Uniform motion on the arbitrary trajectory

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Abstract

In this paper we show that, it's possible the uniform motion on the arbitrary trajectory and number of examples that can be cited for uniform motion is infinite. As an application of this method, we can conclude uniform circular motion as an especial state. Also as another application, we investigate Archimedean spiral so that particle moves by constant speed on this path. We believe, this procedure illustrates the application of simple mathematical tools to physics, a barrier that some student have difficulty to overcome.

Keywords: Uniform motion, uniform circular motion, Archimedean spiral.

Resumen

En este trabajo se muestra que, es posible que el movimiento uniforme sobre la trayectoria arbitraria y el número de ejemplos que pueden citarse del movimiento uniforme es infinito. Como aplicación de este método, podemos concluir que el movimiento circular uniforme es un estado especial. También como otra aplicación, investigamos la espiral de Arquímedes así que la partícula se mueve con velocidad constante en este camino. Creemos, que este procedimiento ilustra la aplicación de herramientas matemáticas simples para la física, una barrera que algunos estudiantes tienen dificultad para superar.

Palabras clave: Movimiento uniforme, Movimiento circular uniforme- Espiral de Arquímedes.

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I. INTRODUCTION

In classical mechanics uniform motion of the motion is said to be constant speed. Generally in the textbooks this kind of motion by the example of uniform circular motion is introduced [1, 2]. It's true that when students are faced with this discussion, they will have a question. If number of examples about uniform motion is limited? We shall show that, according to the mathematical nature of this motion, not only number of examples is limited but they are infinite.

II. METHOD

We insist to use a mathematical approach. Because this procedure illustrates the application of simple mathematical tools to physics, a barrier that some student have difficulty to overcome.

Suppose a particle moves at the arbitrary curve by

$$y = f(x).$$

And with speed V. the velocity vector is

$$\vec{V} = V_x\hat{\imath} + V_y\hat{\jmath}.$$

The vertical component of velocity can be writing

$$V_y = \frac{dy}{dt} = \frac{dy}{dx}\frac{dx}{dt} = f(x)'V_{x.} \qquad (1)$$

Based on the relation (1), we get the speed of particle as follow

$$V = |\vec{V}| = \sqrt{V_x{}^2 + V_y{}^2} = V_x\sqrt{1 + (f(x)')^2}. \qquad (2)$$

Then the components of velocity are as follow

$$V_x = \frac{|\vec{V}|}{\sqrt{1 + (f(x)')^2}}, \qquad (3)$$

$$V_y = \frac{f(x)'|\vec{V}|}{\sqrt{1 + (f(x)')^2}}. \qquad (4)$$

By the same way the components of acceleration are achieved

$$a_x = -\frac{|\vec{V}|^2 f(x)' f(x)''}{(1 + (f(x)')^2)^2}, \qquad (5)$$

$$a_y = \frac{\left|\vec{V}\right|^2 f(x)''}{(1+(f(x)')^2)^2}\,. \qquad (6)$$

III.APPLICATIONS

A. Linear path

Suppose a particle moves at straight line by $y = mx + b$ With slope b and m is constant. The speed of this particle is V. Then according to relation (3) we get

$$\frac{V}{\sqrt{1+m^2}}\,.$$

Since the second derivative of equation of path is zero so

$$a_x = 0\,.$$

We can conclude in uniform motion on a straight line acceleration is always zero.

B. Uniform circular motion

We consider a particle moves at constant speed V in a circular path with radius of R. As we know, both the velocity and acceleration are constant in magnitude, but both change their directions continuously [1]. This situation is called uniform circular motion. The equation of path is

$$f(x) = (R^2 - x^2)^{\frac{1}{2}}. \qquad (7)$$

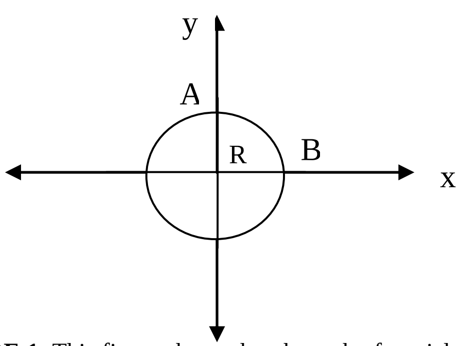

FIGURE 1. This figure shows that the path of particle that moves with constant speed.

We calculate the components of velocity and acceleration in the point of A(0, R)

$$V_x = V \quad , \quad V_y = 0,$$
$$a_x = 0 \quad , \quad a_y = -\frac{V^2}{R}. \qquad (8)$$

Also the components of velocity in the point of B(R, 0) are

$$V_x = 0 \quad , \quad V_y = -V \quad ,$$
$$a_x = -\frac{V^2}{R} \quad , \quad a_y = 0\,. \qquad (9)$$

C. Archimedean spiral

The Archimedean spiral is a spiral named after the 3rd century BC Greek mathematician Archimedes. It is the locus of points corresponding to the locations over time of a point moving away from a fixed point with a constant speed long a line which rotates with constant angular velocity [2]. Following figure shows that The Archimedean spiral.

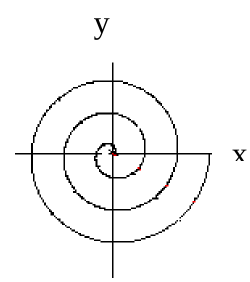

FIGURE 2. The Archimedean spiral.

We suppose that particle moves on the above trajectory with constant speed V. Now we want to solve this problem by the mentioned method. The equation of Archimedean spiral is

$$y = x\tan\left(\frac{\sqrt{x^2+y^2}}{a}\right). \qquad (10)$$

Where *y* is vertical axis and *x* is horizontal and *a* is constant. For simplicity we use from Maclaurian theorem. Then according to Maclaurian theorem we have

$$y = \frac{x^2}{\sqrt{a^2-x^2}}\,. \qquad (11)$$

According to the relations (3) and (4) we have

$$V_x = \frac{V}{\sqrt{1+\frac{x^2(2a^2-x^2)^2}{(a^2-x^2)^3}}} \quad , \qquad (12)$$

$$V_y = \frac{V}{\sqrt{1+\frac{(a^2-x^2)^3}{x^2(2a^2-x^2)^2}}} \quad . \qquad (13)$$

Again according to the relations (5) and (6) can be calculated the components of accelerations.
For example with *x*=*a* we obtain

$$V_x = 0 \quad , \quad V_y = V\,. \qquad (14)$$

VI. CONCLUSIONS

Number of examples about uniform motion is infinite. Also solved examples shows that use of mathematical procedure can be useful.

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