Modelling Football Penalty Kicks

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Abstract

In modern football the penalty kick is considered a golden opportunity for the kicker to register a goal. The kicker is virtually unchallenged by any opposing player except the goalkeeper who stands on the goal-line 12 yards away. Therefore, the kicker has an overwhelming advantage. Maximising on this advantage is of paramount importance since penalties in many instances, determine the outcome of games. This paper analyses the variables involved in a penalty kick and attempts to devise the best method to kick a penalty to ensure a very high success rate. The two fundamental components of a penalty shot are the angle at which the shot is kicked and the velocity of the shot. A feasible range of angles is established using right angled triangles and trigonometric ratios. Also, the sides of these triangles are calculated using Pythagoras theorem. Velocities are calculated using simple projectiles motion equations. Numerical methods are employed to find the range of velocities for the respective angles. The penalty kicks modelled in this thesis are high velocity shots placed in areas of the goal that are difficult for goal-keepers to reach. These results inform coaches about the techniques used to kick a penalty with the required trajectory. Players can practise these techniques to develop mastery. It is also important to mention the educational impact this project can have on the teaching of calculus to undergraduates. Interest is generated with the use of real world examples that appeal to students who like sports and provides a foundation for research in Applied Mathematics. This can be described as a simple and stimulating introduction to the technique of Mathematical Modelling.

Keywords: Penalty kick, goalkeeper, angle, velocity, trajectory, football, mathematical modelling.

Resumen

En el Futbol moderno el tiro de penalti es considerado la oportunidad de oro para que el jugador que lo patea lo registre como un gol. El tirador es virtualmente insuperable en cuestiones de oportunidad por cualquier jugador del equipo opuesto excepto por el portero el cual permanece sobre la línea de goleo a 12 yardas de distancia de el punto de tiro. Por lo tanto, el tirador tiene una oportunidad sobresalientemente insuperable. Maximizar esta ventaja, es un asunto de suma importancia puesto que los penaltis en muchas circunstancias, determinan el destino del partido. Este artículo analiza las variables involucradas en un tiro de penalti e intenta diseñar el mejor método para tirar un penalti y así asegurar una muy alta razón de éxito en el tiro. Un rango adecuado de ángulos es establecido usando triángulos de ángulos con orientación derecha y razones trigonométricas. También, los lados de estos triángulos son calculados usando el Teorema de Pitágoras. Las velocidades son calculadas utilizando sencillas ecuaciones de movimiento para proyectiles. Varios métodos numéricos son utilizados para hallar el rango de velocidades que corresponden a sus respectivos ángulos. Los tiros de penalti modelados en esta tesis son tiros a alta velocidad colocados en áreas de la portería que son difíciles de alcanzar para los porteros. Estos resultados informan a los entrenadores acerca de las técnicas usadas para anotar un penalti con la trayectoria requerida. Los jugadores pueden practicar estas técnicas para volverse maestros. Es importante también mencionar el impacto educacional que este proyecto puede tener en la enseñanza de cálculo para los estudiantes universitarios. El interés en el tema es generado con el uso de ejemplos de la vida real que atraen a los estudiantes a los que les gusta el deporte y provee del fundamento para la investigación en Matemática Aplicada. Esto puede ser descrito como una simple y estimulante introducción a la técnica de Modelación Matemática.

Palabras clave: Penalti, portero, ángulo, velocidad, trayectoria, fútbol, modelos matemáticos.

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I. INTRODUCTION

Taking into account the physical and technical abilities of modern professional football players it would appear that the 12 yard- spot kick is a certain goal. However, this is by no means the case since the best players on the planet are unable at times, to convert their spot kicks. This problem seems to magnify itself in penalty shootout situations to decide matches, which end with a drawn result in normal playing time. An example of this was the 2006 World Cup match between Portugal and England that ended goalless in regulation time. In the penalty shootout contest Portugal scored 3 out of their first 4 kicks and England scored only

1 out of their first 4 kicks. Portugal advanced to the semi-finals with a 3- 1 victory on penalty kicks.

Most, if not all players chosen to take penalty kicks in professional football are considered specialist 'Penalty Kick' takers. When a player takes a few paces back and then moves forward to kick a football placed on the penalty spot he does not usually think (unless he happens to be a mathematician) which part of his foot he wants to kick the ball with, what angle with respect to the starting point he wants the ball to travel or how much force he must apply to the shot to get the right velocity. What most players think before they kick a penalty is where they want to place the shot i.e. the direction.

At this point we present here a calculus- based model for football penalty kicks that address some of the short-comings of penalty kicks. We begin by stating objectively, that some players shoot and miss penalty kicks because they are shooting the ball at the wrong angle, a simple place to start and we will extend it later. Some of the more interesting facts that we'll discover by refining and interpreting our model are:

1. The best way to shoot 'penalty kicks' is as close as possible to either vertical post; on the ground or in the air just under the cross bar.
2. The velocity the ball must travel with to beat the goalkeeper.
3. The margin of error in angle and velocity of the shot that results in a goal depends on where the goalkeeper moves and the keeper's ability to cover the range of his goal- line and how quickly.
4. It is much more important to kick the ball at the right angle than with the right velocity.
5. The part of the foot that makes contact with the ball and how the rest of the body follows through with the motion influences the trajectory of the ball.

A. Mathematical Modelling

Before we attempt to model a football penalty kick, it would be useful to tell you what we mean by mathematical modelling: Mathematical modelling is the process of formulating real world situations in mathematical terms.

Mathematical modelling takes observed real- world behaviours or phenomena and describes them using mathematical formulae or equations. All the formulae you see in a typical science class are mathematical models. Mathematical models can be found everywhere, not only in science, but also in the social sciences, and even business. For instance, people make a healthy living by modelling the stock market. By constructing mathematical models, we can often explain real- world behaviours; predict how sensitive real- world situations are to certain changes. The following is a summary of the standard steps for constructing a mathematical model:

1. Identify the problem. What do you want to find out?
2. Derive the model. Identify the constants and variables involved. Make assumptions about what variables to include in the model. Determine the interrelationships between the variables.
3. Solve the equations and interpret the model.
4. Verify the model. Does it answer the original problem? Does it match up to real world data?
5. Refine the model. If the model is not satisfactory refine it by removing some of the earlier assumptions.

II. OUR FIRST MODEL: THE BEST ANGLE

It is true for most models, including this one, that trying to include every possible physical effect immediately is rather cumbersome and unrealistic especially if you want to be able to solve the model. The modelling process typically begins with the construction of very simple models which are easy to solve. Models are then refined to make them more realistic, which in turn requires the introduction of more complex mathematics in order to solve them. At the end the model should be refined enough to describe reality as close as possible while still being solvable. This refinement process will be demonstrated as we go through the modelling procedure.

A. Problem Definition

When observing penalty kick takers taking penalty kicks, we notice that the possibility of them scoring depends on their shots being placed within the framework of the goal-line. It seems reasonable that the range to score is determined by the angle at which the ball was kicked with respect to the horizontal and vertical axis of the ball's path.

Let's identify the physical constants of the problem [7].

TABLE I. The physical constants of the problem.

PHYSICAL CONSTANT	SYMBOL	VALUE
Width of goal area	w	8 yards
Height of goal area	h_t	8 ft
Horizontal distance from the centre of the goal area to the penalty spot	l	12 yards
Circumference of the ball	c	28 inches
Diameter of the ball	D_b	8.92 inches
Weight of the ball	w_b	0.94 lbs
Acceleration due to gravity	g	-32 fts^{-2}

Given that the circumference of the ball is 28 inches, the diameter will be 8.92 inches. Therefore, the radius will be just 4.46 inches. A shot entering the goal with the centre of the ball 5 inches from the goal post will appear to graze the post on entering. However, such a shot leaves no margin for error. Hence, this shot will score if the player gets it absolutely spot-on. A better area to aim for is one foot from the centre of the ball to the goalpost.

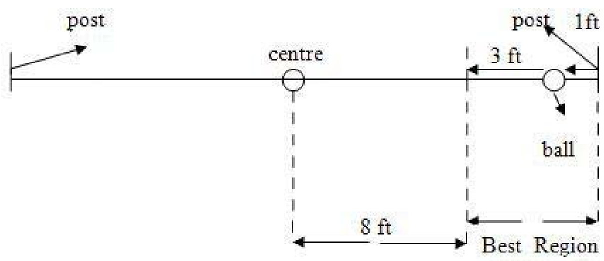

FIGURE 1. Best area to place a Penalty on the ground.

Figure 1 shows the best area to place a penalty shot on the ground. Now, the reason that the best area starts 8 ft from the centre of the goal-line is that a goalkeeper must stand in position at the centre of the goal-line. A fully stretched keeper in a horizontal position with arms outstretched will be 1.25 times the keeper's height. Therefore, a 6.5 ft keeper has a reach of 7.625 ft. 8 ft from the centre of the goal-line will be just outside such a keeper's reach. In professional football there are no goalkeepers over 6.5 ft tall which makes this area a good place to start. Let's look at the diagram more closely to identify the best region to allow for error on the ground in Figure 2.

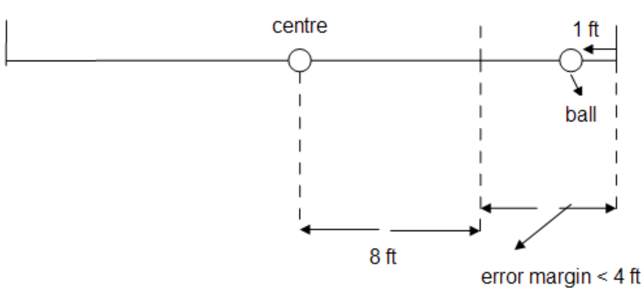

FIGURE 2. Best region to allow for error on the ground.

So far, we have only considered the best shot on the ground. We need to also consider the best shot in the air just under the cross bar. Let's look at Figure 3 to identify the best region for a penalty shot in the air with error margin.

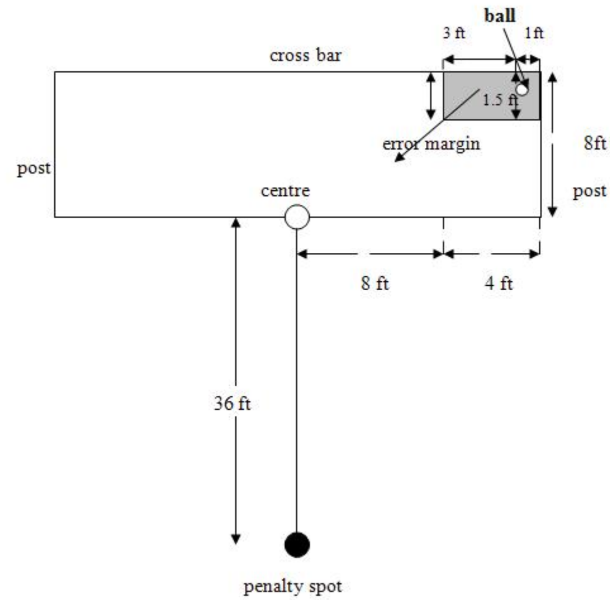

FIGURE 3. Best region for a Penalty in the air with error margin.

The shot in the air has two components of error a vertical and horizontal component. The vertical component tells us that the height of the ball must be at least greater than 6.5 ft (*i.e.* higher than the tallest goalkeeper) and the horizontal similar to the error on the ground. These two will combine to create a region very difficult if not impossible for a goalkeeper to cover during a penalty shot. In Figure 4 we illustrate the best shots on the ground and in the air.

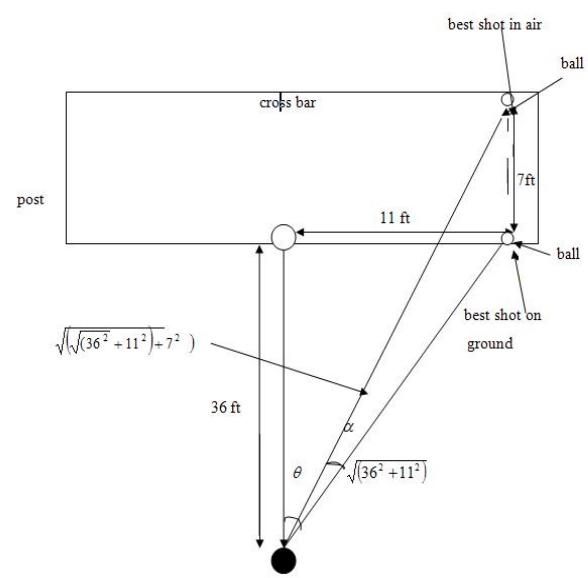

FIGURE 4. Best shot on the ground and in the air.

$$\frac{opp}{hyp} = \sin\theta ,$$

$$\frac{11ft}{\sqrt{\left(36^2+11^2\right)ft}}=\sin\theta\,,$$

$$\theta=\sin^{-1}\left(\frac{11}{\sqrt{36^2+11^2}}\right)=16.99^{\circ}\,,$$

θ is the best angle on the ground

$$\frac{opp}{hyp}=\sin\alpha\,,$$

$$\frac{7}{\left(\sqrt{37.64^2+7^2}\right)}=\sin\alpha\,,$$

$$\alpha=\sin^{-1}\frac{7}{\left(\sqrt{37.64^2+7^2}\right)}=10.53^{\circ}\,,$$

α is the best angle in the air. When watching football players shoot penalty kicks we notice that sometimes they make small errors as shown in Figures 2 and 3 and still score the penalty. It seems reasonable that the amount of error that the player can make and still have the shot score depends on the initial angle at which the ball was kicked. We'll therefore begin by defining the problem as follows.

Given a football player, what is the best angle for him to shoot a penalty? It should be carefully noted that the angle for a shot on the ground differs from the angle for a shot in the air. The previous diagram illustrates this using θ and α as the respective angles for ground and aerial shots.

Furthermore, Figure 4 is designed to facilitate a right-footed kicker since it is a known fact that a right-footed kicker controls and executes better to his right [17]. Therefore, the angles and lines drawn on the diagram are symmetric and can be reflected about the 36 ft line.

B. Deriving our first model: identify the constants and variables

The physical constants: dimensions of the goal; distance of penalty spot from goal; circumference of the ball; weight of the ball. The goal is a rectangle 8 yards across by 8 ft high. The penalty spot is 12 yards from the centre of the goal-line. The ball is 28 inches in circumference and 0.94 lbs in weight. We identified simple right angle triangles in the model with sides that were calculated using Pythagoras Theorem and angles calculated using trigonometric ratios.

C. Deriving our first model: simplifying assumptions

We make the following assumptions for our first model:

1. Allow only 'shots that travel in a straight line and enter the goal in the area illustrated by the error margin?' We do this to account for a small range of angles that keeps things fairly simple.
2. Ignore air resistance. The effect of air resistance is minor compared to the mathematical complexity it adds to the model.
3. Ignore any curl on the ball. Curl changes the straight line path the ball travels. Since we are only allowing side-footed shots and ignoring air resistance, we'll also ignore curl.
4. There is no error in the initial shooting velocity. We are assuming that some football players have problems shooting penalty kicks because they are shooting at the wrong angle. Therefore, our first model concentrates on errors in the release angle only.
5. The best shots are the two that are illustrated by the diagram. That is, the model will be one in which the initial velocity is the velocity that would allow the ball to enter the goal in these two positions.

Remember, though, that to begin with, the model should be a simple one – one that is easy to solve and interpret. Later, in the refinement, stage, the model will become more realistic and some of these assumptions will be removed.

D. Deriving the first model: mathematical interrelationships between the variables

The objective of this section is to derive a mathematical formula that expresses the amount of error a player can make in the release angle in terms of the other variables identified above. We'll do this by taking standard projectile motion equations that are derived from Newton's second law of motion. Instead of finding one long formula for the amount of error that the player can make before failing the score the penalty, it is better to break down the equation into separate parts and put things back together later. We start by resolving the initial velocity v_0 into horizontal and vertical components as

$$v_H=v_0\cos\theta_0\,, \tag{1}$$

$$v_v=v_0\sin\alpha_0 \tag{2}$$

Here θ_0 and α_0 are the initial release angles. It is important to note that the best penalty on the ground has no acceleration due to gravity or, by assumption, any air resistance. Thus the horizontal equation of motion is

$$x(t)=vt\,. \tag{3}$$

Here $x(t)$ stands for distance, v for velocity and t for time. Substituting our initial horizontal velocity into the equation we obtain

$$x(t)=\left(v_0\cos\theta_0\right)t\,. \tag{4}$$

Using the parameter l as the horizontal distance from the penalty spot to the goal-line and letting T be the time it takes to get there, we substitute $x(T)=l$ into Eq. 4 and obtain for our model

$$l = (v_0 \cos\theta_0)T \, . \tag{5}$$

Remember, we are finding the initial velocity needed v_0 for two shots that *i.e.* one on the ground and one in the air. The one on the ground only has a horizontal component Eqs. 1, 3, 4, and 5 with release angle θ_0. However, the shot in the air has both a horizontal and vertical component. Therefore, we take θ_0 as our starting point and model α_0 as the angle of elevation. Now, we write the equations for the shot in the air

$$v_0 = v_0 \cos\alpha_0 \tag{6}$$

$$v_0 = v_0 \sin\alpha_0 \, . \tag{2}$$

Here, as before, $x(t) = vt$ where $x(t)$ stands for distance, v for velocity and t for time. Substituting our initial horizontal velocity into the equation we obtain

$$x(t) = v_0 \cos\alpha_0 t \, . \tag{7}$$

Using k as the horizontal distance from the penalty spot to the goal-line at an angle θ_0 from the centre of the goal-line and letting T be the time it takes to get there we substitute $x(t)=k$ into Eq. 7 and obtain for our model

$$k = v_0 \cos\alpha_0 T \, . \tag{8}$$

Similarly, the vertical equation of motion is given by

$$y(t) = v_0 t + 0.5gt^2 = v_0 \sin\alpha_0 t + 0.5gt^2 , \tag{9}$$

where $g = -32 ft\, s^{-1}$ $(-9.8 m\, s^{-2})$ is the acceleration due to gravity. Substituting $y(T) = h,$ the vertical distance as shown in figure 4, into the above equation we obtain for our model

$$h = v_0 \sin\alpha_0 T + 0.5gT^2 , \tag{10}$$

where h is the vertical distance from the goal-line on the ground to the best spot in the air just under the cross-bar. Solving Eq. 8

$$T = \frac{k}{v_0 \cos\alpha_0} \tag{11}$$

and substituting it into Eq. 9, we find the initial velocity v_0 needed, for a given initial angle α_0, so that the ball goes in the direction illustrated in figure 4 :

$$v_0 = \frac{k}{\cos(\alpha_0)} \sqrt{\frac{-g}{2(k\tan(\alpha_0) - h)}} . \tag{12}$$

We note that this formula is designed to work for a limited range of α_0. If we let this range vary widely then we may end up with an aerial shot that may hit or go over the cross-bar. Also, it may result in a shot going too low or below 6.5 ft which is the boundary for the error-margin.

Our calculated range using vertical heights of 6.92 ft and 7.58 ft instead of 7 ft in the formula for α_0 under figure 3 is $10.42^o \leq \alpha_0 \leq 11.38.^o$ Notice that the formula gives real values only for $k\tan\alpha_0 - h > 0,$ as g is negative.

Let us now determine the range of θ_0 the angle for the shot on the ground. It is important to note that the initial angle θ_0 on the ground also works for the shot in the air. However, remember for the shot in the air we are fixing θ_0. Using the formula under figure 4 for θ and horizontal distances from the centre of goal-line of 8.42 ft and 11.58 ft instead of 11 ft we obtain the range $13.16^o \leq \theta_0 \leq 17.83^o$

In modelling, it is always good practice to note the range that your parameters can take. Otherwise you may unwittingly attain solutions which turn out to be nonsensical.

We make special note of the physical range of θ_0 so that the required shot would not hit or pass on the outside of the post. Also we don't want the shot to pass too near to the centre of the goal-line. By considering both ranges of θ_0 and α_0 we cater for our margin of error. Remember, we are aiming for a 100% success rate in terms of scoring a penalty. Therefore, the given ranges of the angles were established to achieve this.

E. Calculating the penalty on the ground

Let's go back a bit to our calculations for the best shots on the ground and the air. The respective angles are $\theta = 16.99^o$ and $\alpha = 10.53^o$. Now since the shot on the ground has no elevation we can work out its velocity using a much easier equation that does not contain angle.

$$v_0 = \frac{\text{distance}}{\text{time}} , \tag{13}$$

where *distance* = 37.64 ft and *time* = 0.3 seconds [16]. Therefore, our calculations yield $v_0 = 125.47$ ft s^{-1}.

Bear in mind that we are rounding off our calculations to 2 decimal places. Our first result shows that 125.47 ft s^{-1} is the ideal initial velocity needed for a strike of the ball to enter the goal on the ground 1 ft inside the post.

F. Calculating the penalty in the air

We now look at the aerial shot. Remember this shot has two components; a horizontal and vertical component. In order to work out the initial velocity for this shot we need to use Eq. 12

$$v_0 = \frac{k}{\cos(\alpha_0)}\sqrt{\frac{-g}{2(k\tan(\alpha_0)-h)}}\,.$$

Substituting $k = 37.64\,ft$, $h = 7\,ft$, $\alpha_0 = 10.53$ and $g = -32\,ft\,s^{-2}$ we get $k\tan\alpha_0 - h < 0$ which suggests that no real value exists by this equation. Clearly, by this equation the initial velocity for the aerial shot is impossible. Even if the value for $k\tan\alpha_0 - h$ was > 0, it would be so small that v_0 would be extremely high *i.e.* not attainable in football. We need to devise a new method to work out this part that would give us realistic results.

Our present model has calculated the velocity required for a shot on the ground. Although we have not yet determined the velocity for the aerial shot we can still discuss both methods of approach. In the first case the angle of the ankle is 0 degrees relative to the horizontal ground and the ball is struck with the inside of the foot. Also the attack angle of the foot against the ball is given 30 degrees relative to the horizontal ground. Thus, the direction of external impulse is also identical to this direction if the direction of the external impulse is toward the centre of the ball. In the second case the action is repeated; the only difference is the angle of the ankle is 20 degrees relative to the horizontal ground. In the first case the ball is struck around its centre. In the second case we need the ball to lift off the ground so the contact is made near the base.

So we have established the best way to kick a penalty in terms of angle. Careful analysis of this side-footed technique or the so called “push shot” gives maximum control to the kicker with respect to angle [20]. However, not too much power can be generated using this method. Therefore, the initial velocity for the flat shot should not be too high or else accuracy would be sacrificed [16]. A shot though, hit at 125.47 ft s^{-1} is quite powerful which would make control a challenge.

It may seem silly to imagine that a player stepping up to take a spot kick and thinking “16.99^o ” or “10.53 ^{o}”, I must shoot at these angles for a shot on the ground or a shot in the air respectively. For some players though, it may have to start this way. Then with practice, making the same shot over and over again it will hopefully become unconscious.

In order to work out the aerial shot we need to consider motion of projectiles and their range. Let us assume that the goal does not contain a net to hold back the shot after crossing the goal-line. For the aerial shot the ball would continue along a curved path until it reaches a maximum height. The ball then follows a symmetric path from when it was kicked after it reaches its maximum until it comes in contact with the ground.

The horizontal distance from where the ball is kicked to where it comes back in contact with the ground is called the range. Figure 5 illustrates the aerial path of the ball from a side view.

Again, the ball at O on the ground is kicked with a velocity v_0 at an angle α to the horizontal. We consider the vertical and horizontal motion separately in motion of this kind and use components.

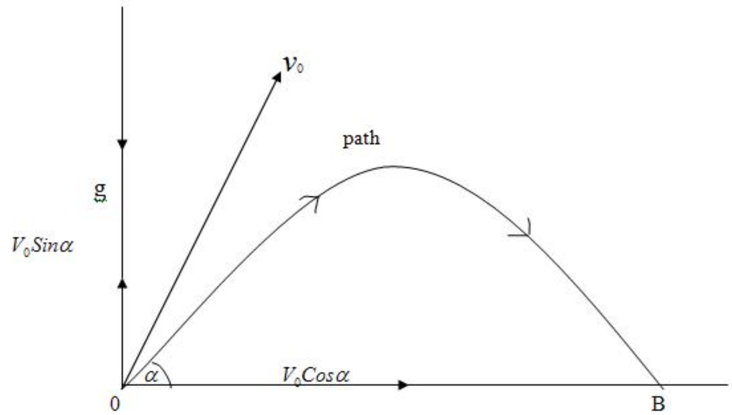

FIGURE 5. Aerial path of the ball from a side view.

Vertical motion: The vertical component of v_0 is $v_0\sin\alpha$, the acceleration a=-g. When the projectile reaches the ground at B, the vertical distance s travelled is zero. So from $s = v_0 t + \frac{1}{2}at^2$, we have

$$0 = v_0\sin(\alpha)t - \frac{1}{2}gt^2. \tag{9}$$

Thus,

$$t = \frac{2v_0\sin\alpha}{g}. \tag{14}$$

Horizontal motion: Since g acts vertically, it has no component in a horizontal direction. So the ball moves in a horizontal direction with an unchanged or constant velocity $v_0\cos\alpha$ because this is the component of v_0 horizontally. So

$$Range\ R = OB = velocity \cdot time$$

Substituting the horizontal component for motion and Eq. 14 for time we get

$$R = \cos(\alpha)\cdot\frac{2v_o\sin(\alpha)}{g},$$
$$R = \frac{2v_o^{\,2}\sin(\alpha)\cdot\cos(\alpha)}{g},$$
$$R = \frac{v_o^{\,2}\sin(2\alpha)}{g}. \tag{15}$$

Our task now is to determine a possible range for the shot to work out v_0. Clearly, after the ball crosses the goal-line at 7ft above the ground it continues to rise. At this point the horizontal distance is 37.64ft. If at this point the ball does not reach it maximum then we can safely say the range will be more than twice 37.64ft. Let us assume that the ball reaches its maximum height 65ft horizontally from the penalty spot well after it crosses the goal-line. Since from this point the path of travel is symmetric, the range will be 130ft. Remember; we removed the net so the ball continues along this path after it crosses the goal-line. Let us illustrate this in the following diagram

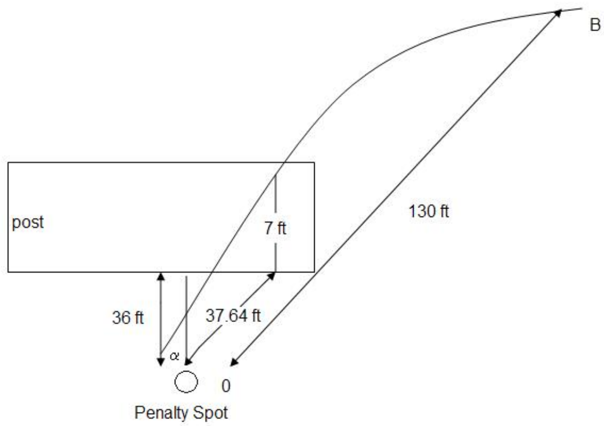

FIGURE 6. Aerial path the ball travels from a front view.

Now, using R = 130 ft, g = 32 ft s^{-2} and α =10.53^{o} we calculate the velocity for the aerial shot.

$$v_0 = \sqrt{\frac{Rg}{\sin(2\alpha)}} \, . \tag{16}$$

Therefore, v_0 = 106. 68 ft s^{-1}.

III. OUR SECOND MODEL: THE BEST TRAJECTORY

Let's improve our model by removing the assumption that the best shots are the ones where the ball enters the goal 1ft inside the post on the ground and 1ft inside the post 7ft in the air. Still keeping the same equations of motion, we now let both the initial velocity v_0 and the distance and initial angle α_0 vary independently and at the same time. Each pair (v_0, distance) or (v_0, α_0) will give the ball a trajectory that results in either a penalty made or a penalty missed.

A. Constructing a feasible region

Again, if we refer back to Figure 2 and Figure 3 we identify the ranges called the error margins. Throughout these regions, we employ the use of numerical methods to describe the ordered pair.

The feasible region of trajectories is the set of all pairs (v_0, distance) that result in a successful penalty kick on the ground and (v_0, α_0) for a successful penalty in the air (using the assumptions on allowable trajectories). Below are the two programs that give us graphs for both the ground and aerial penalties. MATLAB was used to design these programs.

```
function penalty
%this program works out the various velocities V0 for ground shot
%for penalty kicks
distopp=8.42; %This is the opposite side to theta. It varies
distadj=36; %This is the adjacent side to theta, it is fixed
hypot=sqrt(distopp^2+distadj^2);%pythagorus theorem work out distance travelled
time = 0.3; %Time needed to beat the Goalkeeper
n=100 %number of points taken
%simple for loop to calculate all the velocity values
%for a reasonable range of distances
for j=1:n
V0=hypot/time; %change the angle to radians
distance(j)=hypot; %store the distance travelled used to calculate velocity
distopp=distopp+0.01; %update the opposite side to theta in 0.01 increments
hypot=sqrt(distopp^2+distadj^2); %calculate next updated distance travelled
vel(j)=V0; %store the velocity calculated for the angle alpha
end
%Now we plot the graph
figure(1)
plot(distance,vel,'*');
title('Graph showing initial velocity Vo versus distance travelled for ground shot')
xlabel('Distance (ft)')
ylabel('Vo (ft/s)')
return
function penalty2
%this program works out the various velocities V0
%For given alpha projectile angles
%for the projectile equation for penalty kicks
R=130; %This is the paramater for the range
g=32; %Acceleration due to gravity
n=100 %number of points taken
alpha=10.42; %first angle taken to calculate the velocity
%simple for loop to calculate all the velocity values
%for a reasonable range of angles alpha
for j=1:n
V0=sqrt(R*g/sin(2*(pi/180)*alpha)); %change the angle to radians
 angle(j)=alpha; %store the angle (in degrees) used to calculate velocity
alpha=alpha+0.01; %update the alpha in 0.01 increments
 vel(j)=V0; %store the velocity calculated for the angle alpha
end
%Now we plot the graph
figure(1)
plot(angle,vel,'*');

title('Graph showing initial velocity Vo versus projection angle alpha for aerial shot')
xlabel('alpha (degrees)')
ylabel('Vo (ft/s)')
return
```

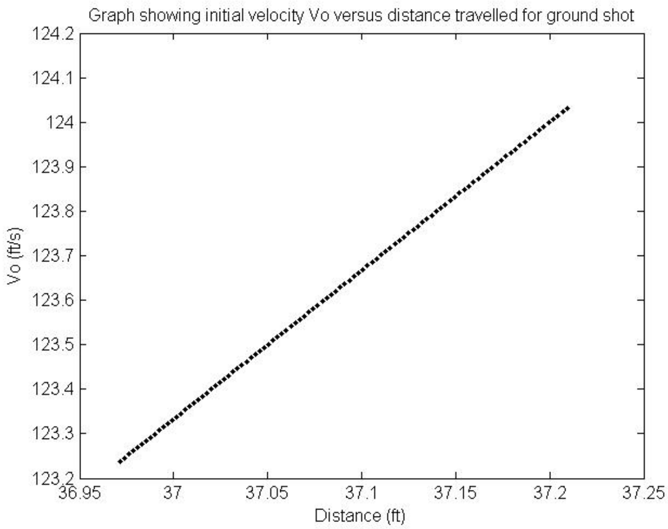

FIGURE 7. Ground Penalty.

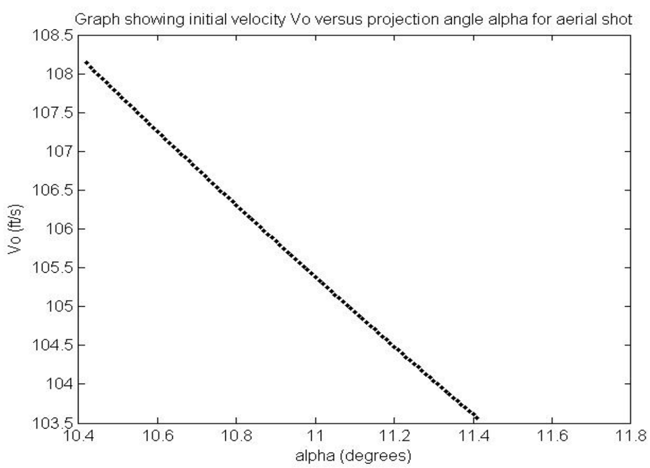

FIGURE 8. Aerial Penalty.

IV. COMPARING OUR MODEL TO REAL-WORLD DATA

A true test of a model's validity lies in its ability to predict real-life behaviours. That is, when modelling you needs to evaluate the model you have after you've created it. It is common knowledge that quite a significant number of penalty shots hit the post and still score. If such shots can be perfected they would undoubtedly be the best since they are furthest from the goalkeeper's reach. However, to aim for the post in order to score makes the modelling much more complex. We would have to consider spin on the ball since spin influences the direction in which the ball is deflected after making contact with the post.

A recent analysis done on penalty kicks of past World Cups dating back to 1982 and following up to 1998 shows the percent of penalty kicks converted [10]. Advancement and elimination of many teams has been decided by a penalty shoot out.

TABLE II. A recent analysis done on penalty kicks of past World Cups.

1982	77%
1986	77%
1990	73%
1994	75%
1998	80%
Average	76%

This analysis has produced results to inform us that 1998 was the most successful for converting penalty kicks. At the 1998 World Cup two second round games (France & Italy, Argentina & England) and a semi final clash (Brazil and Holland) ended up in penalty shoot outs. A further 18 penalty kicks were awarded during regulation play making a total of 46 penalty kicks in the whole tournament. Thirty seven of the 46 penalty kicks were scored (80%). The startling thing about this statistics is that the success rate has varied very little since 1982.

Our models focus primarily on the "push" shot. However, at the 1998 World Cup penalty kicks were broken down into three categories. The first is the "push" shot, the second the "driven" shot and the third the "cut" shot. The cut shot was the most popular by far. 27 of the 46 kicks used this technique with a success rate of 89%. The push shot was used 14 times with a success rate of 71%. The least popular and least successful was the driven shot. Out of 5 attempts only 3 goals were scored. However, none were saved. Both misses hit the crossbar.

Not surprisingly most of the kicks were taken with the right foot (the base of our model). Ten players shot with their left foot. It is worth noting that 70% of left foot shots went to the goal keeper's left-hand side. 67% of right footed players shot to the goal keeper's right hand side (contrary to the model). Perhaps more significant is an examination of the "cut" shot. Twenty-one right-footed players used this technique resulting in 20 shots going to the goal keeper's right-hand side (opposite to that of the model). Five of 6 left footed players shot to the goal-keepers left hand side.

Two important things these results bring to our problem. They are essentially, the cut shot being the "best shot" and if you're right footed go left and vice versa. It is important to note however, that the cut shot does not follow a straight line path as illustrated in Figure 3. The path is curved which would give rise to more complex equations to solve. Also, different curved paths may bring about the same end result. This would in turn impact greatly on the player and his ability to execute.

In football penalty kicks pit the goalkeeper against a lone striker in a mentally demanding contest. Once the penalty taker strikes the ball, it takes 0.3 seconds to hit the back of the net – unless the goalkeeper can somehow get his body in the way [17]. Modern changes in football have allowed goalkeepers to move on their goal-line before a penalty kick is taken. This would make it possible for a goalkeeper to save a penalty shot described in our model by covering the area on that particular side of the goal. If the goalkeeper moves prior to the shot and then stretches sideways his reach can extend to the post or even beyond. This puts the onus on the kicker to kick to the other side in order to score.

Game theory, applied to the problem of penalties, says that if the striker and the goalkeeper are behaving optimally, neither have a predictable strategy. The striker might favour his stronger side, of course, but does not mean that there will be a pattern to the bias. Game theory also says that each choice of shot should be equally likely to succeed, weighing up the advantage of shooting to the stronger side against the disadvantage of being too predictable. If shots to the right score three- quarters of the time and shots to the left score half the time, you should be shooting to the right more often. However, as you do, the goalkeeper will respond and shots to the right will become less successful and those to the left more successful.

Ignacio Palacios-Huerta, an economist at Brown University, found that individual strategists, out of 42 top players whom he studied, only three departed from game theory's recommendations-in retrospect, they succeeded more often on one side than the other and would have been better altering the balance between their strategies.

Professionals such as the French superstar Zinedine Zidane and Italy's goalkeeper Gianluigi Buffon are apparently superb economists. These two players are absolutely unpredictable and, as the theory demands, they are equally successful no matter what they do, indicating that they have found the perfect balance among the different options.

In professional football either turf grasses are used to cover the surface of the pitches or artificial grass surfaces made from non-abrasive fibres allowing the ball to sit in the surface, controlling ball roll and allowing freedom of movement for the rubber which adds resilience to control ball bounce and absorb impact [29]. However, the latter is recognised by the governing body- FIFA- for only matches below World Cup Finals level. Artificial surfaces facilitate the smooth movement of the various skills in kicking because these surfaces are resistant to wear and tear. The variables involved in a penalty have a stable environment to perform within.

Turf grasses on the other hand are subject to wear and traffic stresses and recovers from their damages slowly [5]. On different occasions when penalties are taken the pitch conditions may not be the same and hence this factors into a penalty kick. Let us assume that these pitches are well maintained in terms of cutting the grass, aeration and drainage. There are areas where soil composition may vary. This would influence movement and control of the ball. Therefore, techniques would have to be adjusted during a penalty kick situation.

Nowadays football is played virtually the whole year around. On top of that professional players not only play at home, but also go abroad and can experience a variety of climates, which they have to be aware of because it will affect the way they play the game. Rain before a match can make the surface very soft, and make it difficult for a player to keep his footing, it also slows down the movement of the ball along the playing surface. More force will have to be applied in kicking a ball on a soaked playing surface than on an ideal surface to get the same velocity.

In cold weather muscles at not at their optimal level, therefore more warm up exercises are recommended to prevent injury. In hot weather, fewer warm-ups are needed and electrolytes are taken in by players to prevent dehydration. The wind can also play a big part on the game as it can influence the direction the ball goes. Kicking the ball off the ground in the wind involves a lot more skill as the player has to take into consideration which way the wind is blowing, because it will carry the ball. This makes the penalty kick particularly difficult. Our models have not taken into consideration these factors which will definitely determine how a penalty kick is taken.

V. PSYCHOLOGY OF PENALTIES

Psychology of football is at the core of being a great athlete. How many great athletes do you know have little confidence in their playing ability? Just the ability to have strong confidence in your game can take you to another level [27].

Penalty kicks and stress was studied in a laboratory simulation. The moment beyond which the probability of the kicker to respond to an early goalkeeper dive was < 50%. Point of no return, in quiet and ideal laboratory conditions, was around 820 ft s^{-1} before kicker – ball contact. Although motivation was generally considered to be critical in the performance of professional players in a stressful penalty situation, this problem has been rarely addressed. The purpose of the study was to investigate the effect of a noisy and participative audience on the performance of volunteers in a simulated penalty kick task.

Twenty – one undergraduate students performed the simulated penalty task as part of a practical on motor control. The image on the computer screen to which participants responded was visible to >70 student spectators, in real time on a large screen. Participants were divided in two teams, competing as if in a penalty shoot out. The audience was encouraged to support or boo participants as they performed.

Unexpectedly, performance under stress saturated at 70%, *i.e.* even if the goalkeeper moved a full 30.49 ft s^{-1} (100 ms^{-1}) sooner than necessary for perfect performance in the laboratory, participants under stress seemed unable to show 100% performance, putting the ball on the same side as the goalkeeper on about >30% of the trials. Failure rates in actual penalties in official games were around 25-

35%, remarkably close to the result obtained in this laboratory simulation. There may have been a link between stress and imagining failure, and that imagining failure inevitably may have contributed to imperfect motor planning, leading to a certain error rate difficult to avoid, at least without adequate preparation.

The moment the referee has placed the ball on the penalty spot and the goalie is gearing up. There may be thousands of people watching as you prepare to take your penalty kick. If you score you're a hero, if the goalie saves your shot then he reigns supreme. But what does all this pressure do to your ability to a perfect penalty kick? Taking a penalty kick in football can be major pressure. You may have practiced countless times perfecting your penalty during football training but the pressure of a real football game is something else again.

The right mind-set is of paramount importance. The key to good sports psychology is not I repeat not "having positive thoughts". You can be positive generally about your play and before a match but optimum sports psychology for pressure situations like penalty kicks means having no thoughts, just pure focus [18]. Penalty takers will often try to fool the goalie. To enter a state of flow or "being in the zone" when taking a penalty shot you need to stop thought. The state of mind you need to have to take world beating penalties is akin to the purest hypnotise trance state. You need to instinctively know how to make the crowd disappear. At the same time all thoughts of success of failure need to fade away.

Remember, a penalty kick is not an activity taken in a vacuum. Our present model does not include the pressure felt by the player who takes the shot and how it influences his ability to perform as required. Therefore, the so called "perfect" penalty shot may vary depending on the situation at hand.

VI. CONCLUSION

At this point we are not in the process of obtaining any new results. Our mission is simply to reiterate what was done and appraise it. By so doing we can outline the usefulness of our project.

The modelling process carried out presents an ideal situation for a player with limited technical ability to execute a well placed and powerful penalty shot. The skill level is basic and the player can maximise on his control. The focus is on which part of the foot makes contact with the ball and the follow through. The two components of the penalty shot are the angle and velocity. If the player gets the right measure of both, then the shot is almost certain to result in a goal.

Apart from the development of penalty kicking, the ideas the model presents in the project are manifested in ways that are interesting and stimulating to students. Therefore, they not only learn about projectile paths and calculus but enhance their knowledge about the world's most popular sport which would increase their interest in the game. In other words this project has a two fold effect in that it is educational and intriguing.

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