# The jumping ring experiment revisited and new possibilities

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**Abstract**
The Thomson ring experiment is revisited with the aim of obtaining higher and higher jump heights. A new way to determine the inductance of the ring and the phase lag between the primary magnetic flux and the current in the ring is introduced. New more effective configurations are presented, up to that of true electromagnetic cannon that fires aluminium disks with great violence. As an interesting by-product, regular geometric patterns of the iron filings used to visualize the structure of the very high magnetic fields inside the coil of the electromagnetic cannon are revealed, indicating the emergence a self-organization phenomenon typical of dissipative systems.

**Keywords:** Physics Education, Thomson ring experiment.

**Resumen**
El experimento del anillo de Thomson se revisa con el objetivo de obtener alturas de salto altas y más altas. Se introduce una nueva manera de determinar la inductancia del anillo y el retardo de fase entre el flujo magnético primario y la corriente en el anillo. Se presentan nuevas configuraciones más eficaces, hasta la del cañón electromagnético verdadero que dispara discos de aluminio con gran violencia. Como un subproducto interesante, se revelan patrones geométricos regulares de las limaduras de hierro para visualizar la estructura de los campos magnéticos más altos dentro de la bobina del cañón electromagnético, lo que indica la aparición de un fenómeno de auto-organización típica de los sistemas disipativos.

**Palabras clave:** Educación en Física, experimento del anillo de Thompson.

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## I. INTRODUCTION

The creator of this fine experiment, also known as the Thomson ring experiment, is Elihu Thomson (1853-1937), inventor, great experimenter and founder of the Thomson-Huston Electric Company.

The first time it was performed was at a congress of the American Institute of Electrical Engineers, and since then it has become one of the experiments most demonstrated when in conferences and lectures one must show electric induction phenomena. One of these was performed in London in 1891 by Lord Rayleigh at a meeting of the Royal Institution during a lecture in commemoration of the centenary of the birth of Michael Faraday since the "jumping ring experiment", as it is universally known, is a sum of the electromagnetic phenomena discovered by Faraday: from the induction laws to the magnetic properties of substances to the forces acting between electric currents flowing into conductors etc.

The set-up is as follows: a long soft iron nucleus is placed in the core of a coil with a high number of turns. It protrudes from the top so that it may be directly connected to the a.c. mains. An aluminium or copper ring is inserted on the nucleus and rests on the coil. When the coil is powered, the ring jumps up to some height. If one retains the ring to prevent the jump, it rises along the nucleus up to a certain height, levitating in equilibrium with its weight.

The jumping ring experiment, besides its simplicity, is at the origin of an immense literature [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11] also because it lends itself to many other experiments. As we shall see, its explanation is far more intriguing than appears at first sight.

## II. THE NAIVE EXPLANATION

This is the recurrent explanation, for example among others in [9]. The a. c. current in the coil produces a variable magnetization of the nucleus, which induces a high short-circuit current in the ring. This current in turn generates a magnetic field which for Lentz's law opposes itself to the field of the nucleus. Accordingly, the ring moves away from the coil. The fact that the current in the ring is very high can be immediately verified: if one prevents the ring from moving from its rest position, it heats up considerably.

Is this the correct explanation? The electric current induced in the ring is proportional to the e.m.f. induced in it by the variation of the magnetic flux, $e = - dF/dt$ and is therefore 90° out of phase with respect to the primary current. These two currents, the primary P and the secondary S are drawn in figure 1.

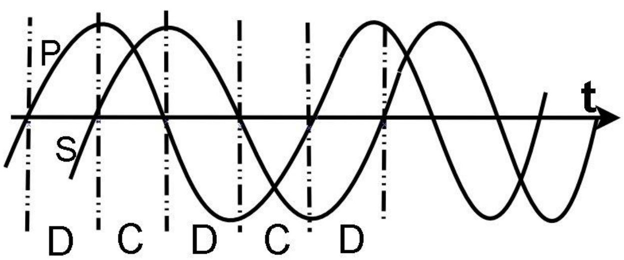


**FIGURE 1.** P: primary current; S: secondary current. D: discordant currents; C: concordant currents.

The force acting between two conductors is proportional to the product of their intensities but, as is seen in the figure, at each quarter period their reciprocal direction changes: in the intervals indicated by C their directions are concordant, and the force acting on them is attractive, while in the intervals marked D they are discordant and the force is repulsive.

In analytical terms: $i_p = I \sin \omega t$ being the primary current, and $\phi_v = \Phi_v \sin \omega t$ the vertical component of the magnetic flux it generates, with $\omega = 2\pi \cdot 50$ rad/s, the angular frequency in Europe. The e.m.f. induced in the ring is, by Faraday's law, $e = - \Phi_v\, \omega \cos \omega t$, and consequently the current circulating in the ring is:

$$i_a = -\frac{F_v}{R} coswt \ , \tag{1}$$

where $R$ is the ohmic resistance of the one-turn coil of the ring. The forces acting between the two currents are proportional to their product:

$$F' = K\left(Isinwt\right)\left(-\frac{F_v w}{R} coswt\right). \tag{2}$$

With a simple transformation and gathering all the constants in H it becomes:

$$F' = Hsin2wt \ .$$

The average of the product of two quadrature sinusoidal components is zero. The overall result is that the ring vibrates at twice the main frequency, but it does not jump. The naive explanation does not explain the phenomenon.

## III. THE CLASSIC EXPERIMENT AND A TENTATIVE EXPLANATION

Figure 2 shows the approximate pattern of flux lines of the magnetic field generated by the primary coil current and by the nucleus which cross the ring. As can be seen, the magnetic field has both vertical and horizontal components. The vertical component is responsible for the short-circuit current circulating in the ring which, as seen above, has no mechanical effect. So, is it the horizontal component that is responsible for the effect ?

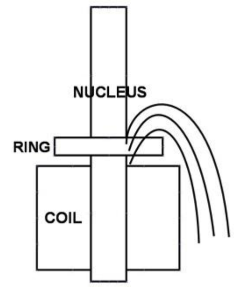


**FIGURE 2.** The flux lines must be imagined as distributed all around the vertical axis of the figure.

In Figure 3 the situation is sketched. According to the left-hand rule: the force-field current corresponding to thumb-index-middle fingers respectively, if the magnetic field is horizontal and radial, the current being tangential, by Lentz's law the force is vertical and directed upwards. When the field, which is alternate, changes direction, the current also changes direction and the force maintains its upwards direction.

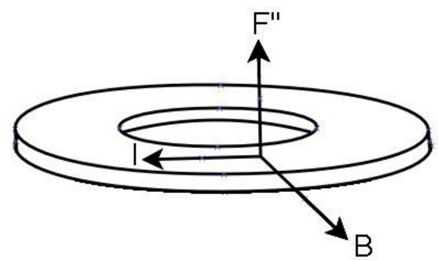


**FIGURE 3.** Force, field and current in the ring.

The analysis even now may reveal what takes place. Let us write the horizontal component of the magnetic field as $B_h \sin \omega t$, in phase with the primary current. The vertical force acting on the ring because of the current given in Eq. (1) is, in modulus:

$$F'' = \left(B_h sinwt\right)\left(-\frac{F_v w}{R} coswt\right) \times L \ , \tag{3}$$

where L is the length of the mean circumference of the ring. This, as seen above, transforms again into the form:

$$F'' = M sin2w\,t\ ,$$

having gathered all constants in M. The situation is actually the same as found in paragraph 2; the ring does not jump. This explanation is again inadequate. The true explanation is more subtle, striking and quantitatively surprising.

## IV. THE CORRECT EXPLANATION

A new element has to be introduced: the self-induction of the ring, as takes place, for instance, in [10], even though the explanation of the phenomenon given there is in our opinion incomplete. Because of its inductance, the current circulating in the ring lags with respect to the inducing field. Introducing the phase lag $\delta$ in the expression of the current in Eq. (1), the force will be the resultant of the two components calculated above, Eq. (2) and (3) in which the phase lag has been introduced. Being formally identical, let us consider the former.

$$F' = \left(Isinwt\right)\left[-\frac{F_v w}{R} cos\,(w t + d)\right] \times L\ ,$$

which, after simple transformation and having incorporated all the constants in N becomes:

$$F\,' = N\left[\left(sin2w\,t\right) cosd + \left(cos^2 w\,t\right) sind\right].$$

The first product inside the square brackets has zero mean, while the second is always positive. The second component *F''* has the same form, and also for it the useful term will be the second one. Eventually, the resultant between these two components will in any case be proportional to sin $\delta$.

Actually, the self induction coefficient of the one-turn coil ring is very small, but since its resistance is also very small, the phase lag, being tan $\delta = X_l /R$, may not be small. The accurate evaluation of $\delta$ is therefore essential.

It is convenient to refer to a specific case because the measurements carried out have revealed an intersting aspect of the phenomenon. The apparatus is photographed in Figure 4 and is the classic one. When powered, the aluminium ring jumps up by about 40 centimeters.

The resistance of the ring can be calculated with high precision, since its dimensions are known and the resistivity of aluminiun is $\rho = 2.82\ 10^{-8}\ \Omega$ m. With the ring used herein [13] we find $R = 1.06 \cdot 10^{-4}\ \Omega$.

A possible way to determine $\delta$ requires knowledge of the inductance of the ring, but the expression to be used for its calculation is not only one. For instance, with the formula given in [6] and without the iron nucleus:

$$L = ½\ \mu_0 D\ [\log_e (8D/d) - 7/4],$$

where $D$ is the external diameter, $d$ the width and e $\mu_0 = 4\pi \cdot 10^{-7}$ H/m is the magnetic permeability of the vacuum. We obtain $L = 1.30\ \mu$H, and then $\delta = 75°$. In ref. [11] the formula given is:

$$L = 0.01257a\ [2.303\ \log_{10} (16a/d - 2)],$$

seemingly equally plausible, and again without the iron, where $a$ is the radius of a wire equal to the mean radius of the ring and $d$ is the diameter of the wire, in this case equal to the thickness of the ring, both expressed in centimeters. With this formula one obtains $L = 0.158\ \mu$H, from which $\delta = 25°$, a value very different from the previous one. The presence of the iron nucleus, whose magnetic permeability is unknown, and the further complication of the uncertainity of the filling factor makes even more uncertain the evaluation of the ring inductance and phase log $\delta$.

**FIGURE 4.** The jumping ring apparatus.

We now describe a procedure, new to our knowledge, that leads to a very accurate determination of the inductance of the ring and phase lag $\delta$. The initial data are as follows. The primary coil [13] has 1200 turns and a resistance of 12 Ω. Measurements of the inductance of the coil with the laminated iron nuclei inserted as shown in figure 4 are performed, the first without the ring in position and the second with the ring in its starting position. The results are:

- Primary inductance without the ring $Lw = 0.331$ H
- Primary inductance with the ring $Li = 0.133$ H

With only these data (see the appendix) it is possible to deduce with precision all the relevant parameters of the experiment by means of purely energetic considerations. The results are summarized in table I.

**TABLE I.** Summary of data and results on the jumping ring apparatus.

| |
|---|
| e.m.f. acting in the ring: 0.183 V |
| Resistance of the ring: $R = 1.06 \cdot 10^{-4}\ \Omega$ |
| Current (lagging $\delta$ with respect to the e.m.f.) $I$ = 1400 A |
| Inductance of the ring: $L$ = 0.245 μH |
| Inductive reactance of the ring: $X_l = 0.77 \cdot 10^{-4}\ \Omega$ |
| Impedance of the ring: $Z = 1.31 \cdot 10^{-4}\ \Omega$ |
| Phase angle between the e.m.f. and the current in the ring: $\delta$ =36° |
| Phase angle between primary flux and current in the ring: 126° |
| Active power to the ring in its starting position: $Ptr$ = 207.2 W |

It must be stressed that with this technique it has been possible to determine the profile of the powers transmitted to the ring as a function of its height on the nucleus. The results are shown in figure 5.

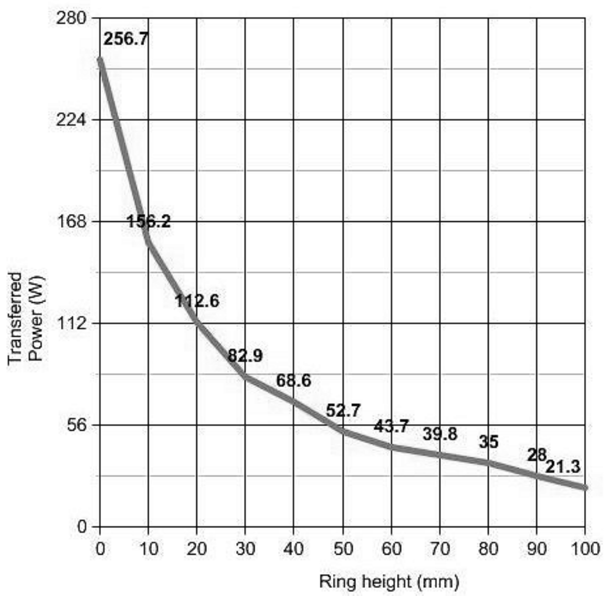


**FIGURE 5.** The dependence of the primary power as a function of the height of the ring along the nucleus

## V. OTHER STORIES: HOW TO MAKE HIGHER JUMPS. THE SIMPLER WAY

As we have seen (see the appendix) the primary current with the ring in its starting position is $I_p$ = 5.06 A but lags by 74° on the voltage. The most obvious way to obtain higher jumps is to increase the primary current. This may be obtained in the simplest way by connecting in series to the coil a capacitor whose capacity is such that the LC circuit in series is resonant at line frequency (50 Hz in Europe). The capacitive reactance of the capacitor must be equal to the inductive reactance of the coil. The value found is $C$ = 76 μF. In this way the circuit offers to the alternating current only its ohmic resistance, and the current will be $I$ = 220V/12 Ohm = 22 A, but with the great advantage that now the current is in phase with the voltage.

The circuit diagram of the apparatus is shown in figure 6. Because of the greatly increased current, to switch on it is now necessary to use a contactor that is energized by the button marked "Fire" in the diagram.

A switch allows connection or exclusion of a 40 μF capacitor to make comparisons between the two configurations. With the capacitor inserted the ring hits the ceiling at a height of 4 meters. Of course the apparatus must be powered for a short time to avoid overheating the coil, and caution must be used in this case.

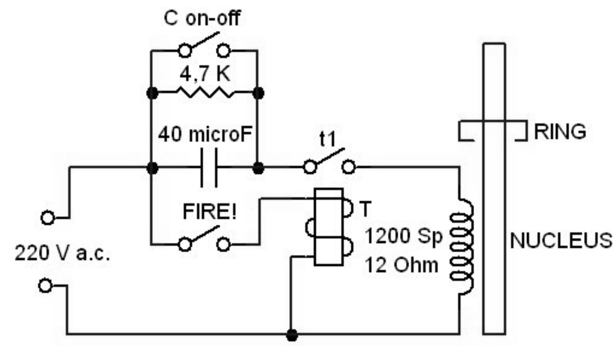


**FIGURE 6.** Circuit diagram of the jumping ring apparatus of Figure 4.

In reality the situation is slightly more complicated. When the ring starts to jump under the action of the electrodynamic force, primary inductance increases until it reaches the value of $L$ = 0.331 H when the ring is no longer in place. This would drive the LC circuit to 50 Hz resonance with a much lower value of the capacity: $C$ = 30 μF. By trial and error it has been found that the maximum jump height is for a capacity of 40 μF. The launch dynamic is likely to be as follows. Initially the circuit is slightly out of resonance; the ring starts to rise. As the ring is lifted, the inductance increases, the circuit reaches resonance and the lifting force also increases, gradually accompanying the motion of the ring. In other words, a force which increases gradually with respect to a strong initial pulse of short duration appears to be more effective.

## VI. HOW TO MAKE HIGHER JUMPS: BRUTE FORCE AND COMPLEX EXPLANATION

Another more violent strategy may be implemented in the apparatus of figure 4. A bigger capacitor whose capacity is $C$ = 200 μF is connected in series with the coil. With this capacitor the height of the jump depends on the instant the button switch "Fire" is pressed. In some cases jumps more violent than the ones in the preceding paragraph are obtained.

This experiment was created by the author to show at least qualitatively a little known phenomenon that occurs in strong dependence on initial conditions and which may evolve in a chaotic way (deterministic chaos). This phenomenon is known as ferroresonance [15].

It can be described as follows. When because of the current peaks the magnetic flux becomes so intense that the ferromagnetic nucleus reaches saturation magnetization, the inductance of the coil, being proportional to the magnetic permeability of the nucleus, falls to very low values. When this occurs, the LC circuit in series may begin resonating at the line frequency with values of the capacity much higher than those calculated in the case of paragraph 4.1. The energy transferred during the rising of the ring may be higher because the primary inductance varies much less and the resonance condition is maintained for a longer time during the rising motion of the ring. Being chaotic, the phenomenon triggers rarely and in very critical conditions.

## VII. HIGHEST JUMPS: THE ELECTROMAGNETIC CANNON

One can obtain much more violent jumps with an aluminium ring if one uses a pulsed unidirectional field. The magnetic field pulse will induce a pulse of current in the ring which in turn will generate a unidirectional opposed magnetic pulse. This pulse is active in making the ring jump. The Lentz law is now at work.

**FIGURE 7.** The Electromagnetic Cannon. The aluminium disk is shown displaced to view the coil. The capacitor is the large black cylinder and the contactor is in the right foreground. The remote control is on the left.

The apparatus, baptized by the author as "the Electromagnetic Cannon", is photographed in figure 7 and its wiring diagram is in figure 8. A 2200 μF capacitor, charged to 320 V (100 J of stored energy) is abruptly discharged on a coil of little resistance and inductance. The peak discharge current is many thousands of amperes. The magnetic field pulse lasts some tens of microseconds, so we have very high values of $d\Phi/dt$. The coil has no ferromagnetic core, so that the magnetic field peak is not limited by the saturation magnetization of the nucleus.

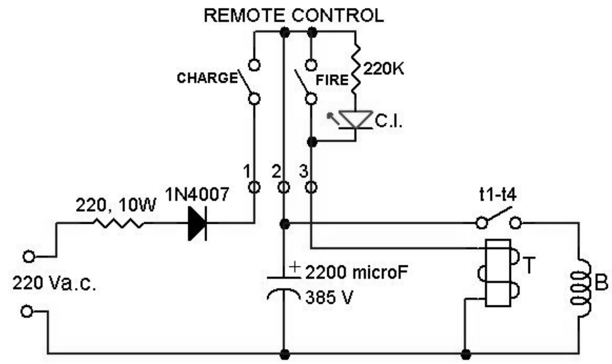


**FIGURE 8.** Circuit diagram of the Electromagnetic Cannon. T is the power contactor with all its contacts t1 ÷ t4 wired in parallel. CHARGE and FIRE buttons on the remote control are self-explanatory. The LED diode C.I. indicates the charge of the capacitor. B is the thin flat rigid coil.

An extensive development program has shown that the best form of the coil is that of a thin flat donut of the same size as the aluminium disc to be launched. The coil has 20 turns of enamelled copper wire of 1.24 mm diameter, an inner diameter of 55 mm, an external diameter of 90 mm and thickness of 3 mm. The coil is completely impregnated with epoxy resin to make it rigid and to avoid damage due to the intense squeezing of electrodynamic forces. The disk, which before the shot rests directly on it, has the same dimensions. The inductance of the coil is 11 μH with the disk in shooting position and di 55 μH without the disk. The resonant frequency of the LC parallel circuit with the disk is 1 KHz; this suggests that discharge time will last a few milliseconds.

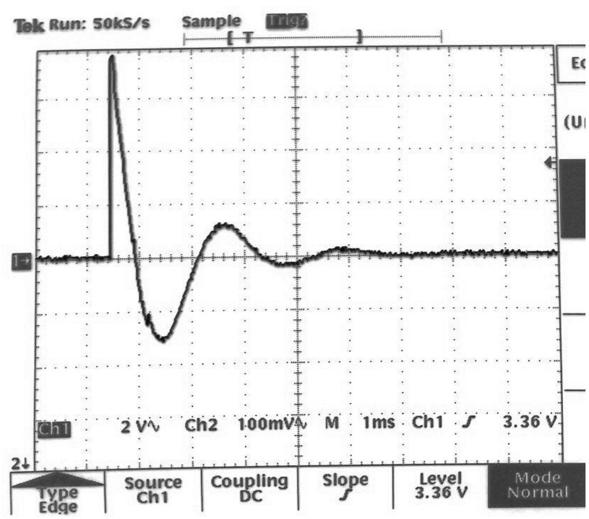


**FIGURE 9.** Oscillogram of the induced voltage in a probe loop resting on the coil when the cannon is fired. Vertical scale 2 V/div; time axis: 1 ms/div

Figure 9 shows the waveform of the induced voltage in a shielded one turn coil resting on the cannon's coil during a shot. This probe of the magnetic field pulse is sketched in figure 10 and photographed in figure 11. As can be seen, the magnetic field pulse rises in some tens of nanoseconds, and so very large currents are induced in the projectile disk. Successively the phenomenon evolves with a strongly damped oscillation with a period of about one millisecond.

The shots are noisy and impressive. The aluminium disk bangs violently against the ceiling 4 meters high. The electromagnetic cannon can shoot other objects, such as the aluminium disks taken from the disassembly of a hard disk, copper rings, short-circuited three or four coils of thick copper wire.

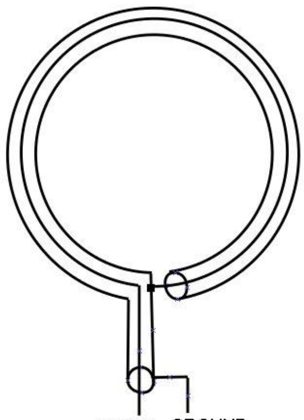


**FIGURE 10.** The probe loop of the flux variations. The loop must be shielded to avoid error voltages by capacitive coupling with the coil wires.

Classic iron filings have been used for the visualization of the magnetic field flux lines above the coil. The filings were distributed uniformly on the surface of the coil. After the shot the filings concentrate in correspondence to the turns of the coil as shown in the photograph in figure 12. The coil behaves like a ring magnet with the poles on the plane surfaces. The magnetic field flux crosses all and only the metal mass of the projectile disk.

Finally, we make an incidental observation which to our knowledge has never been previously reported. In figure 12 it can be seen that in correspondence to the centre of the coil the iron filings sometimes assume regular geometric arrangements that recall Benard cells, a spontaneous auto-organization phenomenon. An hexagonal arrangement appears.

This fact suggests that under these conditions of very intense magnetic flux the filings are to be considered as a dissipative system, an open thermodynamic system that when crossed by increasing energy fluxes evolves toward the spontaneous creation of anisotropy, i.e. of structures in which the order increases and the entropy decreases (*neghentropy*).

**FIGURE 11.** The coil of the Electromagnetic Cannon and the shielded probe loop.

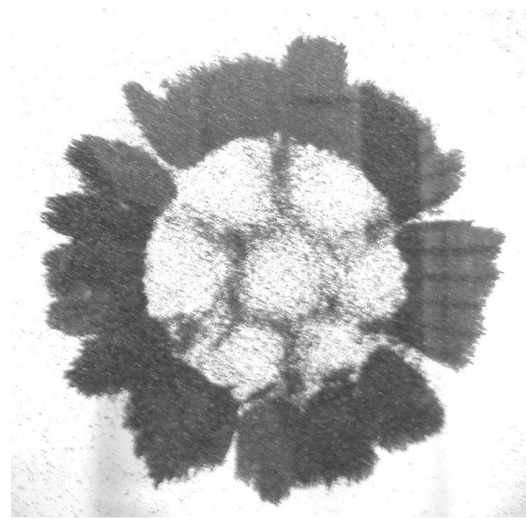

**FIGURE 12.** Distribution of the iron filings on the coil of the Electromagnetic Cannon as a result of a shot. The small circle on the left at 8 o'clock corresponds to one of the terminal wires of the coil which at that point is vertical.

## VIII. CONCLUSIONS

In this work we conducted an extensive analysis of the operation of Thomson's jumping ring experiment to obtain new ideas and suggestions on suitable electromagnetic arrangements for making more and more spectacular jumps.

In the classic jumping ring experiment the explanation of the phenomenon is not straightforward Lentz repulsion, but is much more subtle and quantitatively surprising. New possibilities were investigated, ranging from the use of the resonance at the line frequency to the exploration of the little-known phenomenon of ferroresonance, ending with a real firing system which we call the electromagnetic cannon, based on the violent and extremely fast discharge of a large capacitor on a coil of very little resistance and inductance and without a ferromagnetic core.

Finally, with the huge impulsive magnetic fluxes that take place in the electromagnetic cannon, the appearance of an unexpected phenomenon was observed: the spontaneous auto-organization of ion filings, used to display the distribution of the magnetic flux, in ordered structures similar to Benard cells, a symptom of the emergence of a dissipative system in the sense introduced by Prigogine

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[13] The coil is Phywe cat. n. 06515-01. The nucleus of the apparatus in figure 4 is a stack of two nuclei of the same firm, cat. n. 06500.00, elements of their modular transformer, 29 x 30 mm$^2$ section by 101mm length. The aluminium ring has an inner diameter of 42.5 mm, an outer diameter of 65 mm and a thickness of 4 mm.

[14] The measurements of the inductance are performed by assembling an LC parallel resonant circuit with the capacitor value precisely known and measuring the resonant frequency. The circuit is connected to a variable frequency sinusoidal generator through a resistor of 10 KΩ in series. An oscilloscope is used to find the maximum amplitude of the voltage at the terminals of the circuit, which occurs at the frequency of resonance [15]. A fine treatment of the phenomenon is available, among many others, in: *Ferracci P*, *Ferroresonance*, Cahier Technique No. 190, Group Shneider.


## APPENDIX

As we have seen, the starting data are: primary coil of 1200 turns, resistance 12 Ω. Resistance of the ring [13] $R$ = $1.06\cdot10^{-4}$ Ω. Primary inductance without the ring $Lw$ = 0.331 H; primary inductance with the ring $Li$ = 0.133 H. We can now calculate the inductive reactances at the frequency of 50 Hz without and with the ring respectively:

$$Xlw = 2p\, f \times Lw = 104W,$$
$$Xli = 2p\, f \times Li = 41.7W.$$

The primary coil impedance, again without and with the ring:

$$Zw = (R^2 + Xlw^2\,)^{1/2} = 104.6W,$$
$$Zi = (R^2 + Xli^2\,)^{1/2} = 43.3W.$$

The respective primary currents:

$$Iw = \frac{V}{Zw} = \frac{220\,V}{104.6\,W} = 2.1\,A,$$
$$Ii = \frac{V}{Zi} = \frac{220\,V}{43.3W} = 5.08\,A.$$

The power factors in the two cases:

$$cosdw = \frac{R}{Zw} = \frac{12W}{104.6W} = 0.115,$$
$$cosdi = \frac{R}{Zi} = \frac{12W}{43.3W} = 0.277.$$

The active powers:

$$Pw = V \times Iw \times cosdw = (220V)\times(2.1\,A)\times 0.115 = 53.1W,$$
$$Pi = V \times Ii \times cosdi = (220V)\times(5.08\,A)\times 0.277 = 309.7W.$$

Consequently, if one prevents the ring from moving from the starting position, the power transferred to the ring is $Pa'$ = $Pa$ – $Ps$ = (309.7 – 53.1)$W$ = 256.6 $W$ (it quickly overheats).

The e.m.f. acting in the circuit of the ring can be calculated, it being the one-turn short-circuited secondary of the transformer which has the coil of 1200 turns as the primary:

$$e.m.f. = 220\text{V}/1200 = 0.183\text{V}.$$

Since in general $P = V^2/Z$, it is now possible calculate the ring impedance:

$$Z = (e.m.f.)^2/Pa' = (0.183\text{V})^2/(256.6\text{W}) = 1.31\cdot10^{-4}\ \Omega,$$

and the phase angle:

$$\cos\delta = R/Z = (1.06\cdot10^{-4}\ \Omega)/(1.31\cdot10^{-4}\ \Omega) = 0.809,$$

from which:

$$\delta = 36°.$$

The inductive reactance of the ring is therefore:

$$Xl = Z \sin\delta = (1.31 \cdot 10^{-4}\ \Omega) \cdot 0.587 = 0.77 \cdot 10^{-4}\ \Omega$$

and the inductance of the ring:

$$L = Xl/2\pi f = (0.77 \cdot 10^{-4}\ \Omega)/314\ \mathrm{Hz} = 0.245\ \mu\mathrm{H},$$

and finally the active power transferred to the ring in its starting position is:

$$Ptr = (e.m\ f.) \cdot I \cdot \cos\delta = (0.183\mathrm{V}) \cdot (1400\mathrm{A}) \cdot 0.809 = 207\ \mathrm{W}.$$

The phase angle $\delta = 36°$ is the phase lag between voltage and current in the ring. Since the voltage in the ring, $-d\Phi/dt$, lags by 90° with respect to the primary current and the primary flux, the phase delay of the current in the ring on the primary current and flux is:

$$delay = \delta + \pi/4 = 126°.$$

The same results hold approximately for rings of slightly different sizes.