Folding three-dimensional model of equipotential surfaces

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Abstract
We present a new model of equipotential surfaces. We can easily construct it at a low cost. Using the model, students come to realize that the gradient of the electric potential is directly proportional to the density of lines of force, and hence the electric field intensity.

Keywords: Equipotential surface, Lines of force.

Resumen
Se presenta un nuevo modelo de superficies equipotenciales. Se puede construir fácilmente a bajo costo. Usando el modelo, los estudiantes llegan a darse cuenta de la relación entre "líneas de fuerza" y "superficies equipotenciales".

Palabras clave: Superficie Equipotencial, Línea de fuerza.

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I. INTRODUCTION

In introductory courses on electricity and magnetism [1, 2, 3], one of the hardest problems for students is the visualization of 'lines of force' from charged particles and their attendant 'equipotential surfaces'. We present here a three-dimensional model of equipotential surfaces in the vicinity of two equal positive charges as shown in Fig. 1.

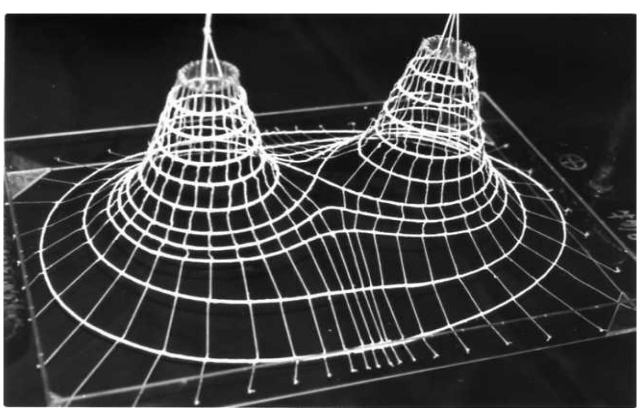

FIGURE 1. Photograph of the model for showing equipotential surfaces for two equal like charges.

Models of such surfaces have been built, in various media [4], for over a hundred years. For example, using the contour map shown in Fig. 2 and a polystyrene board elements reproducing the different contours can be cut out and glued together to form the potential surfaces, or plaster of Paris could be used. As these models are bulky, the model below has been developed to simplify storage.

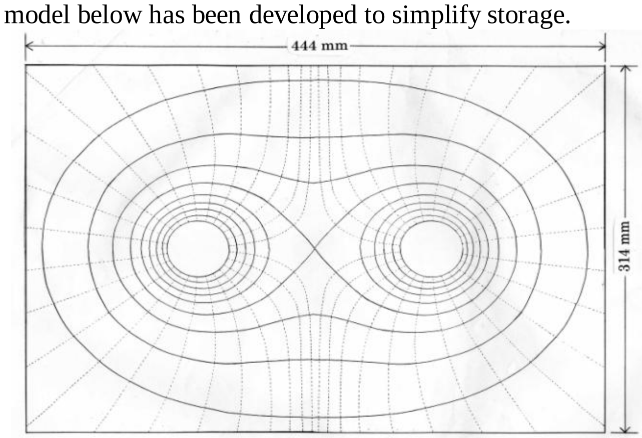

FIGURE 2. Lines of force (broken curves) and equipotential surfaces (full curves) for two equal like charges.

II. CONSTRUCTION

Let us now describe the construction of this model. An acrylic plate is cut into two strips (A: 40mm × 320mm) and two additional strips (B: 40mm × 444mm) and small holes are drilled into each strip. The position of the drilled holes is shown in Fig. 3.

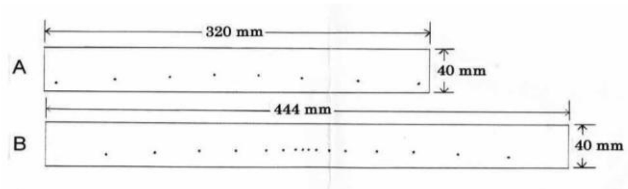

FIGURE 3. The position of the drilled holes in each strip.

A frame is constructed using these four strips glued together with ethylene dichloride. The four corners of the frame are reinforced with four triangular acrylic plates (see Fig.4).

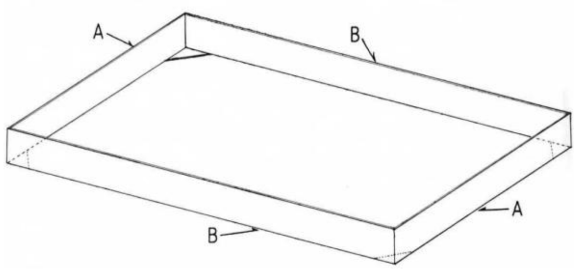

FIGURE 4. The acrylic frame.

Wires of diameter 1mm are shaped to the equipotential surfaces, with reference to Fig. 2, which shows equipotential surfaces (full lines) for two equal like charges, and both ends of each wire are soldered. In this way, the outlines of 15 surfaces are made by wires. Then, 48 lengths of kite string are provided. One end of each string is passed through each hole in the frame and knotted to it. The wire-shaped equipotential surfaces are then connected by the length of string so that the strings are normal to the wires and the difference in potential between the two nearest equipotential surfaces corresponds to 20mm in height with reference to Fig. 2. The potential is zero at the bottom of the frame, and the broken lines in Fig. 2 represent lines of force. The other end of each string is connected to an acrylic disk 48mm in diameter. Lastly, the wires and strings are painted white so that they can be seen clearly.

When the model is used, each of the two disks is suspended by another string from a strong rigid support. When the model is collapsed for storage, all surfaces fit into the 320mm × 450mm × 40mm frame.

For two equal unlike charges, we can construct a model like that shown in Fig. 5.

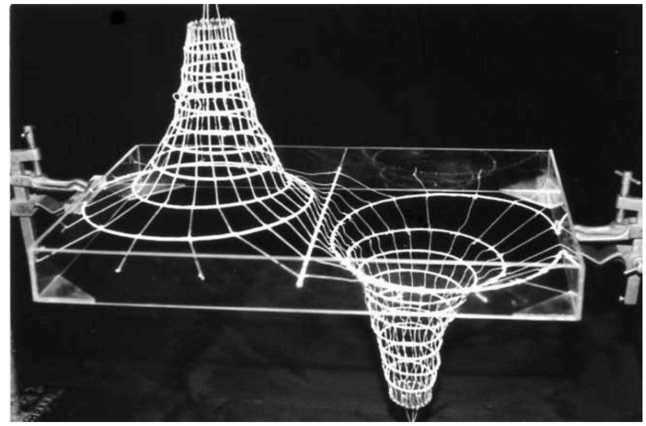

FIGURE 5. Photograph of the model for showing equipotential surfaces for two equal unlike charges.

III. PROGRAM

FIGURE 2 is obtained by the following program.

$$q=1000\,;\ \varepsilon_0=8.854\times10^{-12}\,;\ a=80\,;\ \hookleftarrow$$

$$f_1=\text{ContourPlot}[\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{(x-a)^2+y^2}}+\frac{1}{\sqrt{(x+a)^2+y^2}}\right),$$
$$\{x,-222,222\},\{y,-157,157\},\ \text{AspectRatio}\rightarrow\text{Automatic},$$
$$\text{ContourShading}\rightarrow\text{False},\text{Contours}\rightarrow 9,$$
$$\text{PlotPoints}\rightarrow 1000],\ \hookleftarrow$$

$$f_2=\text{ContourPlot}[\left(\frac{x-a}{\sqrt{(x-a)^2+y^2}}+\frac{x+a}{\sqrt{(x+a)^2+y^2}}\right),$$
$$\{x,-222,222\},\{y,-157,157\},\ \text{AspectRatio}\rightarrow\text{Automatic},$$
$$\text{ContourShading}\rightarrow\text{False},\text{Contours}\rightarrow 24,$$
$$\text{PlotPoints}\rightarrow 1000],\hookleftarrow$$

$$\text{Show}[f_1,\ f_2],\hookleftarrow$$

The program is obtained by computer software "Mathematica" [5]. Where q is a charge, ε_0 is the permittivity of free space, a is half of the distance between two charges, f_1 is the expression for 'equipotential surfaces', and f_2 is the expression for 'lines of force'.

The position of the drilled holes in each strip in Fig. 3 is also obtained by computer software "Mathematica".

IV. CONCLUSION

We present the models for showing equipotential surfaces for two equal like charges and two equal unlike charges. As they are folding models, they take up less space. We can put them into the acrylic frame.

The models show us the following facts:

1. The gradient of the electric potential is directly proportional to the density of lines of force.
2. The electric potential has a constant value as we move along a direction perpendicular to lines of force.
3. For two equal like charges the potential is zero at the infinite point, and for two equal unlike charges the potential is zero at the axis of symmetry.

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