Bowing effect on energy conservation in an incline experiment

Chunfei Li
Department of Physics, Clarion University, Clarion, PA 16214.

E-mail: cli@clarion.edu

(Received 15 December 2011, accepted 28 February 2012)

Abstract

Incline experiments with a car travelling down dynamic track are performed to demonstrate mechanical energy conservation under the assumption that there is no friction. Data analysis revealed cases when mechanical energy seems to increase as the car travels down incline, contradictory to the expectation. A detailed examination leads to the conclusion that this is caused by the slight downward bowing of the dynamic track, the details of which are reported here. This result provides an explanation to the seemingly contradictory result. It can also be used to stimulate students to perform in-depth data analysis.

Keywords: Energy conservation, Incline, Bowing.

Resumen

Se realizaron experimentos de inclinación con un coche que circulaba por la pista dinámica para demostrar la conservación de la energía mecánica con el supuesto de que no hay fricción. El análisis de datos reveló casos en que la energía mecánica parece aumentar a medida que el coche se desplaza por una pendiente, en contradicción con la expectativa. Un examen detallado conduce a la conclusión de que esto es causado por la inclinación ligeramente hacia abajo de la pista dinámica, cuyos detalles se presentan aquí. Este resultado proporciona una explicación para el resultado aparentemente contradictorio. También se puede utilizar para estimular a los estudiantes para realizar un análisis en profundidad de datos.

Palabras clave: Conservación de la energía, inclinación, inclinado.

PACS: 01.40.Di, 01.40.Fk, 01.50.My, 01.55.+b **ISSN 1870-9095**

I. INTRODUCTION

Mechanical energy conservation is an important part in introductory physics and a variety of experiments are used to demonstrate this principle [1, 2, 3]. Among these, many of them are based on the usage of a car moving down an incline. Under the assumption that there is no friction, the theory predicts that when an object moves from one point to another, the mechanical energies, many times the summation of gravitational potential and translational/rotational kinetic energies, are conserved. In reality, friction exists and causes a net loss of mechanical energy, implying that a reduced total mechanical energy is not a surprise in this type of experiment. However, if a seemingly increased mechanical energy is observed, it deserves special attention. In a student experiment, the calculated value of mechanical energy of a car travelling down an inclined dynamic track was found to increase and the reason was analyzed here.

II. EXPERIMENTAL SET-UP

A picture and a schematic diagram for the experimental set-up are shown in Fig. 1. A PASCO dynamic track was raised in one end and a car is released from the top. The motion of the car was measured by a Vernier motion sensor. θ is used to represent the incline angle, which is measured by PASCO angle indicator. The distances from the sensor to the car and to the end of the track are called x and x_0, respectively. Upon releasing a car at rest from the top, the position and speed of the car at different x values from the motion sensor were measured. The total mechanical energy (*TotE*) can be calculated as follows. The heights of the car relative to the leveled ground at different x positions can be calculated from the following Eq. (1)

$$h = \sin\theta(x_0 - x). \quad (1)$$

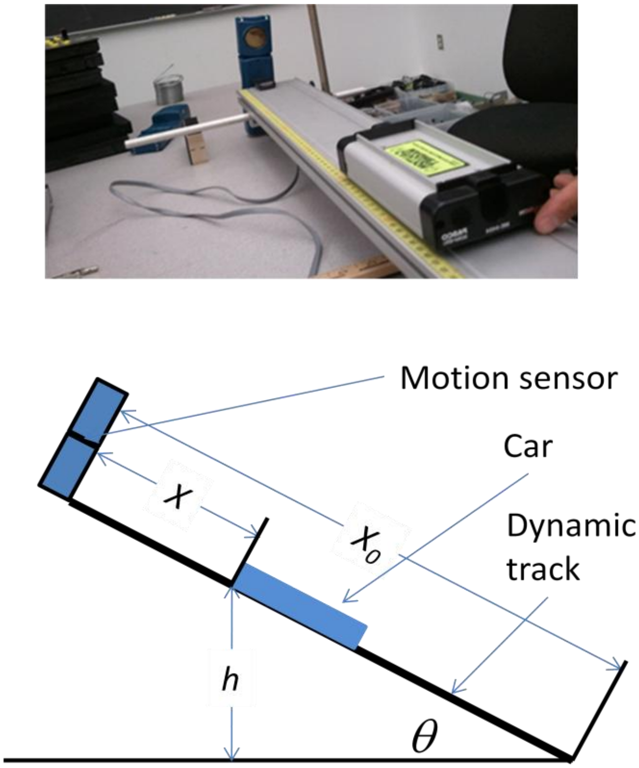

FIGURE 1. A picture and a schematic drawing of the experimental set-up. Parameters used in the text are defined in the schematic drawing.

The speed *v* can be read directly from Logpro, the software for motion sensor. Taking the leveled ground as the zero point, the gravitational potential energy *PE* has a value of *mgh*. The translational kinetic energy (*KE)* is $mv^2/2$. Here, *m* and *g* represent the mass of the car and free fall acceleration, respectively. The total mechanical energy *TotE* is the summation of *KE* and *PE*.

Students are required to perform three experiments: at incline angles of 6^o and 3^o without load on the car and at incline angle of 3^o with load. Then, values of *x*, *v(m/s)*, *h(m)*, *KE(J)*, *PE(J)*, and *TotE(J)* are either read from the software or calculated for each experiment and a table as shown in Table I is filled. The position *x* ranges from approximately 0.6m to 1.1m with a step of approximately 0.1m. *TotE* is used to evaluate the mechanical energy conservation. The value of *TotE* at the first recorded position was taken as the reference point in calculating energy difference. Taking the case of incline angle = 6^o as an example, the first recorded position is x = 0.607m. The energy percentage difference (% diff.) at x = 0.998(m) is calculated from [*TotE(x=0.998m)* – *TotE(x=0.607m)*]/*TotE(x=0.607m)*. As the car travels down incline, a positive % difference implies an apparent mechanical energy increase. Here, the word apparent is used to emphasize that mechanical energy increase in the present experimental setting should not happen.

III. UNEXPECTED DATA AND EXPLANATION

Out of eight student groups, six groups observed that the total mechanical energy remains the same or decreases. The results of these six groups are easy to understand. They simply correspond to the case of virtually no friction or some friction to cause the mechanical energy to decrease. Two groups observed apparent energy gain, which deserves further discussion. Since such data is not expected, the experiment was repeated by the present author and similar data was obtained. Table I shows typical result showing apparent mechanical energy increase. There are four features in such data set. First, the apparent energy gain is observed in both 6^o and 3^o experiments. Second, the % difference increases as the car travels further down incline. Third, the magnitude of the % difference is higher for the case of 3^o than that of 6^o. Fourth, for the two 3^o experiments, with and without load, the % differences are approximately the same.

TABLE I. A sample data set showing the apparent mechanical energy increase.

Incline Angle = 6°		Mass of Car = 0.502 kg		Mass of Load = 0 kg		Total Mass = 0.502 kg
x (m)	V (m/s)	h (m)	KE (J)	PE (J)	TotE (J)	% diff.
0.607	0.467	0.177	0.0547	0.870	0.925	[redacted]
0.691	0.657	0.168	0.108	0.827	0.936	1.1
0.803	0.846	0.156	0.180	0.770	0.949	2.6
0.895	0.972	0.147	0.237	0.722	0.959	3.7
0.998	1.10	0.136	0.304	0.669	0.973	5.2
1.114	1.24	0.124	0.386	0.610	0.996	7.6

Incline Angle = 3°		Mass of Car = 0.502 kg		Mass of Load = 0 kg		Total Mass = 0.502 kg
Position	V (m/s)	h (m)	KE (J)	PE (J)	TotE (J)	% diff.
0.605	0.337	0.0887	0.0285	0.436	0.465	[redacted]
0.688	0.488	0.0844	0.0598	0.415	0.475	2.1
0.800	0.633	0.0785	0.100	0.386	0.487	4.7
0.903	0.743	0.0731	0.139	0.360	0.498	7.2
0.981	0.804	0.0690	0.162	0.340	0.502	7.9
1.110	0.948	0.0623	0.226	0.306	0.532	14

Incline Angle = 3°		Mass of Car = 0.502 kg		Mass of Load = 0.5 kg		Total Mass = 1.002 kg
Position	V (m/s)	h (m)	KE (J)	PE (J)	TotE (J)	% diff.
0.595	0.318	0.0892	0.0507	0.876	0.927	[redacted]
0.699	0.509	0.0838	0.130	0.823	0.952	2.8
0.816	0.623	0.0777	0.194	0.763	0.957	3.3
0.885	0.732	0.0741	0.268	0.727	0.996	7.4
1.003	0.843	0.0679	0.356	0.666	1.022	10
1.091	0.923	0.0633	0.427	0.621	1.048	13

These summarized features lead to the prediction that this apparent mechanical energy gain is introduced by some systematic error, rather than random human errors. There are two possible explanations. First, it may be caused by the downward bowing of the dynamic track. Second, it may be introduced by the underestimation in reading the incline angle. A detailed explanation of these two ideas is provided in the following. Fig. 2 shows an exaggerated schematic drawing for the case of a bowed dynamic track. If the track is bowed, the car moves on the solid line arriving at a height *h(bowed)*. If the track is straight, the car moves on the dotted line, arriving at a height *h(straight)*. The speeds are denoted as *v(bowed)* and *v(straight)* for the two cases, respectively. Obviously, *h(straight)>h(bowed)* and *v(straight)<v(bowed)*. These are equivalent to state that at *x*, *PE(straight)>PE(bowed)* and *KE(straight)<KE(bowed)*. If no friction, *KE(straight)+PE(straight)* is equal to *KE(bowed)+PE(bowed)*. And both of them are equal to the total mechanical energy at the reference point, the first recorded position. However, unaware of the bowing, the total energy calculation in student experiment was performed by adding *KE(bowed)* and *PE(straight)*. *KE(bowed)+ PE(straight)* > *KE(bowed)* + *PE(bowed)* = *TotE*(at reference point), which explains the apparent mechanical energy gain. The magnitude of the apparent mechanical energy gain ΔE can be calculated as follows

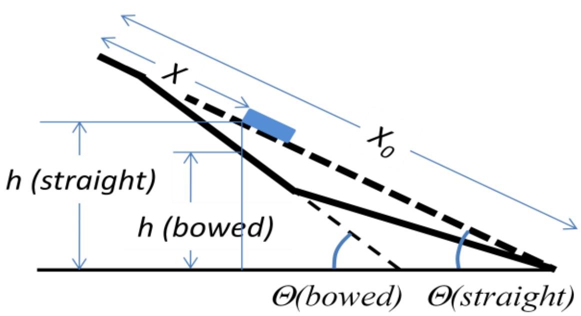

FIGURE 2. An exaggerated schematic diagram illustrating the effect of a downward bowed track.

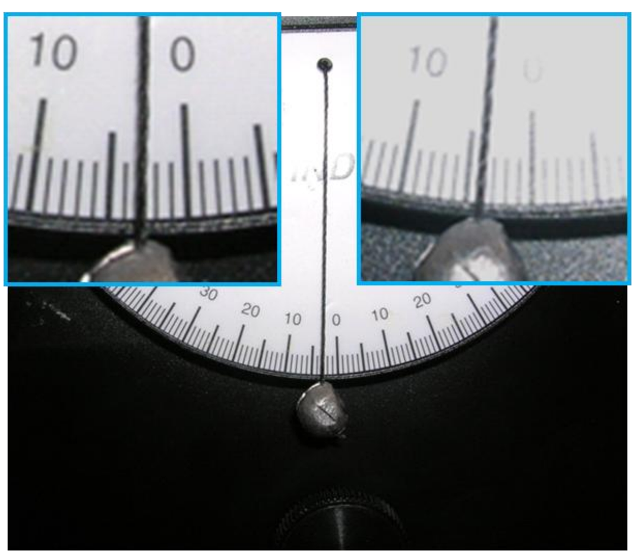

FIGURE 3. Reading of angle indicator at different locations.

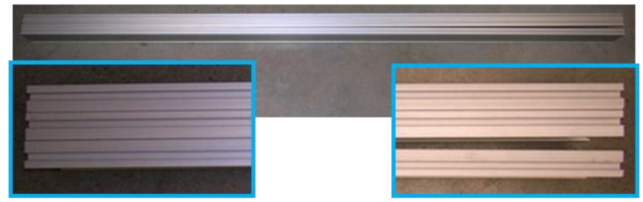

FIGURE 4. Pictures of three dynamic tracks placed next to each other.

$$\Delta E = [KE(bowed)+ PE(straight)] - [KE(bowed)+ PE(bowed)] = PE(straight) - PE(bowed) = mgx\cdot[\sin(\theta(bowed)) - \sin(\theta(straight))] \approx mgx\cdot\cos(\theta(straight))\cdot\Delta\theta. \quad (2)$$

$\Delta\theta = \theta(bowed) - \theta(straight)$ represent the incline angle difference caused by track bowing. These considerations explain the observed features well. First, since the apparent mechanical energy gain is caused by the bowing of the track, it should be independent of the nominal incline angle of the track. This is in agreement with the observed feature one. Based on Eq. (2), ΔE is linearly proportional to *x*, which explains the observed feature two. For a fixed $\Delta\theta$, ΔE increases with the decrease of $\theta(straight)$, which explains the enhanced effect for the case of 3^o relative to that of 6^o described above as feature three. It is believed that the bowing of the track pre-exist, independent of the incline angle and car load, explaining the observed feature four.

The second possible explanation to the apparent mechanical energy gain is as follows. Based on this idea, the track is straight, but the reading of the incline angle is lower than the true value $\theta(true)$ because of human error. If one replaces $\theta(bowed)$ with $\theta(true)$, the same argument described above holds true and the apparent contradict is solved. However, this idea is not favored because of the following reasons. First, the angle reading was checked by multiple persons. Second, remember that if a group observes apparent energy gain in 6^o experiment, the same group observes an enhanced effect in the 3^o experiment. It is hard to imagine a group of multiple members read the angles with lower values consistently in all their experiments. The degree of misreading can be estimated. For this purpose, different incline angles are tried in the calculation of *TotE* until the % difference is close to zero, which is expected for an ideal case. The angle that makes the apparent mechanical energy increase to be zero is called $\theta(fit)$. $\theta(fit)$ obtained from the data shown in Table I is 2^o higher than the recorded one, independent of the recorded angles are 6^o or 3^o. $\theta(fit)$ corresponding to the data of other group is in the order of 1^o different from the recorded one. With the instrument provided, such a misreading is unlikely.

Further efforts were made to confirm the bowing of the track. First, the bowing of suspicious track was examined by angle indicator, where the angle indicator was moved downward from the top and the angle change was monitored. A typical picture of the angle indicator is shown in Fig. 3. The upper left inset shows an enlarged picture when it is placed on top of the track for 3^o incline angle experiment. Another picture was taken when the angle indicator is approximately 0.8m away from the top and is shown as an inset in the upper right corner of Fig. 3. It is clear that the incline angle has changed from 3^o to 4^o, consistent with previous analysis. Further, the bowing was examined by lining up three tracks, two good ones and one "bad" one, next to each other as shown in Fig. 4. In the picture, the two good ones are on top and the "bad" one is on the lower part. The left ends have been pressed together so that there are no gaps between tracks. An enlarged picture of the left end is shown on the lower left corner of Fig. 4. On the right end, there is no gap between the two good tracks while a gap is observed between the "bad" one and the good ones. The inset on the lower right corner of Fig. 4 shows the gap clearly, illustrating the bowing of the "bad" dynamic track.

The results and analysis presented here is helpful for teaching. Although solid evidence can be provided as shown here in Figs. 3 and 4, one to two degree bowing will not be noticed to an unprepared mind. In performing mechanical energy conservation experiments, it is easy to remember and recognize the effect of friction, which causes the total mechanical energy to decrease. When faced with an apparent mechanical energy gain as described here, people become frustrated and tend to simply blame human errors. Further, teachers can actively use such results to remind students that all the analysis of experimental data is based on some assumptions, which are true only to certain extent [4]. The deformation of bowing is most likely caused by improper usage and handling. Therefore, the present result also serves as an reminder that extra care should be taken in using these instruments.

IV. CONCLUSION

In an energy conservation experiment of an assumed frictionless car travelling down an incline of a dynamic track, apparent mechanical energy gain up to approximately 10% was observed. The effect is more enhanced with the 3^o incline angle than that of 6^o. The magnitude of energy gain increases with the distance the car travels. The phenomenon is explained successfully as the effect of bowed track. This result is helpful to others who perform similar experiments.

REFERENCES

[1] Niculescu, A., *A Rolling Sphere Experiment*, Phys. Teach. **44**, 157-159 (2006).
[2] Taylor, R. and Noll, E. D., *Controlling All Variables in an Experiment*, Phys. Teach. **36**, 115–117 (1998).
[3] Calvini, P., *Dynamics of a tape that unwinds while rolling down an incline*, Am. J. Phys. **51**, 226-230 (1983).
[4] Amato, J. C. and Williams, R. E., *Turning a Common Lab Exercise into a Challenging Lab Experiment: Revisiting the Cart on an Inclined Track*, Phys. Teach. **48**, 322–323 (2010).