A question about anti-reflective coating

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Abstract

We discuss the electric fields and light energy reflected by anti-reflective coating and the interference of the electric fields. By emphasizing that the light energy is determined by the total, rather than individual, electric field, we clarify the confusion about how the anti-reflective coating increases the transmission energy. An example shows that the main electric fields of destructive interference on a coated surface are from the first and second reflections.

Keywords: Anti-reflective coating, interference, electric field, light energy.

Resumen

Se discuten los campos eléctricos y energía de la luz reflejada por la capa anti-reflejo y la interferencia de los campos eléctricos. Al hacer hincapié en que la energía de la luz es determinada por el total, en lugar de un campo eléctrico individual, se puede aclarar la confusión acerca de cómo la capa anti-reflejo aumenta la energía de transmisión. Un ejemplo muestra que los primeros campos eléctricos de interferencia destructiva sobre una superficie cubierta son de las primeras y segundas reflexiones.

Palabras clave: Capa Anti-reflejante, interferencia, campo eléctrico, energía de la luz.

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It is well known that the anti-reflective coating can reduce the reflection greatly, and in the ideal situation, can reduce the reflection by 100% [1]. Such coatings are important in modern optical equipment since it usually consists of quite a few lenses that cause reflection loss. The simplest form of anti-reflective coating was discovered by Lord Rayleigh in 1886. The optical glass available at the time tended to develop a tarnish on its surface with age, due to chemical reactions with the environment. Rayleigh tested some old, slightly tarnished pieces of glass, and found to his surprise that they transmitted more light than new, clean pieces. Interference-based coatings were invented in November 1935 by Alexander Smakula (1900, Ukraine–17 May 1983, Auburn, Massachusetts, USA), who was working for the Carl Zeiss optics company. Anti-reflection coatings were a German military secret until the early stages of World War II [2]. The physical principle behind the interference-base coatings is the destruction interference of the reflection light. However, a question arises: how can the reflection be reduced by the interference as the light has been reflected at the first and then interfere after that? [3]. In other words, the light is already reflected before the interference occurs so the reflection cannot be reduced.

The answer here is: there is essential difference between electric field, E, of the light and the energy of light, $u = \varepsilon E_2$. (Actually here u is the energy density). In Fig. 1, suppose that the electric fields for reflection beams are: $E_1, E_2, E_3, \ldots$, then the sum $E_{total} = \Sigma_i \mathrm{E}_i$ can be zero when their amplitudes and phases are right, since it is a vector sum. But the sum of the energy $U = \Sigma_i u_i$ can never be zero because it is a scalar sum and each term is positive. Therefore, when calculating light energy at certain point in space, one must use the E_{total}, instead of getting energy u_i from each component of E_{total}. That is, we cannot say that there is energy u_1 for E_1, energy u_2 for E_2, energy u_3 for $E_3, \ldots$ Rather we must say that there is an electric field $E_{total} = \Sigma_i E_i$ and energy there is $u = \varepsilon E_{total}^2$ Actually we can see in mathematic:

$$u = \varepsilon E_{total}^2 = \varepsilon\left(\mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \ldots\right)^2,$$

$$\neq \varepsilon\left(\mathbf{E}_1^2 + \mathbf{E}_2^2 + \mathbf{E}_3^2 + \ldots\right) = u_1 + u_2 + u_3 + \ldots,$$

Here we consider an example, shown in Fig. 1 that a light beam, with electric field E_0 and energy u_0, is incident from air to be reflected by the interface of air/MgF_2 and interface of MgF_2/Crown glass. The indices of three media are $n_0 = 1$ (air), $n_1 = 1.38$ (MgF_2) and $n_2 = 1.52$ (crown glass) respectively, and the reflection coefficients between n_0 and n_1, and n_1 and n_2 are r_1 and r_2, respectively. That is:

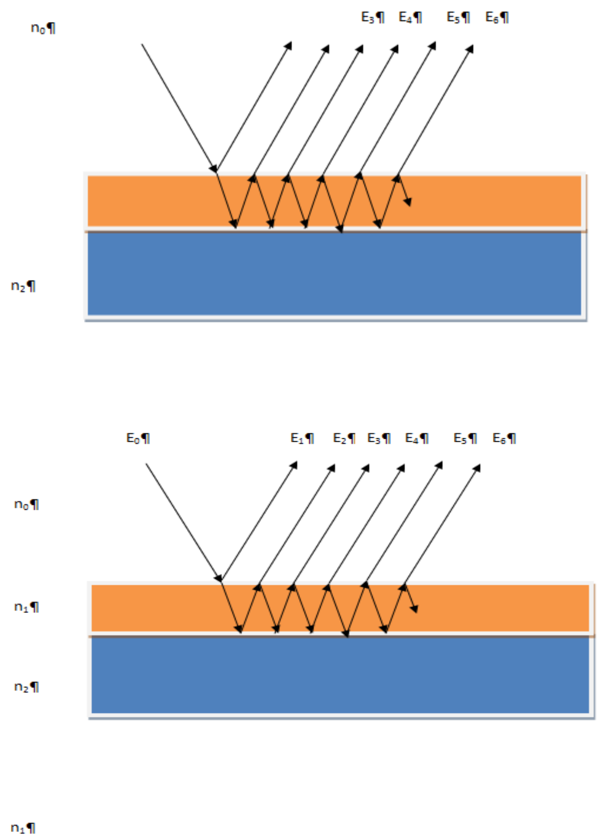

FIGURE 1. Light beam is reflected many times on two interfaces; reflected light beams interfere to form constructive/destructive pattern. Notice that the reflection is electric field instead of energy.

$$r_1 = \left(\frac{n_1 - n_0}{n_1 + n_0}\right)^2 = \left(\frac{1.38 - 1}{1.38 + 1}\right)^2 = 0.160^2 = 0.0256\,,$$

$$r_2 = \left(\frac{n_2 - n_1}{n_1 + n_2}\right)^2 = \left(\frac{1.52 - 1.38}{1.38 + 1.52}\right)^2 = 0.0483^2 = 0.00233\,.$$

Also the transmission coefficient from n_1 to n_0 (same as from n_0 to n_1) is:

$$T = 1 - r_1 = 0.9744\,.$$

Therefore, the electric fields of reflected beams are:

$$E_1 = \sqrt{r_1}E_0\,,$$
$$E_2 = \sqrt{r_2}TE_0\,,$$
$$E_3 = \sqrt{r_1 r_2}TE_0\,,$$
$$E_4 = r_1 r_2^{3/2}TE_0\,,$$
$$\ldots$$
$$E_{n+1} = \sqrt{r_1 r_2}E_n\,.$$

Substituting values of r_1, r_2, and T into them yields $E_1 = 0.160E_0$, $E_2 = -0.0470E_0$, $E_3 = 0.000363E_0$… In the second and third columns of Table I, the electric fields as the order of reflections are listed with $n_1 = 1.38$ and 1.233 respectively. We can see that E decreases fast as the reflections go on. Actually only the first 2 terms are important in the interference – that's why some textbooks just consider the interference of the first two reflections [4].

TABLE I. The reflected electric fields change with the number of reflections.

number of reflection	relfected E ($n_1 = 1.38$)	reflected E ($n_1 = 1.233$)
1	0.159663866	0.104295797
2	-0.047045187	-0.10316353
3	-0.00036262	-0.001122197
4	-2.79504E-06	-1.22071E-05
5	-2.15E-08	-1.33E-07
6	-1.66059E-10	-1.44444E-09
7	-1.28E-12	-1.57E-11
8	-9.86587E-15	-1.70917E-13
9	-7.60E-17	-1.86E-15
10	-5.8615E-19	-2.02241E-17
11	-4.52E-21	-2.20E-19
12	-3.48243E-23	-2.39307E-21
13	-2.68E-25	-2.60E-23

(Note: Negative values of E mean the direction is opposite to the E in the first reflection).

In the case that $n_1 = 1.38$, the sum of the reflected E is $0.112E_0$ which produces the energy of $u = 0.0128u_0$. Without the anti-reflective coating, however, the reflection E would be is $0.206E_0$ which produces the energy of $u = 0.0426u_0$. We can see with this coating the reflected energy is about 33% of that of uncoated surface. In the case that n_1 is the geometrical mean of n_0 and n_2 [5],

$$n_1 = \sqrt{n_0 n_2} = 1.233\,,$$

(if we can find material with such a low index of refraction), then

$$r_1 = \left(\frac{n_1 - n_0}{n_1 + n_0}\right)^2 = \left(\frac{1.233 - 1}{1.233 + 1}\right)^2 = 0.1043^2 = 0.01088\,,$$

$$r_2 = \left(\frac{n_2 - n_1}{n_1 + n_2}\right)^2 = \left(\frac{1.52 - 1.233}{1.233 + 1.52}\right)^2 = 0.1043^2 = 0.01088\,,$$

$$T = 1 - r_1 = 0.98912\,.$$

Then the sum of the reflected E is $2.27\times10^{-6}E_0$ which produces the energy of $u = 5.16\times10^{-12}u_0$. In theory the reflected energy should be exactly zero so here the nonzero value is caused by numerical calculation error. In the 3rd column of Table 1, we can see that still, the first and 2nd reflections give the most important electric fields.

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