions might actually be posed in the real-world situation, and their solutions require the application of Physics.

[1] This is an adaptation of a framework proposed by Palm [19] for describing the concordance between word problems in mathematics education and tasks in the real world beyond the mathematics classroom.

III. THE EXISTING MODELS OF THE PROBELM SOLVING PROCESS

Several models of the problem solving process in Physics exist. One dominant model is that of Reif and colleagues [11, 12, 21] which concerns effective human problem solving in general. The procedures contained in their model should be used in conjunction with a domain-specific knowledge base, such as mechanics. The model, as expounded in [12], divides problem solving into four stages:

1. Problem description and analysis;
2. Construction of a solution to the problem;
3. Assessing the solution; and
4. Exploiting the solution

The knowledge base which is specific to the problem domain in question facilitates these stages. It contains declarative knowledge of concepts and principles as well as specific procedures facilitating their use.

In the first stage, the original problem is redescribed in a way that facilitates the subsequent search for its solution, including identifying and organizing relevant knowledge and describing it in convenient symbolic form. The resulting problem description assists the construction of the solution by limiting the domain of search and by allowing a ready application of the problem solver’s knowledge base. Reif and Heller subdivide this stage into two fairly distinct stages. The one sub-stage, which we will call 1A, is where everyday knowledge is used to generate a basic description readily interpretable by the problem solver. The aim of this description is merely to translate the original problem into a form clearly describing the situation specified and the information to be found. This includes using diagrams and/or statements to describe the specified situation about the system and its properties, introducing convenient symbols, and identifying those denoting unknown values. The other sub-stage, which we will call 1B, is where the problem solver uses his/her specialized knowledge about the domain in question. This leads to a theoretical problem description in which the problem is described in terms of the concepts of the particular domain. This means that the entire body of theoretical knowledge about this domain is accessible when the solution of the problem is implemented. The theoretical analysis of a problem is usually followed by a qualitative analysis of the problem in which the main implications of applicable principles are explored qualitatively. Such an analysis may facilitate the subsequent solution by suggesting possible approaches and by helping to interpret physically the results of the mathematical analysis. When substage 1A and 1B is completed, the way is paved for the actual construction of the solution. Here, Reif and Heller focus mainly on generally applicable methods, such as constraint satisfaction. After a solution is constructed, how satisfactory it actually is in terms of *e.g.* completeness and internal consistency is assessed.

To solve a real-world problem, the problem solver has to go through the Reif and Heller’s four stages listed above.

Sub-stages 1A and 1B play a particularly important role when solving real-world problems because these two steps take the problem from the real-world situation and put it into the realm of Physics. However, as will be argued below, their model, which in principle is applicable to all types of problem solving, is too general and simple to capture the complexity of translating a real-world problem into Physics.

Two recent models of the problem solving process, one developed in [14, 15], and the other in [13], modify the model by Reif and colleagues. These two models focus greatly on the role of Physics knowledge in the problem solving process, but neither of them is designed to characterize the possible challenges and obstacles students encounter due to the real-world aspect of the problem solving.

IV. MODELLING IN PHYSICS

Seeing problem solving as a modelling process allows us to focus particularly on the translation from the real world

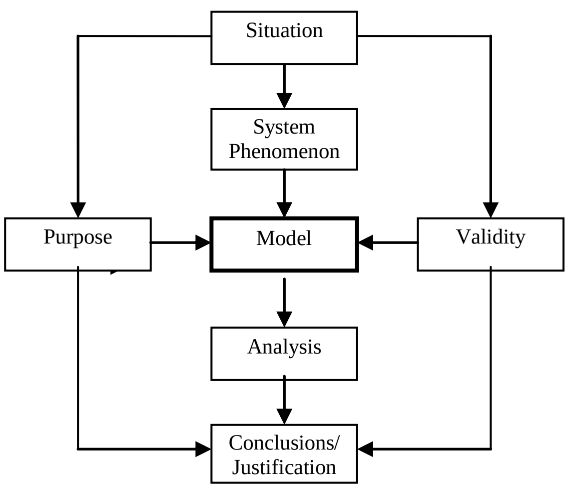

FIGURE 3. Modelling in Physics according to Hestenes (1995).

to the physical world. According to Hestenes [24], modelling in Physics typically begins with a real-world situation. Then the following steps are taken:

A. The system to be modelled is *identified* along with its relevant properties (identification of variables). This leads to the construction of a system schema and the selection of property descriptors.

B. A *model* is then constructed or selected and adapted from a collection of available models. The intended use or purpose greatly influences this step and a variety of purposes govern variations of the modelling process.

C. Empirical determination of the model's *validity* is obtained by comparing the model with the system in the original situation. This can involve designing and performing an experiment or simply checking the answer to a problem.

D. The previous step provides *justification* for the conclusions about the system and the situation which are drawn from the model.

E. In order to extract conclusions from the model, an analysis of the behavior of the model is required.

The model deployment scheme described in Fig. 4 shows that Halloun [25] builds on Hestenes' ideas, though some of the stages are reformulated and several new ones added. As will be argued below, the most important additional stages for the purpose of this article is the stage 'Paradigmatic choice: What theory? What model(s)?', which Halloun describes as follows, 'The problem solution would begin with the choice of an appropriate theory within the context of a specific scientific paradigm (*e.g.* the choice of Newton theory, Euler theory, or Hamilton-Jacobi theory for classical mechanics situations), followed by the choice of appropriate models' [25, p. 150-151]. Two distinct stages are involved in Halloun's paradigmatic choice. In the first stage, the overall theory is chosen and the appropriate model is chosen subsequently. The latter stage involves choosing an appropriate principle, *e.g.* conservation of energy, within the theory. Since these two stages may cause different difficulties for the students, we consider them to be separate steps and call them the 'paradigmatic choice' and the 'principle and concept choice,' respectively.

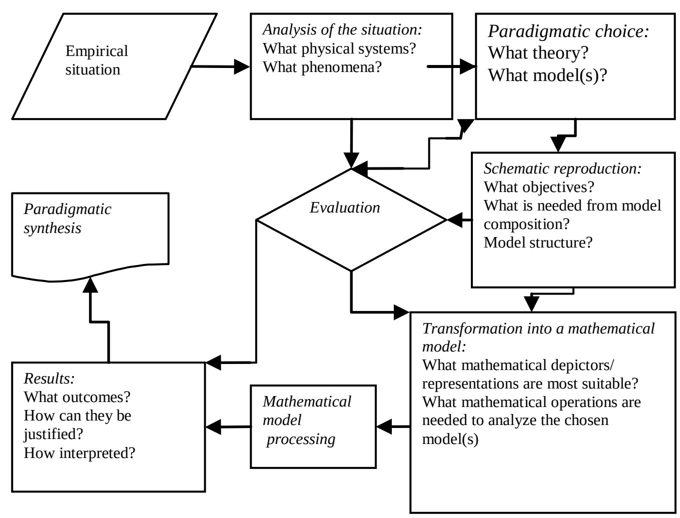

FIGURE 4. Model deployment according to Halloun (2004).

V. THREE EXAMPLES

To assist and illuminate our analysis, we consider solutions to three specific, real-world problems.

A. The power plug problem

The power plug problem can be solved using the following steps [17]:

1. A loose connection is a point in an electric circuit where the metal on one side of the connection is not in complete contact with the metal on the other side, effectively

reducing the cross-sectional area of the wire at the joint. Consequently, higher than normal resistance occurs at the joint, thus, causing it to heat up. For our purpose, the effect of the loose connection is for it to act as a resistor, so that when a current flows through the resistor, heat is generated. The heating of the loose connection must be equal to the power generated by the flow of the current through the resistor. Similarly, we can represent the effect of the water heater on the system using a resistor. Regarding the position of the resistors, it is natural to assume that the loose connection resistor is connected in series with the water heater, which is an external component. The role of the water heater is to affect the voltage drop over the former, since they are placed in a series.
2. We now have a physical model describing what is happening. It follows from the previous considerations that the relevant physical theory is electric circuit theory.
3. Within this theory, Joule's law can be used to calculate the heat generated due to the current flowing through a conductor or, equivalently, the power dissipated in a resistance. Moreover, we know that the current through a series of resistors is the same for each resistor.

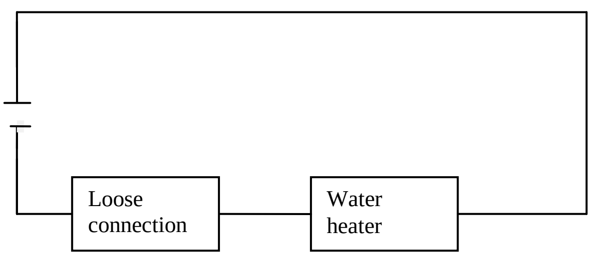

FIGURE 5. Electric diagram for the power plug problem.

4. These physical ideas can be turn into a mathematical model. First, we have to describe the situation in mathematical terms. We denote the resistance of the loose connection by R_l, and the current through it, I. Then we use Joule's law to calculate the power dissipated in the loose connection:

$$W_l = R_l I^2. \tag{1}$$

This does not solve the problem, because we want to reduce W_l to known quantities and I is unknown, while the mains voltage is assumed to be known, namely 220V, 110V, or whatever is the value in the country of residence. One might be tempted to simply use Ohms law for voltage, $V=RI$ with R and V over the loose connection to eliminate I, but that does not work because we do not know the voltage over the loose connection. We call the resistance of the water heater R_h. Noting that for a series of resistors, the current through them is the same we thus replace the above with:

$$W_l = \frac{R_l V^2}{(R_h + R_l)^2}. \tag{2}$$

5. We now want to find for what values of R_l, W_l attains its maximum value. Thus we find the derivative with respect to R_l is given by:

$$\frac{dW_l}{dR_l} = V^2 \frac{(R_h + R_l - 2R_l)}{(R_h + R_l)^2}. \tag{3}$$

The power plug attains the maximum heating for the values of R_l when this is equal to 0. This happens when $R_l = R_h$.
6. Hence, we obtain:

$$W_{l,\max} = R_h \frac{V^2}{(R_h + R_h)} = \frac{1}{4}\frac{V^2}{R_h}. \tag{4}$$

7. The power delivered to the water heater W_h is V^2/R_h so the maximum loss is proportional to this power, which seems reasonable.
8. We can rewrite the maximum loss to get:

$$W_{l,\max} = \frac{1}{4} W_h. \tag{5}$$

Hence, the worst case scenario is that a fourth of the dimensioned power of the water heater is lost as heat due to the loose connection.

B. The cannon problem

The cannon problem can be solved as follows [16]:
1. The cannon's firepower is a measure of the destructiveness of its projectiles. We define the firepower to be the kinetic energy of the projectile when it leaves the barrel muzzle (we could also have chosen the muzzle speed). The ignition of the cannon gun powder causes an explosion in the cannon that expands the air below the projectile, which propels the projectile until the air fills the entire volume of the barrel.
2. The explosion and expansion of the gas in the cannon can be described with thermodynamics, while mechanics is appropriate for the motion of the projectile in the barrel.
3. We can use thermodynamics to calculate the work done by the expanding gas on the projectile during its motion through the barrel. Using the work theorem of mechanics we can relate this work to the kinetic energy. We furthermore have to make an assumption about the expansion process. We will assume that the explosion occurs without an exchange of heat between the gas and the barrel, *i.e.* it is an adiabatic expansion, but other reasonable assumptions could have been made as well.
4. Let V_0 be the small volume behind the projectile and V_L the volume of the barrel. Let P_0 and P be the pressures of the gas right after the explosion and during the expansion, respectively. We assume that the barrel is a cylinder and let

L_0 denote the length of the barrel corresponding to the initial volume V_0 and L the entire length of the barrel. Since we assume that it is an adiabatic process, we know that the pressure during the expansion is:

$$PV^{\gamma} = P_0 V_0^{\gamma}. \tag{6}$$

Here γ is a characteristic constant of the gas.

The work done on the projectile is

$$W = \int_{V_0}^{V_L} PdV. \tag{7}$$

According to the work theorem, the kinetic energy gained by the projectile in the barrel is:

$$W = \Delta K. \tag{8}$$

Since the projectile is initially at rest ΔK is equal to the kinetic energy of the projectile when it leaves the cannon muzzle; this implies that the firepower is equal to *W*.

5. Evaluating the integral in Eq. 7, we get an expression for the work:

$$W = V_0^{\gamma} P_0 \int_{V_0}^{V_L} V^{-\gamma} dV = \frac{V_0^{\gamma} P_0}{\gamma - 1}\left(\frac{1}{V_0^{\gamma-1}} - \frac{1}{V_L^{\gamma-1}}\right). \tag{9}$$

This can be rewritten in terms of the barrel length, using the assumption that it is a cylinder:

$$W = \frac{V_0 P_0}{\gamma - 1}\left(1 - \left(\frac{L_0}{L}\right)^{\gamma-1}\right). \tag{10}$$

6. For the firepower we obtain:

$$K = \frac{V_0 P_0}{\gamma - 1}\left(1 - \left(\frac{L_0}{L}\right)^{\gamma-1}\right) = U_0\left(1 - \left(\frac{L_0}{L}\right)^{\gamma-1}\right). \tag{11}$$

7. Here $\frac{V_0 P_0}{\gamma - 1}$ is identified as the internal energy of the gas after the explosion U_0. Hence, the firepower is equal to U_0 when the barrel length is long. This seems reasonable because in that case all the energy released by the explosion should be transferred to the projectile as none of it is lost to the surroundings as gas expansion.

8. The cannon's firepower dependence on the length is given by Eq. (11).

C. The water tap problem

The water tap problem can be solved in the following way:

1. We assume first that the column has a circular cross-section. The width of the column is then the diameter of this cross-section. The water leaves the tap with a certain flow rate. We assume that in a horizontal section of the water column, the water has a uniform flow rate. Gravity causes the water to accelerate as it falls, thus increasing the flow rate as we go down along the column. Furthermore, we assume that the horizontal shape of the water column does not change.

2. Since this problem involves the follow of water, fluid dynamics can obviously be applied.

3. In order to apply these ideas, we need to make some choices. We neglect turbulence and assume that the flow is laminar. Since we assume a uniform flow rate through a horizontal cross-section, we get a continuity equation. Moreover, we assume that the viscosity can be neglected, so that Bernoulli's equation (a way of taking energy conservation into account) can be applied to the situation. Furthermore, we assume that the pressure down the water column is constant.

4. These ideas can be turned into a mathematical model. Let the tap be placed at the height h_0 above the sink and let it have a circular cross-sectional area of A_0. We denote by v_0 the flow rate of the water leaving the tap. Bernoulli's equation states that the quantity $\frac{1}{2}\rho v^2 + \rho g h + p$, where v is the velocity; ρ is the water density; g is the gravitational acceleration; and h is the height above the sink, is constant down along the water column. In particular, the equation for what occurs just after the water has left the tap is:

$$\frac{1}{2}\rho v_0^2 + \rho g h_0 + p = const. \tag{12}$$

The atmospheric pressure has a constant value, p. So:

$$\frac{1}{2}\rho v^2 + \rho g h + p = \frac{1}{2}\rho v_0 + \rho g h_0 + p. \tag{13}$$

The continuity equation gives that the cross-sectional area A times the flow rate v is constant down the column:

$$A \cdot v = const. \tag{14}$$

In particular, at the tap we get:

$$A_0 \cdot v_0 = const. \tag{15}$$

Hence:

$$A \cdot v = A_0 \cdot v_0. \tag{16}$$

The horizontal cross-section is circular, so $A = \pi d^2$ with d being the diameter.

5. These equations can be analyzed mathematically. Bernoulli's equation implies that:

$$v^2 - v_0^2 = 2g(h_0 - h). \tag{17}$$

Hence:

$$v = \sqrt{2g(h_0 - h) + v_0^2}. \tag{18}$$

Combining this equation with the continuity equation yields:

$$A = \frac{A_0 \cdot v_0}{v} = \frac{A_0 \cdot v_0}{\sqrt{2g(h_0 - h) + v_0^2}}. \quad (19)$$

We then use that the cross-sectional area of the water is circular, so $d = \sqrt{\frac{A}{\pi}}$.

6. We find that the width, *i.e.* the diameter, changes with height as

$$d = \frac{\sqrt{A_0 \cdot v_0}}{\sqrt{\pi}\left(2g(h_0 - h) + v_0^2\right)^{1/4}}. \quad (20)$$

Here v_0, A_0, and h_0 are constants of the system.

7. We find that the column gets thinner as we go down the column, a result that we should expect from our real world experience.

8. We get that the width changes with height according to Eq. (20).

VI. PROBLEM SOLVING AS MODELLING

It is evident that solving these three problems requires executing the actions that researchers see as characteristic of real world problem solving: the problem solver has to make the decisions, assumptions, approximations and idealization that are relevant for the problems.

Hestenes' and Halloun's diagrams of modelling in Physics can be adapted to model the process of solving real-world problems. Hestenes' diagram forms the backbone, but in modified form. First of all, Halloun's 'paradigmatic choice' is relevant for the problem solving process and some of Halloun's titles are more indicative than Hestenes', so they have also been chosen. On the other hand, two of Halloun's boxes, 'Schematic reproduction' and 'Paradigmatic synthesis,' represent unnecessary complications for the present purpose and are consequently left out here. All this leads to the diagram in Fig. 6, which shows the problem solving process.

The process begins with a real-world situation in which some problem is formulated. As this is typically not formulated in Physics terms, the problem needs to be put into a form that is amenable to investigation using Physics. To do so, making an initial *analysis of the situation* is necessary. The aim of this step is the identification of the physical system and the phenomenon that are to be modelled. This is done by identifying the relevant features of reality, selecting the objects, relations and so on that are relevant for the modelling. This process is typically based on specialized knowledge about the domain as well as physical knowledge. In this process the system is delineated from the context and some idealizations are done, explicitly or implicitly.

For the power plug problem and the cannon problem, this initial analysis is the first step. For the power plug case, the analysis has three parts. First, the effect of the loose connection on the situation can be represented by an electrical resistor and this resistor should be placed in a series with the water heater. Second, the water heater does in fact play a role because it affects the current going

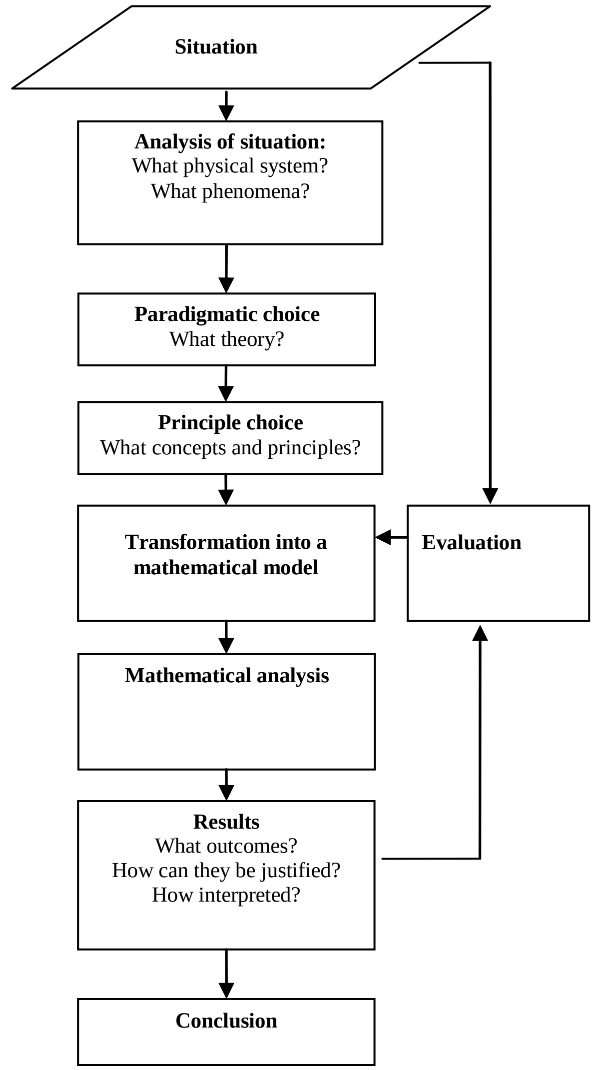

FIGURE 6. Problem solving as modelling.

through the loose connection. Third, the problem solver needs to identify relevant theoretical or special knowledge, *e.g.* that the heating is equivalent to the power generated by the current flowing through the resistor. The cannon problem requires an analysis of how the everyday notion of firepower should be interpreted in physical terms as kinetic energy (or perhaps as the muzzle speed) of the projectile. Moreover, it should be realized that the explosion of the gun powder propels the projectile.

In the next step, the machinery of Physics is brought to bear on the delineated system. This requires that the problem solver makes a *paradigmatic choice* of appropriate physical theory. This means choosing a way of seeing the problem in physical terms and this choice subsequently affects how the problem is approached.

For the three problems, the paradigmatic choice is done in step 2. For the power plug problem and the water tap problem, this choice follows immediately from the initial analysis. For the cannon problem, however, the solver needs to find a strategy for determining the muzzle speed of the projectile. Which physical discipline would solve the problem? One might think that the problem could be attacked by purely mechanical means, for instance, by making a more or less ad hoc assumption about the acceleration of the projectile in the barrel. A more satisfactory solution, along the above lines (several other reasonable solutions with other types of expansion of the gas are possible), requires a certain realization on the part of the problem solver, namely that the solution to the problem involves a combination of a mechanical view point and a thermodynamic one. This requires the combination of two physical disciplines.

When the physical theory has been chosen, the solver has to choose a *physical principle* within this theory. This corresponds to step 3 for the three problems. In the power plug problem, the solver needs to realize both that Joule's law can be used to find the heat generated by the current flowing through a conductor as well as that the current through a series of resistors is the same for each resistor. The cannon problem requires that the solver realizes that the thermodynamic work can be used to obtain the mechanical kinetic energy. In the water tap problem, Bernoulli's equation or another version of energy conservation as well as the continuity equation are the basis of the solution to the problem.

The choices of paradigm and physical principle should lead to a physical model of the phenomenon in question. The physical model obtained can then be *transformed into a mathematical model*. Hestenes and Halloun do not provide many details about this stage, so it is necessary to extend their description. The setting up of the mathematical model requires a mathematization, *i.e.* a translation of the physical structure into a mathematical structure. This involves a description of the situation in mathematical terms (the volumes, pressures, lengths, etc.) as well as writing the mathematical equations described in the paradigmatic choice. During this stage, the abstract laws and principles, often mathematically formulated, are applied to the specific situation by an identification of the general mathematical structure of the situation. Thomas Kuhn's [26] discussion of Newton's second law can serve as an example. This law, Kuhn writes, is usually formulated as $f=ma$. In order to apply it, the solver needs to adapt it to the situation. For the free-fall, the laws reads: $mg = m\frac{d^2s}{dt^2}$; for the simple pendulum, it is transformed to: $mg\sin\theta = -ml\frac{d^2\theta}{dt^2}$; for a pair of coupled harmonic oscillators, it becomes two equations, the first being: $m_1\frac{d^2s}{dt^2} + k_1s_1 = k_2(d + s_2 - s_1)$, etc. In all these situations, the problem solver has to identify the relevant mathematical quantity, whether it is the height above the Earth, the angle, or the position along a pendulum trajectory of the object.

This is stage 4 for the three examples. The power plug problem requires that the solver realizes which variable will solve the problem and which are unknown. Moreover it is important to realize that Ohm's law for the voltage, $V=RI$ with R and V over the loose connection to eliminate I, do not work because we do not know the voltage over the loose connection. We call the resistance of the water heater R_h. In order to solve the cannon problem, the solver has to realize that the barrel length is related to barrel volume. The mathematization process for the water tap problem is quite involved. In order to apply Bernoulli's equation and the continuity equation, a mathematical preparation of the situation is needed that specifies the parameters that can describe the shape of the water and the flow of the water.

The model resulting from the previous deliberations can be *analyzed mathematically* using mathematical methods to obtain mathematical results and conclusions. These methods can be either analytical or numerical, *i.e.* they can involve the use of a calculator or a computer.

This is stage 5. For the power plug, it consists in finding a maximum of the equations, while an integral must be evaluated for the cannon problem. For the water tap, the equations must be manipulated to give the right answer.

The mathematical analysis leads to *results* for the behavior of the model. For the power plug problem and the cannon problem, this is stage 6.

The entire process is *evaluated* during the process with respect both to the original purpose as well as validity. The purpose largely governs the modelling process. The level of detail in the model is, for instance, governed by the purpose. Sometimes only a crude model is needed, while at other times a more elaborate model is relevant. The model furthermore needs to be valid; this is achieved by comparing the model with the system in the original situation.

For the power plug problem, it is reasonable that the value is proportional to the power of the water heater, while the result for the cannon problem is studied in a limit.

Finally, a *conclusion* about the real-world situation is drawn based on the obtained results.

VII. DISCUSSION

By seeing problem solving in Physics as a modelling process, the proposed framework is based on a systemic modelling of the challenges encountered by students solving real-world problems.

It is evident that the model indeed captures the stages of the process of solving the three example problems provided. Since there is nothing special about these three problems, it is reasonable to assume that the model captures the process of solving real-world problems more generally. Moreover, the model allows a focus on aspects of special importance for real-world problems that are rarely covered by previous models. For all three problems, the initial analysis of the situation clearly plays an important role in the process of solving them. This is particularly true for the power plug problem, where, for example, this analysis is crucial and it is not all trivial to determine the relevant aspects of the loose connection. This aspect is much less present in the pendulum problem in Fig. 1, as illustrated by Reif and Heller's (1982) four-stage model analysis of the problem solving process. A basic description can immediately be given with diagrams and symbols and the goal stated as the determination of the value of parameter. Subsequently, a theoretical description could be provided. In contrast, in the real-world problem, considerations concerning how to prepare the situation to set up the theoretical description are required.

Another special feature of the present description of the problem solving process compared to previous approaches is the paradigmatic choice. For standard problems in Physics, the paradigmatic choice is of little importance as the problems have been prepared in such a way that it is rarely unclear which physical theory applies to the problem. Most Physics students are probably undoubtedly aware that the solution to the pendulum problem in Fig. 1 requires mechanics. This may not be the case for real-world problems as is particularly evident with the cannon problem. This problem requires an initial analysis of how the everyday notion of firepower should be interpreted in physical terms as kinetic energy (or perhaps as the muzzle speed) of the projectile. Here the problem solver needs to decide which physical discipline would solve the problem. One might think that the problem could be attacked by purely mechanical means, for instance, by making a more or less ad hoc assumption about the acceleration of the projectile in the barrel. A more satisfactory solution, along the above lines (several other reasonable solutions with other types of expansion of the gas are possible), requires a certain realization on the part of the problem solver, namely that the solution to the problem involves a combination of a mechanical view point *and* a thermodynamic one. This requires the combination of two physical disciplines. It thus requires that the solver is capable of realizing that the thermodynamic work can be used to obtain the mechanical kinetic energy.

The third aspect that is taken into account in the present model of the problem solving process is the mathematization process. The setting up of the mathematical model requires mathematization, *i.e.* a translation of the physical structure into a mathematical structure. This involves a description of the situation in mathematical terms as well as applying the Physics principle to the situation. Standard problems have typically been translated into mathematical language, which is not the case for real-world problems. For the power plug problem, we arrive at Eq. 1 when the mathematization is complete. Physical insight is required to realize that this equation is not sufficient. I is not an independent variable and focusing on V is preferable. At this point the water heater is taken into account to get Eq. 2.

Previous research on student difficulties with problem solving can be incorporated into the framework presented here. The initial analysis of the situation involves what might be called a framing of the problem situation (see [27]), *i.e.* the determination of a certain perspective that guides the problem solver's interpretation of the situation. The framing of the situation constitutes a fundamental understanding of the situation. Based on established theories as well as more informal Physics, the framing includes a selection of the situation's relevant features as well as cutting out of irrelevant features. Sometimes this is based on more or less formal Physics ideas about what is going on and sometimes assumptions, *i.e.* 'less-than-fully established propositions that are used as a basis for continuing a problem solving process' [9, p. 87]. The framing is often a prerequisite for deciding which specific variables would be useful to answer the problem and what Physics concepts and principles could be applied to determine that variable. In the case of the power plug, *e.g.* the framing consists in pointing out that the loose connection can be represented as an electrical resistor with a certain constant resistance. This framing defines a certain perspective on the situation, which is particularly important for real-world problems as they are not prepared in advance.

Fortus [9] stresses that making assumptions is crucial when transforming a real-world problem into a well-defined question. He finds that two types of assumptions are involved: (a) assumptions about the Physics variables and the principles involved in the problem; and (b) assumptions about the absolute or relative magnitudes of the variables. Fortus emphasizes that assumptions of the first type are always involved when solving any Physics problems, whether standard or real world. Heller and Hollabaugh [3] point out, however, that standard textbook problems often specify the unknown variable in the last sentence, thus removing the necessity of making decisions about which specific variable would be useful to answer the problem. The identification of what exactly to look for in terms of physics quantities is much more straightforward for standard problems, with their formalized nature, than for the less formalized real-world problems. It is also natural to assume that this identification may cause difficulties for the students.

It has been widely recognized that a major obstacle faced by novices when they try to solve a standard Physics problem is translating from the verbal statement of the problem to a mental representation of the problem in terms of Physics (see, *e.g.* [1, 14, 15]). An adequate mental representation of the problem is the first requirement for successful problem solving. In order to obtain such a representation, one needs to infer deep features with the use of relevant background knowledge. Understanding why and how a feature is important thus involves conceptual knowledge of the problem. However, novices' representations are lacking and pose an impediment to their problem-solving proficiency [28, 29, 30]. Studies by Finegold & Mass [31] and McMillan & Swadener [32] indicate another difficulty. Poor problem solvers fail or are unable to carry out the necessary qualitative analysis to construct an adequate representation. We should expect real-world problem solvers to encounter difficulties similar to the ones found for standard problems when they attempt to answer the well-defined question arrived at in the process described above.

The difficulties of making paradigmatic choices have not been researched. The reason could be that this step, which is crucial for real-world problem solving, seems to be much easier for most standard problems since their formulations often reveal which physical theory is relevant. Concerning the principle choice and concept choice, several empirical studies show that both cause difficulties for standard problems. Hardiman, Dufresne, and Mestre [33] found that beginning Physics students who had completed a Physics course had trouble identifying major laws or principles that could be applied to solve a problem. Other studies show that while students may be able to state the definitions of scientific concepts, they often do not know what to do to apply these definitions in specific cases [34, 35].

The intricacies of the mathematization process, which is critical to solving real-world problems, have not been studied to a large extent. While not intending to characterize difficulties, Redish [36] argued from a theoretical point of view that the ability to use mathematics in solving Physics problems requires something other than what is learned in the mathematics classroom. In particular, physicists interpret and use equations in a different way than mathematicians, because they combine conceptual physics and mathematical symbolism. On a much more specific level, Clement, Lochhead, and Monk [37] found that college students working on a word problems in mathematics had difficulties translating the English words from the problem statement into algebraic expressions.

To conclude, we provide a few remarks about the potential applications of the framework presented. This paper offers insights into the behavior of Physics students when faced with real-world problems. The identification of difficulties generic to the problem-solving process opens up the possibility of predicting where in the process of solving a given problem obstacles of different types might be expected to arise. This understanding can contribute to the planning of teaching, such as the identification of necessary prerequisite skills as well as the design of activities that scaffold the development of problem solving competency, in particular the design of tasks that address the challenges and obstacles presented in the framework. Finally, the framework, when it has been refined and further examined, may be useful as an instrument in research on problem solving in Physics. The work presented in this paper has advanced our progress towards achieving these goals.

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