How can formulation of physics problems and exercises aid students in thinking about their results?

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Abstract

It has recently became common that the authors of the physics textbooks describe, in general terms, the most important steps students have to follow in order to solve problems. The last step is usually a recommendation to "think about the result" in order to find out if it is reasonable. Nevertheless, the standard formulation of exercises is such that "thinking about the result" is likely to be left out or, in the best scenario, it can be used only to fix students' careless math errors. In fact, it seems that the physics textbook authors have on their minds precisely this role for the "thinking about the result". Even in the cases when the physical evaluation of the mathematically correct results is required explicitly, the students might not have the knowledge necessary to evaluate it in an appropriate way. In this article, a better way to formulate physics exercises is proposed. In such a formulation the evaluation of the result cannot be avoided.

Keywords: Physics problem solving, steps of expert, critical thinking, textbook errors, design of physics problem.

Resumen

Recientemente se volvió común que los autores de los libros de texto describan, en términos generales, los más importantes pasos que los estudiantes tienen que seguir para resolver problemas. El último paso es usualmente una recomendación "piensa sobre el resultado" con el fin de determinar si es sensato. Sin embargo, la formulación estándar de los ejercicios es tal que es probable que el "pensamiento sobre el resultado" se omite o, en el mejor caso, se usa solamente para arreglar los errores matemáticos que los estudiantes hacen por descuido. De hecho, parece que los autores de libros de texto de física tienen en su mente precisamente este papel del "pensamiento sobre el resultado". Incluso, en los casos en que se requiere explícitamente la evolución física de los resultados que son matemáticamente correctos, puede ser que los estudiantes no tengan el conocimiento necesario para evaluarlos de manera apropiada. En este artículo, se propone una mejor manera de formular ejercicios de física. En tal formulación la evolución de los resultados no se puede evitar.

Palabras clave: resolución de problemas de física, pasos del experto, pensamiento crítico, errores en los libros de texto, diseño de problemas de física.

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I. TWO STANDARD RECOMMENDATIONS FOR THINKING ABOUT THE RESULTS: CHECKING MATH OR PHYSICS?

Problem solving, as an important part of physics teaching and learning, has received in the last few decades the deserved attention of the research community [1]. The most important result of this research has been the astonishing difference between experts and novice problem solving strategies. As a pedagogical consequence, it recently became common that physics textbooks provide a summary of problem-solving steps which seem to be in resonance with the ones used by experts. Usually the last step is an explicit recommendation of thinking about the meaning of the results obtained in the calculations performed. Roughly speaking, these recommendations fall in two different categories.

Students should have some kind of ready-to-use knowledge in order to judge the validity of the results (three different versions of this type of recommendation are given in the Table I).

Students have to estimate the order of magnitude in order to judge the mathematical validity of the result (two different wordings of this recommendation can be found in the Table II).

TABLE I. Recommendations to activate ready-to-use knowledge in order to judge the validity of the results.

"…When you arrive at a number, think about it. *Does it make sense?* If you find that it takes 3 min to drive from New York City to Los Angeles, you have probably made a mistake" [2].
"… *Consider whether the result is reasonable*. That is, does the answer have an appropriate magnitude? (This means is it "in the right ball park.") For example, if a person's calculated mass turns out to be 2.30×10^2 kg, the result should be questioned, since 230 kg corresponds to a weight of 506 lbs" [3].
"After you have finished your calculations, always check whether *the answer is plausible*. For instance, if your calculation yields the result that a diver jumping off a cliff hits water at 3000 km/h, then somebody has made a mistake somewhere!" [4].

TABLE II. Recommendations to carry out order-of-magnitude estimation in order to judge the validity of the results.

"Think carefully about the result you obtain: Is it reasonable? Does it make sense according to your own intuition and experience? A good check is to do a rough *estimate* using only powers on ten…" [5].
"As a final check, you should consider whether your answer is reasonable.
Does your result have the proper order of magnitude? You may even carry out a quick order-of-magnitude *estimate* as a way to confirming your work" [6].

No one doubts that both of these recommendations are very useful in eliminating the negative effects of careless errors in algebraic manipulations or incorrect formula usage. Therefore, it is worthwhile to explicitly ask students to use them and make them an important part of problem-solving sessions, homework and exams.

Nevertheless, the first type or recommendation could only be used in a small fraction of standard exercises. While it is likely that students will know that the "3-minute travel time between New York and Los Angeles", "230 kg mass for a human" and "3000 km/h diver's speed" are not feasible in the real world, for many physical situations used in numerical exercises the corresponding real-world facts are simple not part of students' knowledge. Two situations in which the reasonable value of the gravitational force is beyond students' knowledge base are presented in the Table III.

TABLE III. Which real-world knowledge is useful to judge if the results are reasonable?

"Two students sitting in adjacent seats in a lecture room have weights of 600 N and 700 N. Assume that Newton's law of gravitation can be applied to these students and find the gravitational force that one student exerts on the other when they are separated by 0.5 m" [7].
"Two supertankers, each with a mass of 7×10^8 kg, are separated by a distance of 2 km. What is the gravitational force that each exerts on the other. Treat them as particles" [8].
Comment: Students are not asked to examine if the results are reasonable or to compare them with a known force in order to have some experience and knowledge about the size of the gravitational force between humans or men-made objects like supertankers. In addition, the applicability of the law of gravitation is, explicitly or implicitly, suggested.

When students don't have at their disposal the necessary real-world knowledge to judge directly the feasibility of the result, they can only use the second type of recommendation and make an order-of-magnitude estimation.

If such estimation agrees with their prior calculations, is the result feasible? Unfortunately, in some cases it is not. Textbook authors and teachers also make errors, supposing physical situations which are not very likely to happen (or are even impossible) in the real world [9, 10, 11, 12, 13].

Although useful in eliminating students' math errors, the above recommendations are not effective against the more serious errors made by physics textbooks authors or teachers in regard to physical feasibility of problem situations.

This is true because students, like authors and teachers themselves, may lack the real-world knowledge which directly contradicts the result. In addition, an order-of-magnitude calculation is essentially a tactic to rapidly check the numerical results of mathematical steps involved and cannot judge the feasibility of the situation supposed. One cannot stress enough that the mathematical correctness of the results has little or nothing to do with its physical feasibility. Namely, student should know that some mathematically possible situations and results are physically impossible due the real-world restrictions.

With the standard design of numerical exercises and only the type of recommendations discussed above, the students are not likely to practice and develop strategies of critical thinking, proudly announced as one of the most important objectives of physics teaching.

II. SHOULD STUDENTS DEAL WITH "WRONG ANSWERS" IN NUMERICAL EXERCISES?

Detecting unfeasible physical situations, implied by carelessly assigned "bad numbers" to some physical quantities, isn't an easy task with clear solution. Taking into account that many authors, reviewers and teachers are highly-qualified physicists, if it were a trivial task, there would be no such errors in physics textbooks. It is frequently impossible to detect an unfeasible physical situation, supposed as a "context" for calculations, without using some, apparently unrelated concepts. A good example of how nontrivially it is to carry out such a task is to examine the feasibility of the situation "two bodies charged with 1 C at 1 m distance", frequently used in physics textbooks as one of exercise for the application of Coulomb's law "application" [10]. In order to show that such a situation is impossible in the real world, one should know facts about electrical strength of air, cold emission and the electrical stress the metals can sustain.

There are authors who think that students should not be given the problems and exercises whose results are, strictly speaking, incorrect. For instance, Blickensderfer [14] argued that some questions, likely to be found in typical textbooks for introductory-level physics courses, have wrong answers because they are usually answered within overly simplified mathematical models.

He suggests that a more appropriate approach in introductory courses would be to change numerical data in order to fit better supposed simple formulas rather than to use more advanced physical and mathematical models (reserved for upper-division part of curriculum).

Other authors, the present one included, have quite the opposite opinion and would rather answer the question in the subtitle with "yes". For instance, Urone made use of this idea in a unique way. In his college-physics textbook, he stressed the importance that students deal with clearly stated "unreasonable result problems":

"*Unreasonable result problems* are unique to this text. They are designed to further emphasize that properly applied physics must describe nature accurately and it is not simply the process of solving equations. For example, if the heat generated by metabolizing an average day's food is retained, a person's body temperature will rise to a lethal level. Thus, physics correctly applied produces in this case a result that is never observed –the student must recognize that the premise of complete heat retention is at fault. These problems are clearly labeled and are found at the very end of the end-of-chapter problems– all other problems in the text produce reasonable results and often contain discussion to emphasize that physics must fit nature. Taken with the careful accuracy of the text and the discussions at the end of worked examples, unreasonable result problems can help students examine the concepts of a problem as well as the mechanics of solving it" [15]..

"... Problems with unreasonable results are included to give practice in assessing whether nature is being accurately described, and to trace the source of difficulty if it is not. This is very much like the process followed in original research when physical principles as well as faulty premises are tested" [16].

One example of Urone's "unreasonable result problem", related to the law of gravitation, is given in the Table IV.

TABLE IV. A mountain with too big mass.

"A mountain 10.0 km from a person exerts a gravitational force on him equal to 2.00 % of his weight. (a) Calculate the mass of the mountain. (b) Compare the mountain's mass with that of the entire earth. (c) What is unreasonable about these results? (d) Which premises are unreasonable or inconsistent? (Note that accurate gravitational measurements can easily detect the effect of nearby mountains and variations in local geology.)" [15, *Problem 8.43, p. 217*].
Answers (a) 2.94×10^{17} kg (b) 4.92×10^{-8} of the earth's mass (c) the mass of the mountain and its fraction to the earth's mass are too great. (d) The gravitational force assumed to be exerted by the mountain is too great [*15, AN-2*].

Although this type of problems is a step in right direction, students still might lack the skills necessary to arrive at the conclusion that the implied mass of the mountain and its fraction to the Earth's mass are too great. If they have to accept blindly the answer, provided by Urone, without being able to reconstruct it or grasp the essence of the reasoning it is based on, the chance for becoming critical thinker is lost forever.

One way to improve this type of problems is to provide students with some hints or guidance in order that they can conclude by themselves what is wrong with the problem in question. This is the approach I used in the critical-thinking rubrics of my secondary-school physics textbooks, named "No creas todo lo que lees" ("Don't believe everything you read") [17, 18].

In the case of the too-massive-mountain exercise, one tactic might be to ask students: what the value of the supposed gravitational force would be if the person were at a distance 1 km from the mountain? According to the law of gravitation, when distance is ten times smaller, the force would be 100 times bigger. In this situation the force would be quite unreasonable twice the person's weight. As such strange forces were never observed near real mountains, students would have, at least, one plausible reason to conclude that the implied mass of the mountain is too big.

III. MORE WAYS TO HELP STUDENTS THINK WHILE SOLVING NUMERICAL PROBLEM

A. A radical proposal: problem situation should be as unspecified as possible in order to promote a research-like approach to the solution

It is well known that the common formulation of numerical problems, in which the students are told what to calculate and are given the necessary data to perform a calculation using a single formula, leads students to adopt an algorithmic approach to problem solving, consisting of basically looking for the right formula. To eliminate blind formula manipulation and to promote research-like behavior of students, Gil-Pérez and his collaborators [19] suggested that an appropriate pedagogical remedy would be to take away all numerical data (one example of this proposal is given in the Table V).

TABLE V. An example of transforming a numerical exercise into unspecified problem situation by taking away numerical data.

Conventional formulation which promotes algorithmic approach to solution
"A frictional force of 10,000 N is exerted on a moving object weighing 5000 kg and traveling at 20 m/s. What will be the speed of the object 75 m after the frictional force was applied."
No-data formulation which promotes a research-oriented approach to solution
"A driver starts braking at the sight of a red traffic light. What will be the speed of the car when it reaches the traffic light?" [19, p. 142].

Facing such a formulation, Gil-Pérez and his collaborators say

"students are forced to ask questions, make hypothesis and, more generally, adopt a problem solving strategy akin to that of scientific research" [19, p. 143].

Although they don't provide details, one might guess that students would have to

(1) specify problem situation;
(2) find, discuss and make assumptions about necessary data (initial velocity, distance, frictional force,…) on which the final velocity depends, and
(3) decide about the mathematical model or formula to be used (for instance, constant-frictional force model of motion during braking).

In other words, they should research (or make an assumption about) everything that is, in standard problems, given to them by textbook authors or their teachers. No doubt, by doing so, students would actively learn many important elements of thinking used by research physicists.

In addition, students will learn something that is very important but usually hidden in standard exercises: that the result obtained depends on the suppositions used. It may be the case that the same problem can have different solutions whose validity depends on how closely the supposed models and data fit real-word features.

B. A less radical proposal: problem situation is specified but the task formulation makes result evaluation necessary

At the present time, most students lack the necessary skills to take a completely unspecified problem situation and transform it into tractable conceptual and numerical exercise. Lacking these skills, students may become discouraged if faced with a problem like the on quoted above. Even if educators opened up a large amount of classroom time for problems such these, they still need to teach the heuristic involving in achieving a reasonable solution.

Instead, one could keep a problem situation partly specified (by giving, for example, some numbers), but still explicitly promote some important features of the scientific process such as decision making and result analysis. With this in mind, the main points of an alternative approach, lying between the standard and radical design, would be:

(1) Avoiding the suggestion of calculating any specific physical quantity;
(2) Wording the problem in such a way that some kind of result evaluation is necessary to answer it.

In other words, the formulation should provide neither an explicit hint about what to calculate nor how to judge the feasibility of the results or situations. With such a vision, a reformulation of the radical version given above might read as follows:

A driver starts braking at the sight of a red traffic light which is at the distance of 30 m. If the speed of the car was 30 m/s, can the driver stop it before it reaches the traffic light?

Although students are now given the initial speed and distance, they must decide what to calculate. Even when they calculate the necessary frictional force (with implied coefficient of kinetic friction), they still cannot answer the question and have to judge if the calculated force is possible for real cars. The answer can be found by analyzing the information about braking distance published weekly in many automobile journals (for instance, *Car and Driver* or *Four Wheels*).

As an other additional illustrative example of this approach, I provide reformulation of two standard numerical exercises related to the law of gravitation, which normally do not require "thinking about the result" (Table VI).

TABLE VI. 1-N gravitational force between two spheres at 1-m distance

"Two identical spheres are to be placed one meter apart. How massive must the spheres be in order to have a mutual gravitational force of 1 N? [20]
"Suppose that two identical spheres, separated center-to-center by 1.00 m, experience a mutual gravitational force of 1.00 N. Compute the mass of each sphere" [21].

Again, one must be careful to formulate the problem in a way that aids critical thinking. One can run the risk of being too unspecified, losing the opportunity for research-like thinking and instead making a problem that is more like a game of playing with numbers (suppose, for example, a problem such as "how big is the gravitational force between two bodies?").

In an attempt to be more balanced, the exercise could read:

The centers of two spheres are at 1 meter distance: Can the gravitational force between them be 1 N?

Students are again obliged to decide themselves what to calculate and would be able to recognize that it is necessary first to find out the implied mass of the spheres. When the mass is found (1,224 x 10^5 kg), in standard formulation quoted above students are led to believe that they have understood everything of worth in the problem, although, in reality, the have gained little physical intuition.

In the reformulated version, students clearly recognize that the mass, although necessary in the path to solution, is not sufficient to provide the answer to the question asked. In consequence, they must think about the feasibility of the specified problem situation, arriving at the additional question: Is it possible that the spheres we deal with have such a mass?

They should "discover" that one way to judge the feasibility of the situation is to calculate the density of spheres. As the radius of the spheres may not be greater than 0.5 m, the implicit density should be:

$$\rho = 2.34 \text{ x } 10^5 \text{ kg/m}^3.$$

In order to decide whether is it possible to have a sphere with such a density in the real world, students should know or, more likely, should find out what is the upper limit on density for elements known existing on Earth. Looking in their textbook or some handbook in the library, they would be able to determine that the osmium is, under normal conditions, the most dense material, having a density of.

$$\rho_{\text{osmium}} = 2.3 \text{ x } 10^4 \text{ kg/m}^3$$

In order to generate one Newton of gravitational force when their centers are one meter apart, the spheres should have a density which is over 10 times greater than the density of osmium. Students may then conclude that the situation suggested is not feasible, if the spheres are to be made of normal materials.

IV. CONCLUSIONS

Although it is becoming more popular to recommend "thinking about the result" at the end of standard textbook problems, this most often takes the form of checking whether on has made an error in calculation. Many authors focus on providing acceptable data, legitimate use of formula, and reasonable physical situation, but forget to recognize that students need to learn to critically evaluate the result for themselves.

To change the focus from checking mathematical validity of the calculated results to evaluating the physical feasibility of the problem situations, it is necessary to provide students with appropriate practice and tasks.

One possibility, advocated by Urone, is to intentionally introduce errors into commonly formulated exercises and to ask students to find out why the calculation gave an unreasonable result.

Another way, presented in this article, is to give students exercises which strongly promote their decision making about what to calculate in order judge the feasibility of the problem situations and their results.

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