Half, Average, and most probable lives of filament lamps

D. C. Agrawal[1], V. J. Menon[2]
[1]*Department of Farm Engineering, Banaras Hindu University, Varanasi – 221 005, India.*
[2]*Department of Physics, Banaras Hindu University, Varanasi – 221 005, India.*

E-mail: dca_bhu@yahoo.com

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Abstract

The three lives of the tungsten filament lamps viz. the half life, the average life and the most probable life are shown analytically to coincide.

Keywords: Incandescent lamps, tungsten filament.

Resumen

Se muestran las tres vidas de las lámparas de filamento de tungsteno. Analíticamente se muestra que la vida media y la más probable, coinciden.

Palabras clave: Lámparas incandescentes, filamento de tungsteno.

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In a previous paper [1] the relevant expressions demonstrating the approximate equality of the half life, average life and most probable life for incandescent bulbs derived by Leff [2] were further elaborated for the benefit of students. However, since Leff's model was an empirical generalization of the radioactive law it does not follow from the principles of statistics and all the said derivations cannot be done analytically. The present note extends such calculations to a physically realistic tungsten evaporation model [3, 4] which can be deduced from the laws of statistics and leads to analytical derivations.

In our evaporation model the thermal ejection of an atom corresponds to "failure" and nondecay of an atom corresponds to "success" for a binomial distribution. By making a normal approximation and assuming that the bulb fails when the undecayed fraction of atoms falls below a critical value we derived the following expression for the survival probability:

$$S(\tau) = 0.5\,[1 + erf(w_c)],\; w_c \equiv 3\,(1-\tau)\,. \qquad (1)$$

Here *erf* stands for the error function [5], the dimensionless time interval $\tau = t/t_{1/2}$, t is experimental elapsed time and $t_{1/2}$ is the half life in hours. By definition, at the half life point $\tau=1$ we have

$$S(1) = ½\,. \qquad (2)$$

Next, to find the most probable life τ_m we report the general rates of change

$$R(\tau) \equiv dS/d\tau = (3/\sqrt{\pi})\exp(-w_c^2),$$

$$\dot{R}(\tau) \equiv dR/d\tau = 18(1-\tau)\,R(\tau). \qquad (3)$$

By setting $\dot{R}(\tau)$ as zero we identify the most probable life

$$\tau_m = 1. \qquad (4)$$

Finally, to determine the average life τ_{av} we use the definition

$$\tau_{av} = \int_0^\infty R(\tau)\,\tau\, d\tau \Big/ \int_0^\infty R(\tau)\, d\tau\,. \qquad (5)$$

This can be evaluated by making the transformation $\tau = (1 - w_c/3)$ in the range $3 \geq w_c \geq -\infty$:

$$\tau_{av} = \int_{-\infty}^{3} \frac{dw_c}{3}\left(1 - \frac{w_c}{3}\right)\frac{3}{\sqrt{\pi}}\exp\left(-w_c^2\right),$$

$$\tau_{av} = \frac{1}{2}\left[1 + erf\left(3\right)\right] - \frac{e^{-9}}{6\sqrt{\pi}},$$
$$\tau_{av} = 1 - O\left(e^{-9}\right) = 1 - O\left(10^{-4}\right). \qquad (6)$$

Hence our expressions for $\tau_{1/2}$, τ_m and τ_{av} accurately coincide within a tiny correction factor of order 10^{-4}. The basic reason for such remarkable equality is that the expression for R (τ) written in Eq. (3) is symmetrical about the point $\tau = 1$. Thus, Leff's remark that "light bulb labels are believable" has been further strengthened by the present note.

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