Effects of variable viscosity and thermal conductivity on heat and mass transfer flow along a vertical plate in the presence of a magnetic field

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Abstract

Heat and mass transfer flow along a vertical plate under the combined buoyancy force of thermal and species diffusion in the presence of a transverse magnetic field is investigated. The boundary layer equations are transformed in to ordinary differential equations with similarity transformations. The effects of variable viscosity and thermal conductivity on velocity profile, temperature profile and concentration profiles are investigated by solving the governing transformed ordinary differential equations with the help of Runge-Kutta shooting method and plotted graphically.

Keywords: Variable viscosity, thermal conductivity, magnetic field.

Resumen

Se investiga el calor y el flujo de transferencia de masa a lo largo de una placa vertical bajo la fuerza de empuje combinado de la difusión térmica y de las especies en presencia de un campo magnético transversal. Las ecuaciones de capa límite se transforman en las ecuaciones diferenciales ordinarias con las transformaciones de semejanza. Los efectos de la viscosidad variable y la conductividad térmica en el perfil de velocidad, perfil de temperatura y los perfiles de concentración son investigados resolviendo las ecuaciones diferenciales ordinarias transformadas con la ayuda del método de Runge-Kutta y se muestran gráficamente.

Palabras clave: Viscosidad variable, conductividad térmica, campo magnético.

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I. INTRODUCTION

Many transport processes occur in industrial applications in which the transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. Many studies have been reported for vertical, horizontal and inclined plate in presence of a transverse magnetic field. The same problem was solved by Elbashbeshy [1] without the effect of mass transfer, variable viscosity and thermal conductivity. In this study the induced magnetic field is neglected. Soundalgekar [2] studied the Effects of mass transfer on free convective flow of a dissipative incompressible fluid past an infinite vertical porous plate with suction. Bhadauria [3] also studied time periodic heating of Rayleigh Benard convection in a vertical magnetic field. Mostafa Mahmoud [4] has studied the variable viscosity and chemical reaction effects on mixed convection heat and mass transfer along a semi infinite vertical plate. Singh [5] presented paper on unsteady hydro magnetic free convection flow past a vertical infinite flat plate.

Recently Kafousias [6] presented an analysis of the effect of temperature-dependent viscosity on free convective laminar boundary layer flow past a vertical isothermal flat plate. Ganesan and Palani [7] studied numerical solution of transient free convection MHD flow of an incompressible viscous fluid flow past a semi infinite inclined plate with variable surface heat and mass flux. The set of governing equations are solved by using an implicit finite difference scheme. Kafousias [8] has studied the heat and mass transfer along a vertical plate in the presence of a magnetic field. Pantokratoras [9] has studied the free convection along a vertical, isothermal plate under the effect of a constant, horizontal, magnetic field. Takhar and Soundalgekar [10] have studied the dissipation effects on MHD free convection flow past a semi infinite vertical plate. Sparrow and Cess [11] presented their research work on the effect of a magnetic field on free convection heat transfer.

In most of the studies, of this type of problems, the viscosity and thermal conductivity of the fluid were assumed to be constant. However, it is known that these physical properties can change significantly with temperature and when the effects of variable viscosity and thermal conductivity are taken in to account, the flow characteristics are significantly changed compared to the constant property case. Hence the problem under consideration, the viscosity and thermal conductivity have been assumed to be inverse linear functions of temperature.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a steady free convection flow of an incompressible viscous electrically conducting fluid past a semi infinite vertical plate, at constant temperature T'_w, in presence of uniform transverse magnetic field. The temperature of the fluid far from the plate is T'_∞. The x axis is taken along the plate in upward direction and y axis is taken normal to it. The gravitational force is directed vertically downward. Since the velocity of the fluid is low, the viscous dissipative heat is assumed to be negligible. Also a magnetic field of constant intensity is assumed to be applied normal to the vertical plate and the electrical conductivity of the fluid is assumed to be small so that the induced magnetic field can be neglected in comparison to the applied magnetic field. In this paper the temperature and concentration of the fluid at infinity are considered to be T'_∞ and C'_∞ everywhere and the temperature and concentration at the plate are T_w and C_w respectively.

A large value of viscosity parameter θ_c implies either γ or $(T_w - T_\infty)$ is small, where T_w is the surface temperature and T_∞ is the temperature at infinity and the effects of variable viscosity can thus be neglected, on the other hand, for smaller values of θ_c, either the viscosity changes remarkably with or the operating temperature is high. We expect a similar dependence of thermal conductivity on temperature.

The governing equations of the fluid flow can be written as:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \,, \qquad (1)$$

$$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + \frac{\partial u'}{\partial y'}\frac{\partial \upsilon'}{\partial y'} + \beta g(T' - T_\infty)$$
$$+ \beta^* g(C' - C_\infty) - \frac{\sigma B^2}{\rho}u', \qquad (2)$$

$$u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \alpha\frac{\partial^2 T'}{\partial y'^2} + \frac{1}{\rho_\alpha C_p}\frac{\partial k}{\partial y'}\frac{\partial T'}{\partial y'}, \qquad (3)$$

$$u'\frac{\partial C'}{\partial x'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} + \frac{\partial C'}{\partial y'}\frac{\partial C'}{\partial y'} \,, \qquad (4)$$

where C' is the concentration, T' the temperature, (β, β^*) are the temperature and concentration coefficients of volumetric expansion, v is the kinematic viscosity, g gravitational acceleration, further (u', v') are the components of velocity along the (x', y') axes, α is the thermal diffusivity and B the magnetic field strength.

The Eqs. (2-4) must be solved subject to the boundary conditions:

$$\left.\begin{array}{l} \text{At} \quad y=0;\ u=v=0,\ T=T_w,\ C=C_w \\ \text{As} \quad y\to\infty;\ u=0,\ T=T_\infty,\ C=C_\infty. \end{array}\right\} \qquad (6)$$

The velocity components along the axes can be expressed as:

$$u' = \frac{\partial \psi'}{\partial y'}, \qquad v' = -\frac{\partial \psi'}{\partial x'}. \qquad (5)$$

Where ψ' is the stream function such that the continuity equation is satisfied.

$$U = \sqrt{g\beta l(T_w - T_\infty)}\,,$$

is a quantity with the dimension of speed and $Gc = g\beta L^3(T_w - T_\infty)/v^2$ is the Grashof number, here L is a typical length along the plate,

We introduce the following non-dimensional variables:

$$\left.\begin{array}{l} \psi' = x^{\frac{3}{4}} f(\eta), \qquad \eta = x^{-\frac{1}{4}} y, \qquad T = \theta(\eta), \\ C = \phi(\eta), \quad x = \frac{x'}{l}, \quad y = \frac{y'}{l}\,Gc^{\frac{1}{4}}, \qquad \psi = \frac{\psi' Gc^{\frac{1}{4}}}{UL}, \\ B^2 = B_o x^{\frac{1}{4}} \quad (B_0 \ \text{Constant}) \\ \theta = \frac{T' - T_\infty}{T_W - T_\infty}, \qquad \phi = \frac{C' - C_\infty}{C'_W - C_\infty}\ . \end{array}\right\} (7)$$

Here θ and ϕ are non-dimensional temperature and concentration. Viscosity and thermal conductivity are inverse linear functions of temperature, following Lai and Kulacki [12], we assume,

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \gamma(T - T_\infty)\right]$$

$$or \quad \frac{1}{\mu} = b\,(T - T_\infty), \quad \text{where} \ \ b = \frac{\gamma}{\mu_\infty}\,,$$

and $T_c = T_\infty - \frac{1}{\gamma},$

again $\frac{1}{k} = (\frac{1}{k_\infty}) \left[1 + k \quad T - T_\infty \right],$

or $\frac{1}{k} = \alpha \quad T - T_r$, where $\alpha = \frac{k}{k_\infty},$

and $T_r = T_\infty - \frac{1}{k}.$

where *b*, α, T_c, T_r are constants and their values depend on the reference state and thermal properties of the fluid i.e. γ and *k*. In general *b*, α >0 for liquids and *b*, α <0 for gases.

The non-dimensional form of viscosity and thermal conductivity parameters θ_c and θ_r can be written as:

$$\theta_c = \frac{T_c - T_\infty}{T_w - T_\infty} = \frac{1}{\gamma \quad T_w - T_\infty}, \qquad (8)$$

$$\text{and} \qquad \theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} \; . \qquad (9)$$

Substituting the Eqs. (7-9) in to Eqs. (2-4), we get

$$f''' = \left[\frac{3}{4} ff'' - \frac{1}{2} f'^2 - Mf' + e\phi \right] \frac{\theta - \theta_c}{\theta_c} + \frac{\theta'}{\theta - \theta_c} f'' \; , \qquad (10)$$

$$\theta'' = \Pr \frac{\theta' \;\; \theta - \theta_r}{\theta_r} \left[\frac{3}{4} f \right] + \frac{\theta'}{\theta - \theta_r} \; , \qquad (11)$$

$$\theta'' = \Pr \frac{\theta' \; \theta - \theta_r}{\theta_r} \left[\frac{3}{4} f \right] + \frac{\theta'}{\theta - \theta_r} \; . \qquad (12)$$

The boundary conditions with the new variables are:

$$\left.\begin{array}{l} \eta=0, \quad f=f'=0, \quad \theta=\varphi=1, \\ \eta\to\infty, \quad f'=0, \quad \theta=\varphi=0. \end{array}\right\} \qquad (13)$$

Eqs. (10) to (12) with boundary conditions (13) describe the heat and mass transfer along a vertical pate in the presence of a magnetic field under variable viscosity and thermal conductivity.

In Eq. (10) $e = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)}$ represents the relative effect of chemical diffusion on thermal diffusion. When *e* = 0 there is no mass diffusion and the buoyancy force arises solely from the temperature difference. Prime denotes differentiation with respect to η.

The governing Eqs. (10) to (12) with the boundary conditions given by Eq. (13) are solved numerically by using the fourth order Runga-Kutta Shooting method.

III. SKIN-FRICTION COEFFICIENT, NUSSELT NUMBER AND SHERWOOD NUMBER

The parameters of engineering interest for the present problem are the local friction coefficient, local Nusselt number and the local Sherwood number which indicate physically wall shear stress, rate of heat transfer and rate of mass transfer respectively.

The equation defining the wall skin friction is

$$C_f = \frac{2\tau_w}{\rho U_\infty{}^2} \qquad (14)$$

where

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} = \mu U_\infty f''(0)(xGc)^{\frac{1}{4}} \frac{1}{L}$$

Finally the skin friction coefficient is given by

$$C_f \sqrt{R_e} = \frac{2\theta_c}{\theta_c - \theta} f''(0) x^{\frac{5}{4}} Gc^{\frac{1}{4}}, \qquad (15)$$

The heat transfer coefficient is given by

$$Nu = \frac{xq_w}{k\Delta T}. \qquad (16)$$

where the heat flux (q_w) at the wall is given by

$$q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0} = -k\theta'(0) \;\; T_w - T_\infty \;\; \left(\frac{Gc}{x} \right)^{\frac{1}{4}} \frac{1}{L}.$$

Hence the Nusselt number is

$$Nu = -\theta'(0) Gc^{\frac{1}{4}} x^{\frac{3}{4}} \frac{1}{L}. \qquad (17)$$

And the Sherwood number is given by

$$Sh = \frac{xM_w}{D_M \Delta C} = -\phi' x^{\frac{3}{4}} Gc^{\frac{1}{4}}. \qquad (18)$$

IV. RESULTS AND DISCUSSION

The systems of differential Eqs. (10) to (12) governed by the boundary conditions (13) are solved numerically by applying an efficient numerical technique based on the fourth order Runga-Kutta shooting method and an iterative method. The numerical method can be programmed and applied easily. It is experienced that the convergence of the iteration process is quite rapid.

Solutions were obtained for Pr = 0.73, and various values of θ_c, θ_r, *M*, Ec and *Sc* respectively. The viscosity-temperature variation and the conductivity-temperature variation are represented by the dimensionless parameter θ_c and θ_r respectively whereas the magnetic field and the concentration field effect are represented by the dimensionless parameter *M* and Sc respectively. When the temperature difference $\Delta T = T'_w - T'_\infty$ positive in our case, the viscosity-temperature parameter θ_c as well as conductivity-temperature parameter θ_r are negative for fluids and positive for gases [12].

The concept of θ_c was first introduced by Ling and Dybbs [13] on their study of forced convective flow in porous medium. The viscosity and conductivity temperature equations can be written as:

$$\mu = \mu_\infty / (1 - \theta_c^{-1}),$$

$$\mathrm{k} = \mathrm{k}_\infty / (1 - \theta_r^{-1}).$$

It is obvious from this expression that for physical quantity θ_c and θ_r cannot take the values 0 and 1. It is experienced that when θ_c and are large, viscosity and thermal conductivity variation in the boundary layer is negligible, but as θ_c and $\theta_r \to 1$ the viscosity and thermal conductivity variation become increasingly significant.

In Fig. 1 we are substituting the values for different parameters like Prandtl number Pr = 0.73, magnetic field parameter *M* = 0.1, ratio of thermal diffusivity to concentration diffusivity *e* = 0.1, thermal conductivity parameter θ_r = -10. Substituting different values of the viscosity parameter θ_c = -12,-10,-8,-6 we observe that the velocity profiles decreases with increasing of θ_c.

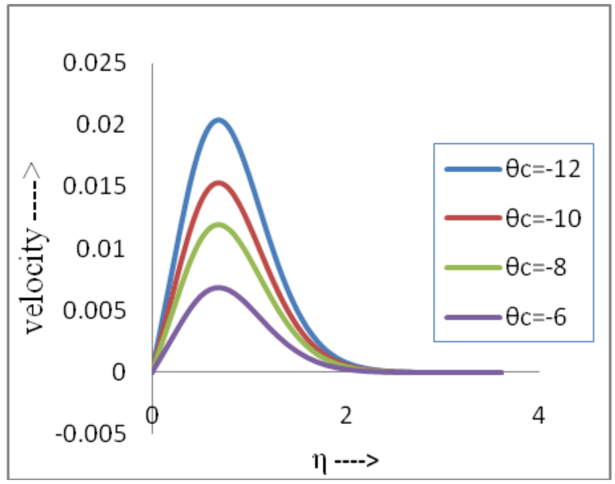

FIGURE 1. Variation of velocity profile with θ_c.

The effect the variable viscosity is not prominent in case of temperature profile. In the Fig. 2 we are observing the effect of concentration profile with the variation of Schmidt number Sc. The values of Sc = 3.10, 4.10, 5.10, 7.10, 9.10 with the values of other parameters Pr = 0.73, Magnetic field parameter *M* = 0.1, ratio of thermal diffusivity to concentration diffusivity *e* = 0.1, thermal conductivity parameter θ_r = -10.

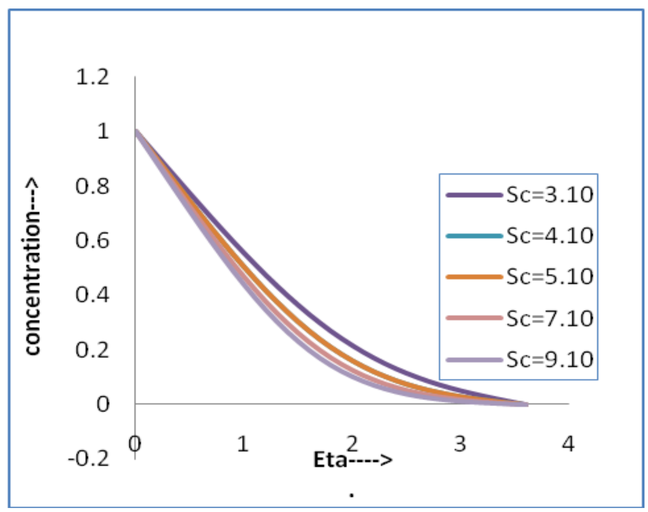

FIGURE 2. Effect of Sc on cocncentration profile.

A rise in Sc strongly suppresses concentration levels in the boundary layer regime. All profiles decay monotonically from the surface (wall) to the free stream. *Sc* embodies the ratio of momentum diffusivity (*v*) to molecular diffusivity (*D*). It is conclude that, an increase in Sc, the concentration decreases. It is observed that the fluids concentration decreases as the mass transfer parameter Sc increases. In the Fig. 3 we study the variations of the velocity profile for various values of the variable thermal conductivity. Here we substitute values of thermal conductivity like θ_r = -12, -10,

-8 and other parameters M = 0.1, e = 0.1, Sc = 1, θ_c = -10, Pr = 0.73 and finally observe that the velocity profile increases with the decrease of the thermal conductivity parameter.

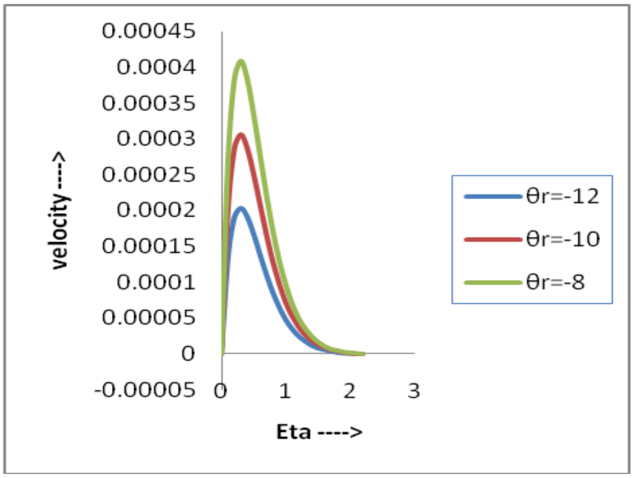

FIGURE 3. Variation of velocity profile with θ_r.

In the Fig. 4 we study the effect of θ_c the variable viscosity parameter on concentration profile. The values of variable viscosity θ_c = -10, -3, -1 has been considered and the other parameters are taken as M = 0.1, Sc = 1, Pr = 0.73, θ_r =-10, e = 0.1. And it is observed that the concentration profile decreases with the increase of the variable viscosity parameter.

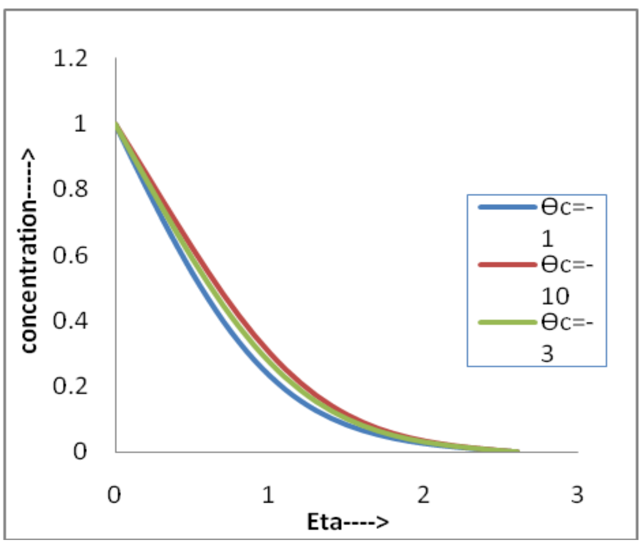

FIGURE 4. Variation of concentration profile with θ_c.

In the Fig. 5 the temperature profile for various values of Prandtl number has been studied. Assuming the values of Pr= 3.73, 5.73, 9.73. Also other parameters are taken as θ_r=-10, θ_c = -10, M = 0.1, e = 0.1 and Sc = 1. It is observed that the temperature profile decreases with the increase of the Prandtl number Pr.

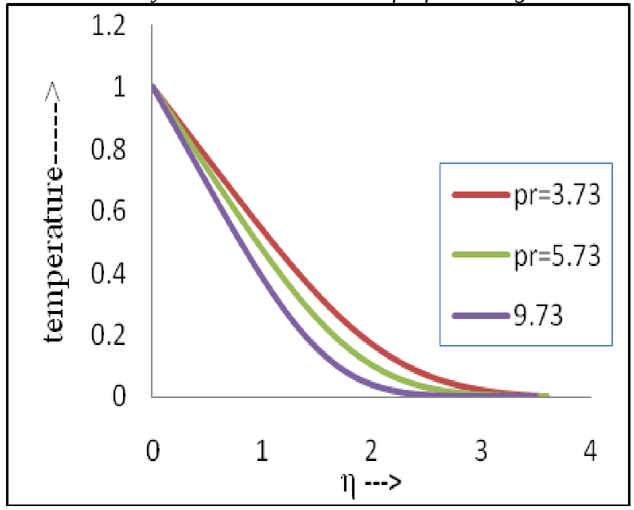

FIGURE 5. Variation of temperature with Pr.

In the Fig. 6 the effect of magnetic field parameter on velocity profile has been studied. We substitute various values of the magnetic field parameter M = 0.3, 0.7, 1, 1.3 and the other values of the parameter has been considered as Pr = 0.73, θ_c = -10, θ_r = 10, E = 0.8, Sc = 1. The velocity profile decreases with the increase of the magnetic field Parameter.

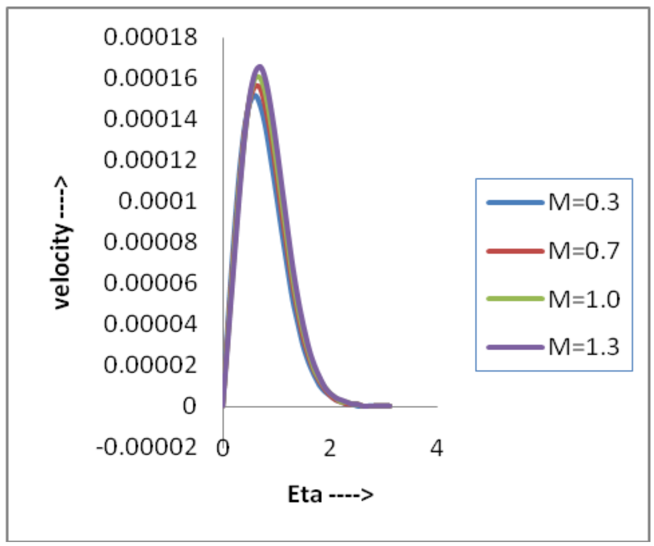

FIGURE 6. Variation of velocity profile with *M.*

The effect of magnetic field parameter M is not prominent in case of variable viscosity parameter θ_r. In the Fig. 7 we observe the effect of Hartmann number M on concentration profile φ. Substituting various values for M = 3.01, 5.31, 7.31, 9.91 and Pr =0.73, Sc=1, θ_c = -10, e=0.1 it is observed that the concentration profile decreases with the increase of magnetic parameter M. Observation is very analogous with the theory because due to the transverse magnetic field a drag force is developed which opposes the flow.

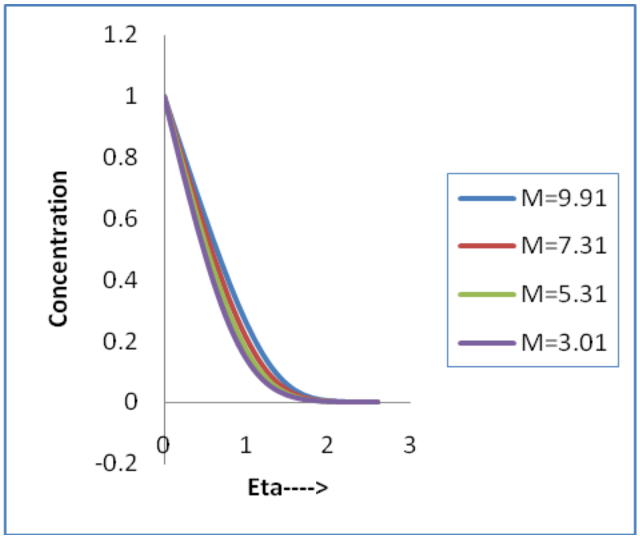

FIGURE 7. Variation of concentration profile with *M*.

It has been investigated that the concentration profile increases with decrease of magnetic field parameter. In fig.-08 the variations of the concentration profile has been observed with the change of Prandtl number Pr. The study reveals that concentration increases with the increase of pr.

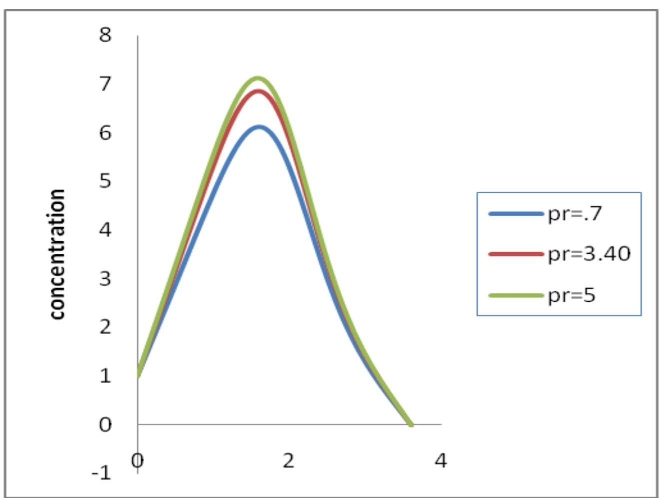

FIGURE 8. Effect of pr on concentration profile.

The variation of skin friction C_f which is also called the drag force is shown for various values of Grashof number Gc, in Fig. 9. The Grashof number in initial stage increases steeply to Gc = 0.07 than the later part which implies that buoyancy force is large within this region.

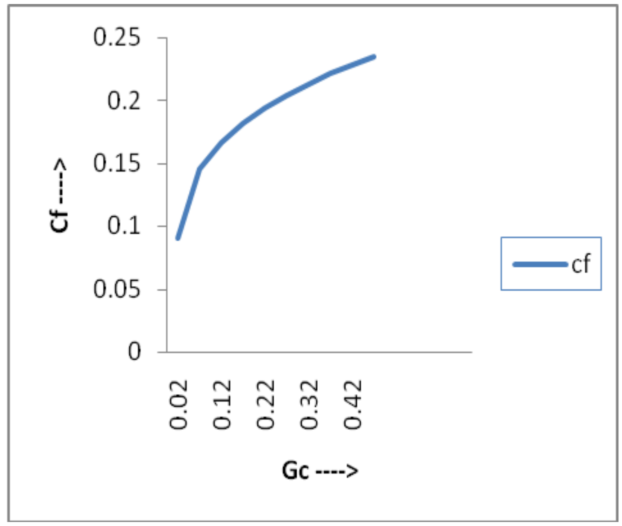

FIGURE 9. Variation of skin friction with Gc.

Missing values of $f''(0), \theta'(0)$ and $\phi'(0)$ for various values of θ_c, θ_r, M, Pr, Sc have been derived. In Table I missing values of $f''(0), \theta'(0)$ and $\phi'(0)$ were found for θ_c = -10, -9, -8, -7, -6, -5, -4, -3, -2, -1. And it is observed that the missing values of $f''(0)$ and $\phi'(0)$ increases while that of $\theta'(0)$ decreases. In Table II it is observed that while increasing the values of θ_r =-10, -9, -8, -7, -6, -5, -4, -3, -2, -1 the missing values of all $f''(0)$, $\theta'(0)$ and $\phi'(0)$ decreases. In Table III it is observed that for increasing values of Pr = 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1, 1.01 the missing values of all $f''(0)$, $\theta'(0)$ and $\phi'(0)$ decreases. In Table IV we observe the missing values of $f''(0), \theta'(0)$ and $\phi'(0)$ for Sc = 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90,1. The study reveals that the missing values of $f''(0)$ and $\phi'(0)$ increases while that of $\theta'(0)$ decreases.

TABLE I. Missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for various values of θ_c.

θ_c	$f''(0)$	$\theta'(0)$	$\phi'(0)$
-10	1.010568	-0.428208	0.690980
-9	1.015436	-0.428262	0.704467
-8	1.020852	-0.428323	0.719720
-7	1.026914	-0.428390	0.737075
-6	1.033745	-0.428466	0.757042
-5	1.041502	-0.428553	0.807498
-4	1.050390	-0.428652	0.839236
-3	1.060680	-0.428767	0.867947
-2	1.072490	-0.428895	0.761868
-1	1.078068	-0.425623	0.332301

TAB LE II. Missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for different values of θ_r.

θ_r	$f''(0)$	$\theta'(0)$	$\phi'(0)$
-10	1.010568	-0.428208	0.690980
-9	1.009744	-0.429265	0.689085
-8	1.008832	-0.430436	0.686999
-7	1.007819	-0.431740	0.683688
-6	1.006686	-0.433201	0.682115
-5	1.005412	-0.434851	0.679234
-4	1.003968	-0.436726	0.675985
-3	1.002317	-0.438878	0.672291
-2	1.00412	-0.441373	0.668056

TABLE-III. Missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for various values of Pr.

Pr	$f''(0)$	$\theta'(0)$	$\phi'(0)$
0.20	0.991532	-0.410215	0.652332
0.30	0.922204	-0.413698	0.644862
0.40	0.984910	-0.417615	0.637557
0.50	0.981648	-0.420629	0.630411
0.60	0.978418	-0.424086	0.623419
0.70	0.975219	-0.430978	0.616577
0.80	0.972051	-0.434413	0.609879
0.90	0.968913	-0.437841	0.603320
1.00	0.965806	-0.441263	0.596897
1.01	0.962727	-0.444677	0.590604

TABLE IV. Missing values of $f''(0)$, $\theta'(0)$ and $\phi'(0)$ for various values of Sc.

Sc	$f''(0)$	$\theta'(0)$	$\phi'(0)$
0.10	0.966661	-0.427368	-0.365875
0.20	0.96696	-0.427369	-0.361054
0.30	0.96732	-0.427369	-0.356191
0.40	0.966769	-0.427370	-0.351281
0.50	0.966806	-0.427371	-0.346337
0.60	0.966843	-0.427371	-0.341344
0.70	0.966880	-0.427372	-0.336306
0.80	0.966919	-0.427373	-0.331222

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