On Students' Misunderstanding of the Basic Concepts of Quantum Mechanics: the case of Algerian Universities

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Abstract

Through many years of teaching Quantum Mechanics in Algerian universities, we have noticed that students face great obstacles in understanding the meaning of the basic concepts of this theory, such as the Wave Function, the Uncertainly Principle, the Superposition Principle and the Complementarity Principle. We believe that these obstacles rise due to the influence of basic concepts in Classical Mechanics which students studied for many years before studying Quantum Mechanics. For example, in Classical Mechanics, the concept of wave and the concept of corpuscle refer to different things, while in Quantum Mechanics both concepts refer to the same thing, namely a **'**quantum particle**'**. In order to study the problem, a questionnaire sheet was presented to concerned students third-year (undergraduate physics students) before they studied the subject and after studying it. We note that these third-year students had had, as an introduction, a deep discussion of the meaning of quantum mechanical concepts for about one hour and a half a week during one month. To compare the results and show the importance of the epistemological and educational aspects, we presented again the same questionnaire to fifth-year physics students (B.Sc degree). We mention that the latter learned Quantum Mechanics without an epistemological introduction of the basic quantum mechanical concepts. Before giving our epistemological and didactical analysis of the results, we give a brief review of a previous didactical study related to the subject made by other authors. We conclude by giving some suggestions which we think are necessary in teaching Quantum Mechanics.

Keywords: Quantum Mechanics, wave function, superposition principle, uncertainly principle, complementarity principle.

Resumen

A través de muchos años de enseñanza Mecánica Cuántica en universidades argelinas, hemos observado que los estudiantes enfrentan grandes obstáculos para comprender el significado de conceptos básicos de esta teoría, como el de función de onda, incertidumbre, el principio de superposición y el principio de complementariedad. Creemos que estos obstáculos aumentan debido a la influencia de los conceptos básicos de la Mecánica Clásica que los estudiantes han estudiado durante muchos años antes de estudiar la Mecánica Cuántica. Por ejemplo, en Mecánica Clásica, el concepto de onda y el concepto de corpúsculo se refieren a cosas diferentes, mientras que en la Mecánica Cuántica ambos conceptos se refieren a lo mismo, es decir, una "partícula cuántica". Con el fin de estudiar el problema, se presentó un cuestionario a los estudiantes de tercer año en cuestión (estudiantes universitarios de física) antes de que se estudiara el tema y después de estudiarlo. Notamos que estos estudiantes de tercer año habían tenido como una introducción, una discusión profunda sobre el significado de los conceptos mecánico cuánticos durante aproximadamente una hora y media a la semana durante un mes. Para comparar los resultados y mostrar la importancia de los aspectos epistemológicos y educativos, presentamos de nuevo el mismo cuestionario a los estudiantes de física de quinto año (licenciatura). Mencionamos que estos últimos aprendieron la Mecánica Cuántica, sin una introducción epistemológica de los conceptos básicos de la Mecánica Cuántica. Antes de dar nuestro análisis didáctico y epistemológico de los resultados, damos una breve reseña de un estudio didáctico previo relacionado con el tema realizado por otros autores. Se concluye dando algunas sugerencias que creemos que son necesarias en la enseñanza de la Mecánica Cuántica

Palabras clave: Mecánica Cuántica, función de onda, principio de superposición, principio de incertidumbre, principio de complementariedad.

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I. INTRODUCTION

In order to show the importance of the subject, we would like to start our work by the following statement: **'**I believe that any lover of nature should study Quantum Mechanics- not its mathematics- but its ideas**'**[1].

In the latter part of the 19th century, most physicists believed that the ultimate description of nature had already been achieved and that only the details remained to be worked out. This belief was based on the spectacular and uniform success of Newtonian mechanics, combined with Newtonian gravitation and Maxwellian electrodynamics, in describing and predicting the properties of macroscopic systems which ranged in size from the scale of the laboratory to that of the cosmos. However, as soon as experimental techniques were developed, to the stage where atomic systems could be studied, difficulties appeared which could not be resolved within the laws and even concepts of classical physics. The new laws and new concepts, developed over the first quarter of the 20th century, are those of quantum theory. But unfortunately the quantum theory interpretations are not easy to understand as the Classical Mechanics interpretation is. There exists a number of contending schools of thought, differing over whether quantum mechanics can be understood to be deterministic and which elements of quantum mechanics can be considered **'**real**'** or else. Although today this question is of special interest to philosophers of physics, many physicists continue to show a strong interest in the subject. The operational definition of the technical terms used by researchers in quantum theory, such as wave functions and matrix mechanics, progressed through intermediate stages. For instance, Schrödinger [2] originally viewed the wave function associated to the electron as the charge density of an object smeared out over an extended, possibly infinite, volume of space. Max Born interpreted it as the probability distribution in the space of the electron's position [3].

Albert Einstein had great difficulty in accepting some of the consequences of the theory, such as quantum indeterminacy. In addition to the unpredictable character of measurement processes, there are other elements of quantum physics that distinguish it sharply from classical physics and which cannot be represented by any classical picture. One of these is the EPR paradox. It is a thought-experiment that was proposed by Einstein, Podolsky, and Rosen [4] and which was performed by Alain Aspect [5]. This experiment demonstrated that if two quantum systems interact and then move apart, their behaviour is correlated in a way that cannot be explained in terms of signals travelling between them at or slower than the speed of light. This phenomenon is known as non-locality, and is open to two main interpretations: either it involves unmediated, instantaneous action at distance, or it involves faster than light signaling.

Major difficulties of interpretation of Quantum Mechanics description whish disturb students' understanding are:

- The problem of non-determinism in quantum theory.
- The uncertainty principle of Heisenberg.
- The interpretation of the superposition principle in quantum mechanics.
- Bohr's concept of complementarity.

We believe also that one of the great obstacles to the good understanding of quantum theory, in addition to the interpretations difficulties, is the mathematical structure of this theory, which is based on fairly abstract mathematics, such as the Hilbert space and the operators on it [6]. This space is composed of infinite complex functions **(**infinite sets**)** and the physical quantities related to the particle are calculated from the operations of operators **(**having no physical meaning) on complex functions belonging to the states' space, where the latter is a subspace of the Hilbert Space. The complex wave functions in this space must be square- integrable functions**.**

$$\int\int\int_{-\infty}^{\infty} \left|\psi\right|^2 dv < \infty \,.$$

In Classical Mechanics and Electromagnetism, on the other hand, the properties of a point mass or the properties of a field are described by real numbers or functions defined on two or three dimensional sets. These have direct spatial meaning, and in these theories there seems to be less need to provide a special interpretation for those numbers or functions.

Because of the difficulties of interpretation, mentioned above, in the basic concepts of quantum mechanics, we wrote a questionnaire sheet related to them. The questionnaire was presented to a sample of 80 students. Our aim was to study the problematic of the students' misunderstanding of basic concepts. We should notice that we did not ask questions about the different schools of interpretation. We only wanted to know what the students understood about the basic concepts of quantum mechanics according to the Copenhagen School interpretation which is the most popular among scientists and is the used interpretation in teaching quantum theory.

The Copenhagen interpretation was formulated by Niels Bohr and Werner Heisenberg while collaborating in Copenhagen around 1927 [7]. Bohr and Heisenberg extended the probabilistic interpretation of the wave function proposed by Max Born [3]. The Copenhagen interpretation rejects questions like **'**where was the particle before I measured its position?**'** as meaningless. The measurement process randomly picks out exactly one of the many possibilities allowed for by the state's wave function.

The essential controversial features of the Copenhagen interpretation are:

- The uncertainty principle (also called the indeterminacy principle) of Heisenberg.
- The Principe of complementarity of Bohr.

With the passage of time, the Copenhagen interpretation has been more specifically identified with a concept known as the **'**collapse of the wave function**'**, also called

'reduction of the_wave packet**'** as formulated by John von Neumann [6].

Today, many physicists are interested in the epistemological and the didactical aspects of Quantum Mechanics. We can classify them into the following categories:

-Some of them are working in pure epistemological Quantum Mechanics researches. These scientists are interested in the interpretations of the basic concepts of quantum theory. The interpretations have started since the rise of quantum theory and many different schools of quantum theory have appeared.

As we saw in the introduction, there exist a number of contending schools of thought differing over whether Quantum Mechanics can be understood to be deterministic and which elements of quantum mechanics can be considered **'**real**'** or else.

-Some others are working on Visual Quantum Mechanics **(**VQM**)** where they present some basic ideas of quantum mechanics by integrating hands-on activities and computer visualization**.** They use (QSAD**)** **'**Quantum Science Across Disciplines**'** which is a software application producing graphical representations of atoms and molecules without requiring students to perform high-level computations. Students can create visual models of different atoms and molecules, predict their behaviour and test those predictions, as example of this kind of work; we suggest the following work [8].

-There are other researchers who are only interested in the didactical aspect of Quantum Mechanics.

Education research in Quantum Mechanics has given a great importance to the investigation and the deduction of the best way to teach and transmit this subject to students (see for example: **'**Students' Concepts of Quantum Physics**'** [9]. All the above researchers concluded that Quantum Mechanics is difficult and abstract. Furthermore, understanding many classical concepts especially waves and optical physics are prerequisites to a meaningful understanding to quantum systems.

In addition to those researchers' conclusion, we add the importance of a good interpretation of the basic concepts of quantum mechanics (see our discussions and analysis in paragraph V).

In our work, we wanted to highlight the importance to the students of having a strong knowledge of the basic quantum concepts.

II. DESCRIPTION OF THE QUESTIONNAIRE

The multiple-choice questions were the following (there is only one right answer in each set):

-The Wave Function*:*

Set 1:

- The wave function associated to a particle is real, *i.e.*, represents a real wave.
- It is only an abstract mathematical tool used to describe particle motion.
- It describes the electron's charge distribution.
- No idea.

Set 2:

- Particle motion is described by a wave packet.
- Particle motion is described by a monochromatic wave.
- Particle motion is sometimes described by a packet wave and other times by a monochromatic wave.
- No idea.

-The Uncertainly Principle:

Set 1:

- This principle is related to the wave aspect.
- This principle is related to the corpuscle aspect.
- This principle is sometimes related to the wave aspect and, other times, to the corpuscle aspect.
- No idea.

Set 2:

- It is possible to measure simultaneously pairs of canonically conjugate variables.
- The measurement of one variable of the **'**pairs conjugate variable' disturbs the second variable measurement.
- There is no relation between this principle and the measurement of one variable of the **'**pairs conjugate variables**'**.
- No idea.

-The Superposition Principle:

Set 1:

- The particle is located in all superposed states before the measurement.
- The particle is located in only one well-defined state before the measurement.
- The particle is located in only even states before the measurement.
- No idea.

Set 2:

- The particle will be found in all superposed states when a measurement is made.
- The particle will be found in only one state when a measurement is made.
- The particle will be found only in the odd states when the measurement is made.
- No idea.

-The Complementarity Principle

Set 1:

- The wave aspect and the corpuscle aspects refer to the same object.
- The wave aspect and the corpuscle aspect do not refer to the same object..
- The wave aspect and the corpuscle aspect refer only sometimes to the same object and other times to different objects.
- No idea.

Set 2:

- A particle behaves as a wave in some cases and behaves as a corpuscle in other cases.
- A particle behaves as a wave and a corpuscle at the same time.

- There is no relation between the particle aspect and the wave aspect.
- No idea.

III. QUESTIONNAIRE RESULTS

The results got from the questionnaire before the students **(**third- year physics students**)** learned Quantum Mechanics were catastrophic. Nearly all students gave wrong answers to the above questions. In fact, most students had **'**no idea**'** about the questions asked (see the average of the different answers in figure 1).

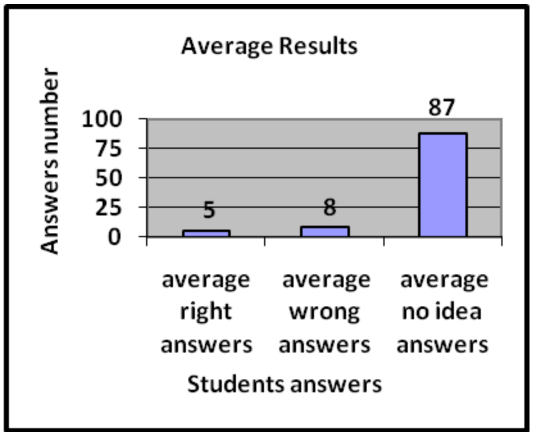
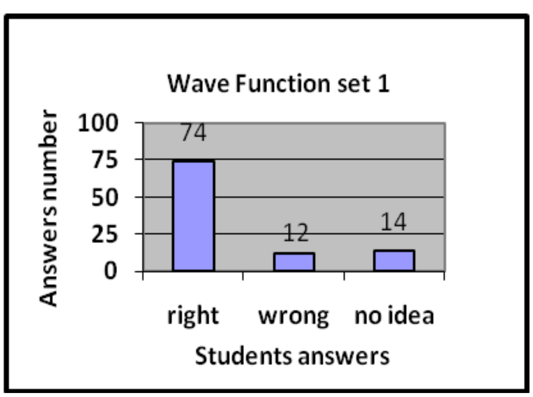


FIGURE 1. Average results of questionnaire application.

Note that we only gave the average results without showing the details of the multiple-choice questions. This meant that the concepts of Quantum Mechanics were new and strange to them.

Before we give the results got from third-year physics students after they had a course on Quantum Mechanics, we should mention that these students had, at the beginning of the course, an epistemological introduction related to quantum theory before they studied its mathematical formalism.

(We note that most universities, at least in our country –Algeria– start teaching quantum theory directly through its mathematical formalism). The results are shown in figures 2.



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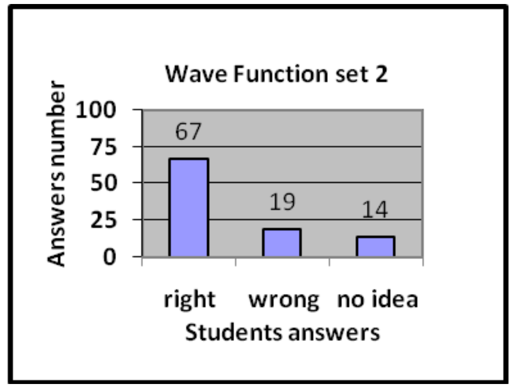

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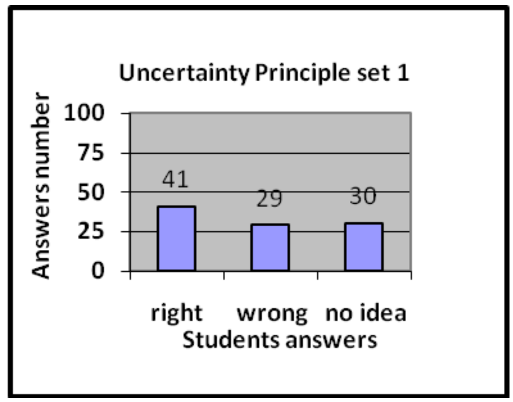

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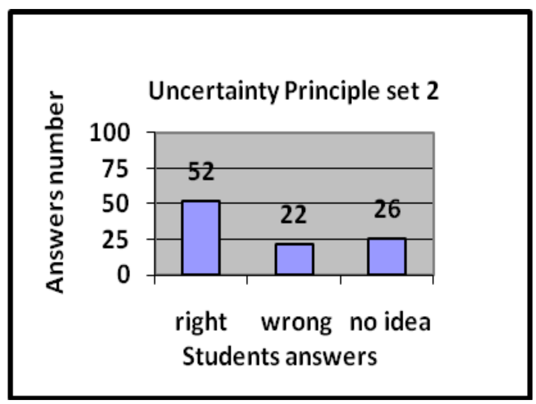

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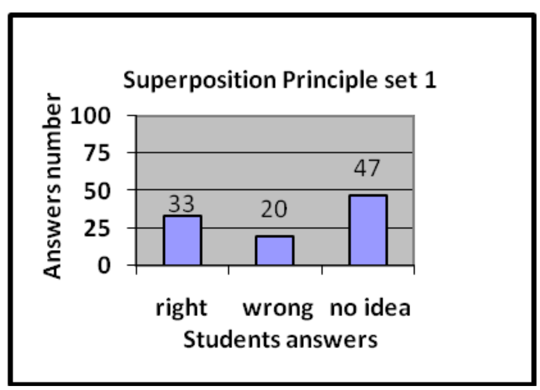

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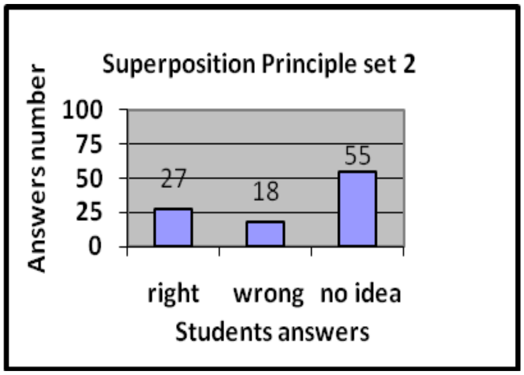

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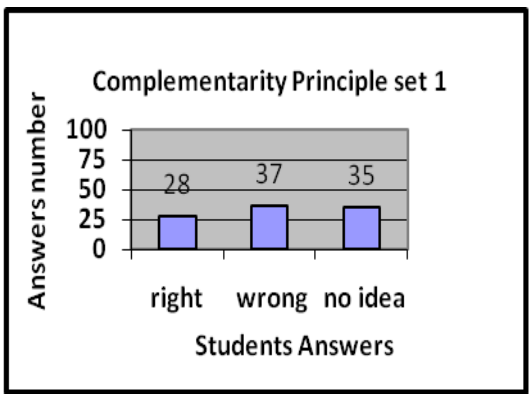

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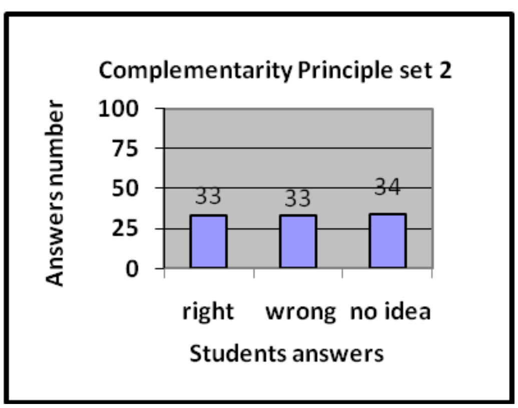

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FIGURE 2. a) Results about Wave function Set 1. b) Results about Wave function Set 2. c) Results about Uncertainty principle Set 1. d) Results about Uncertainty principle Set 2. e) Results about Superposition principle Set 1. f) Results about Superposition principle Set 2. g) Results about Complementarity principle Set 1. h) Results about Complementarity principle Set 2.

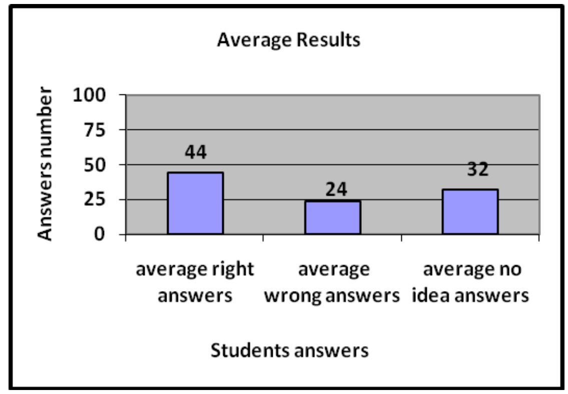
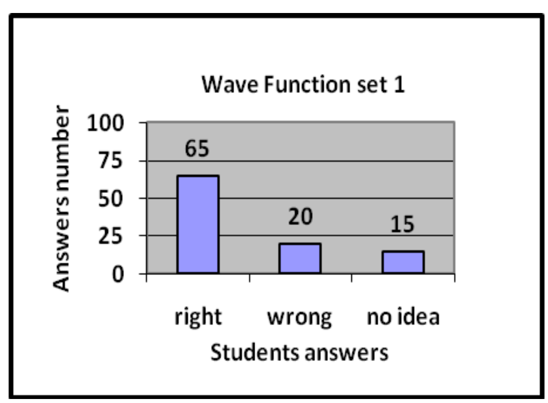


FIGURE 3. Average results of questionnaire application.

IV. DISCUSSIONS AND ANALYSIS

We can conclude from the results in the histogram graphics that these results are not good enough for a physics science course, especially for third-year physics students. We can also say that most students did not well understand the basics concepts of quantum mechanics, even though those who were questioned took, at least one month of 1.5 hour a week course in Quantum Mechanics. During this course, students had an introduction on how Quantum Mechanics interprets the motion of particles and many experiments were deeply discussed according to the different schools of interpretation. Now we would like to give other results of the same questionnaire presented, this time, to fifth-year physics students (40 students) in order to see the impact of introducing an epistemological study with discussions of the basic quantum mechanics concepts, as an introduction, at the beginning of the study of quantum theory (see figures 4).

We should mention that these students had an (Introduction to Quantum Theory) course in their third academic year and an (Advanced Quantum Mechanics) in their final year (fifth year). But these students learned quantum theory without an epistemological introduction (deep discussions of the basic concepts). They start their quantum mechanics course directly with its mathematical formalism.



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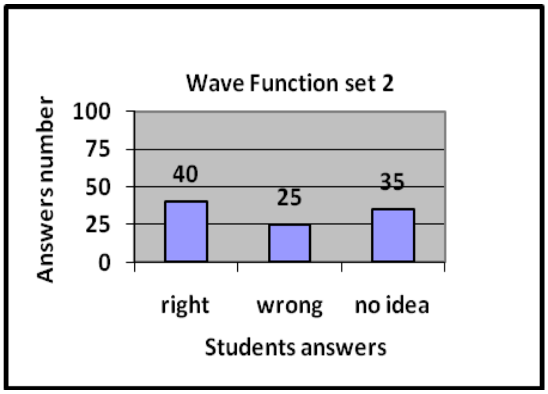

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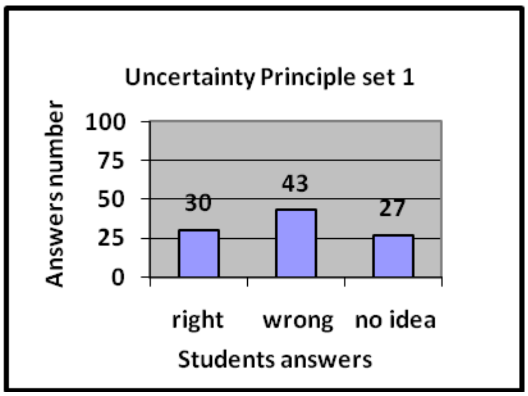

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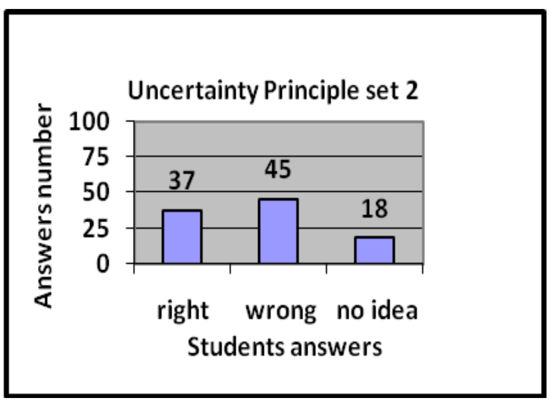

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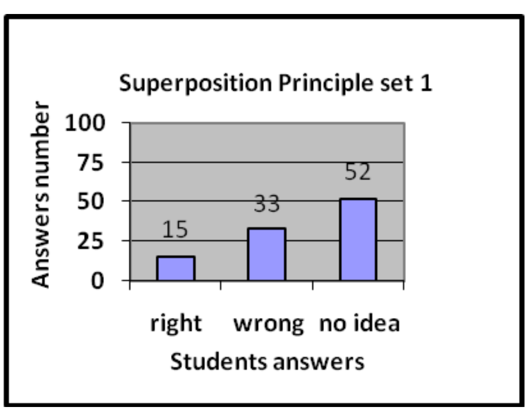

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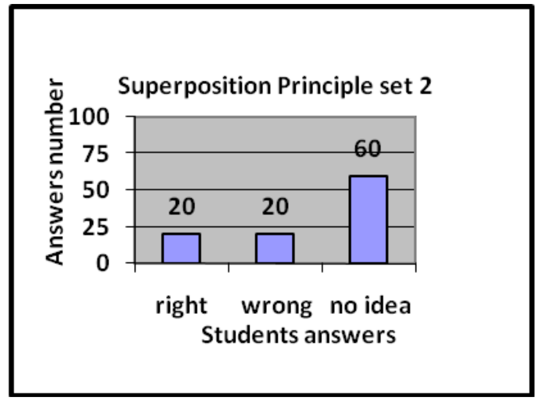

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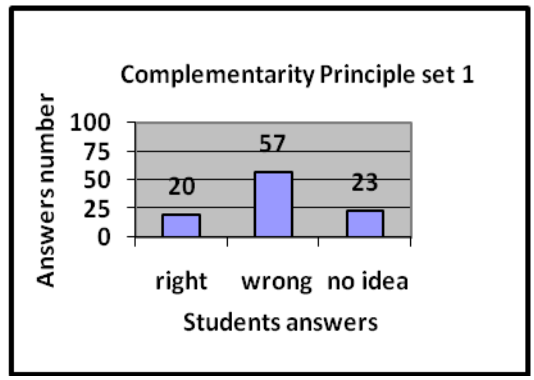

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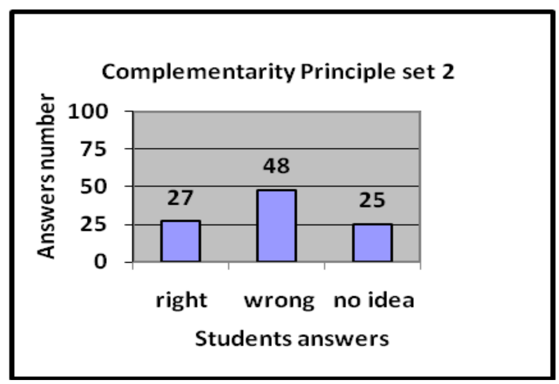

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FIGURE 4. a) Results about Wave function Set 1. b) Results about Wave function Set 2. c) Results about Uncertainty principle Set 1. d) Results about Uncertainty principle Set 2. e) Results about Superposition principle Set 1. f) Results about Superposition principle Set 2. g) Results about Complementarity principle Set 1. h) Results about Complementarity principle Set 2.

When we compare the two results (third-year-students' results and fifth-year-students' results), we can conclude that the basic quantum mechanics concepts are not easy to assimilate, but a good epistemological introduction will improve the assimilation of the theory. The results of third-year physics students who had a epistemological introduction (see the average results in figure 3 and average results in figure 5) were better than those of the fifth-year physics students who had not any epistemological introduction, but they are still not good enough. We sum up the causes of these results as follows.

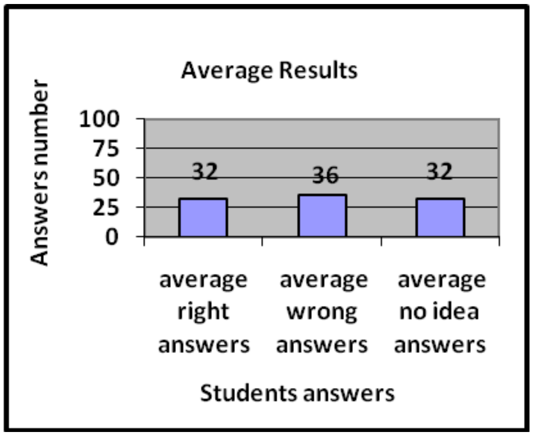

FIGURE 5. Average results of questionnaire application.

The laws of quantum mechanics cannot be derived, any more than can Newton's laws or Maxwell's equations. Unfortunately, the quantum mechanics description of nature is too abstract. More than that, there are a number of unfamiliar concepts in the basic skeleton of the theory which our students could not be familiar with, and it was difficult to appreciate the significance of any one of them until the others are understood. We now discuss the questionnaire questions one by one.

A. The Wave Function

Much of the discussion centres around it because it has a purely abstract significance. For example, an electromagnetic field carries power to moving objects, whereas the wave in quantum mechanics carries nothing more than tangible information about the particle, and the methods for tapping this source of information are somewhat indirect.

Students, when studying this theory for the first time, face many difficulties, because they are not used to deal with concepts and quantities which cannot be calculated directly, and which do not have a direct physical meaning.

B. The Uncertainty Principle

Heisenberg's uncertainty principle is one of the most important aspects of the Copenhagen interpretation. It is also an aspect of quantum mechanics which has received a large amount of attention in the literature. It was the focus of the famous Bohr-Einstein debate. Heisenberg's uncertainty relations are a direct consequence of the character of the solutions of the Schrödinger equation, solutions which are functions of products of conjugate variables such as $\boldsymbol{k.r}$ and $\boldsymbol{E.t}$. The uncertainties in the conjugate variables at any instant must have their product greater than Plank's constant. Heisenberg recognized that if it were possible to measure the conjugate variables simultaneously with great accuracy, the quantum mechanics would collapse. This essential principle in quantum mechanics, never seen before by the students, is not easy to be understood.

C. The Superposition Principle

In its simplest form, this principle reduces to two statements:

-If a system can be in the states described by the wave function ψ_1 and ψ_2, it can be in all states described by the wave functions constructed from ψ_1 and ψ_2 by the linear transformation: $a_1\psi_1 + a_2\psi_2$ where a_1 and a_2 are arbitrary complex numbers.

-If one multiplies a wave function by an arbitrary non vanishing complex number, the new wave function will correspond to the same state of the system (homogeneity).

This principle differs essentially from the superposition of vibrations in classical physics, which the students are more familiar with.

The superposition of a vibration onto itself leads to a new vibration with a large or a smaller amplitude. Moreover, in the classical theory of vibration, there is a rest state for which the vibration amplitude vanishes everywhere. However, if the wave function in quantum theory vanishes everywhere in the space, there is no state present.

D. The Complementarity Principle

The Principe of Complementarily was first enunciated by Bohr. The wave-particle duality is just one of many examples of complementarity. The idea can be summarized as follows: objects in nature are neither particles nor waves. A given experiment or measurement which emphasizes one of these properties necessarily does so at the expense of the other.

An experiment properly designed to isolate the particle properties, provides no information on the wave aspects. Conversely, an experiment properly designed to isolate the wave properties, provides no information about the particle properties. The conflict is thus resolved in the sense that irreconcilable aspects are not simultaneously observed in principle. Other examples of complementarity aspects are the position and linear momentum of a particle, the energy of a given state and the length of time for which that state exists, the angular orientation of a system and its angular momentum, and so on. The quantum mechanical description of the properties of a physical system is expressed in terms of pairs of mutually complementary variables or properties. Increasing precision in the determination of one such variable necessarily implies decreasing precision in the determination of the other.

We have noticed though our experience in teaching this theory those students cannot understand the wave- particle duality because of their background studies. We mean the influence of some basic classical mechanics concepts, which they studied for many years. In classical mechanics, the wave aspect and the particle aspect are completely different and they can never concern the same object.

V. SUGGESTIONS

We conclude from our results that it is more important to have a conceptual understanding of the abstract ideas expressed in the symbolic language of mathematics. The suggestions which we think are necessary in helping students have a good understanding of quantum mechanics concepts are:

1- Either we add an extra course, distinct from all other physics courses, to the curriculum, which should be related to the epistemological aspects of quantum mechanics, where the concepts and the interpretations of the different schools of thought are deeply discussed before the introduction of the mathematical formalisms.

2 –Or, we give, in the beginning of the quantum mechanics course, a historical account of this theory, and show how it developed. We are convinced that deep discussions with students about the different schools of interpretation will increase their understanding faculties.

When we make sure that students begin to assimilate the basic concepts of Quantum mechanics, it will be easy to move on to the mathematical tools related to the subject and its description.

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