Solar luminous constant versus lunar luminous constant

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Abstract

The Sun illuminates the earth directly during the day while it provides a dim light indirectly via moon during night. The measure of this light is illuminance and it is defined in photometry as the total luminous flux or apparent intensity of light hitting or passing through a surface. Both these constants are equivalent to solar constant but with the power at each wavelength being weighted according to the spectral luminous efficiency of the human eye. Theoretical expressions and numerical estimates for these are presented and compared with the reported values satisfactorily.

Keywords: Sun, blackbody radiation, visible light, reflection, moon, solar luminous constant, lunar luminous constant, pedagogic theory.

Resumen

El Sol ilumina la tierra directamente durante el día, mientras que proporciona una luz tenue indirectamente a través de la luna durante la noche. La medida de esta luz es la iluminancia y se define en fotometría como el flujo luminoso total o intensidad aparente de la luz que llega o pasa por una superficie. Ambas constantes son equivalentes a la constante solar, pero con la potencia en cada longitud de onda se ponderarán con arreglo a la eficacia luminosa espectral del ojo humano. Se presentan expresiones teóricas y las estimaciones numéricas de estos y se comparan con los valores reportados de manera satisfactoria.

Palabras clave: Sol, radiación del cuerpo negro, luz visible, reflexión, luna, constante luminosa solar, la constante luminosa lunar, teoría pedagógica.

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I. INTRODUCTION

The Sun illuminates the earth [1, 2] directly during the day while it provides a dim light indirectly via moon during night. The measure of this light is illuminance and it is defined in photometry as the total luminous flux or apparent intensity of light hitting or passing through a surface. It is analogous to the radiometric unit watts per square metre, but with the power at each wavelength weighted according to the luminosity function [3], a standardized model of human brightness perception. The SI unit of illuminance is lux which is equivalent to one lumen per square meter.

Everyone including the students and teachers of physics utilizes these natural lights and some attempts have been made in the past to determine their values experimentally in pedagogic journals [1, 2]. Theoretical attempts to estimate them are confined to only research journals [4] or Handbooks [5]. The aim of the present paper is to estimate solar luminous flux which reaches on the earth directly and that reflected by moon indirectly.

II. THEORY

A. Prelimanaries

The solar energy is electromagnetic in nature which is characterized by wavelength λ, frequency ν and velocity c satisfying the relation

$$c = \lambda\nu, 0 \le \lambda \le \infty, \infty \le \nu \le 0. \qquad (1)$$

The electromagnetic spectrum [6, 7] extends from below the radio frequencies at the long-wavelength end through gamma radiation at the short-wavelength end covering wavelengths from thousands of kilometers down to a fraction of the size of an atom. Assuming that the Sun has as an uniform temperature T over its surface the Planck's radiation law [7, 8] says that

$$I(\lambda,T)d\lambda = \frac{\varepsilon(\lambda,T)\, A(2\pi hc^2)d\lambda}{\lambda^5\left[\exp(hc/\lambda kT)-1\right]}\ W. \qquad (2)$$

$I(\lambda,T)d\lambda$ is the power radiated between the wavelengths λ and $\lambda + d\lambda$, *A* is the surface area, ε is the emissivity and the constants *h* and *k*, respectively, are Planck's constant and Boltzmann's constant. For simplicity, considering the Sun to be an ideal blackbody ($\varepsilon = 1$) the solar flux *Q* emitted over all the wavelengths from the unit area ($A = 1$ m^2) of the Sun is

$$Q = \int_0^\infty I(\lambda,T)d\lambda = \sigma T^4 \ \text{W/m}^2, \qquad (3)$$

where σ is the Stefan-Boltzmann constant. When this flux reaches the earth [9] this is diluted by the factor

$$f = \frac{R_S^2}{d^2}, \qquad (4)$$

Here R_S is the radius of the Sun and *d* is the yearly mean distance between the earth and the Sun. The diluted value of above mentioned solar flux is known as Solar Constant and can be written mathematically as

$$S = \sigma T^4 f \ \text{W/m}^2. \qquad (5)$$

B. Solar luminous constant

It is well known that the wavelength region $\lambda_i = 380$ nm to $\lambda_f = 760$ nm corresponds to the visible light; however the human eye is not equally sensitive to all wavelengths in this region. Rather its spectral efficiency [3] is highest at wavelength $\lambda_m = 555$ nm and becomes vanishingly small outside this interval. This behavior is quantified by spectral luminous efficiency $V(\lambda)$ for photopic vision which is plotted [10] in figure 1. Also, at wavelength $\lambda_m = 555$ nm the electromagnetic radiation of 1 W provides a luminous flux of 683 lumens (L). The number 683 was once referred to as the "mechanical equivalent of light" in the literature [3]. Hence, according to (3) the luminous flux emitted at the surface of the Sun but with the power at each wavelength being weighted by multiplying it with $683V(\lambda)$ is given by

$$Q(\lambda_i \to \lambda_f) = \int_{\lambda_i}^{\lambda_f} \frac{683S(\lambda)(2\pi hc^2)d\lambda}{\lambda^5\left[\exp(hc/\lambda kT)-1\right]}. \qquad (6)$$

This is diluted by the factor [9] *f* when it reaches the surface of the earth giving the value of solar luminous constant as

$$SLC = Q(\lambda_i \to \lambda_f)\, f. \qquad (7)$$

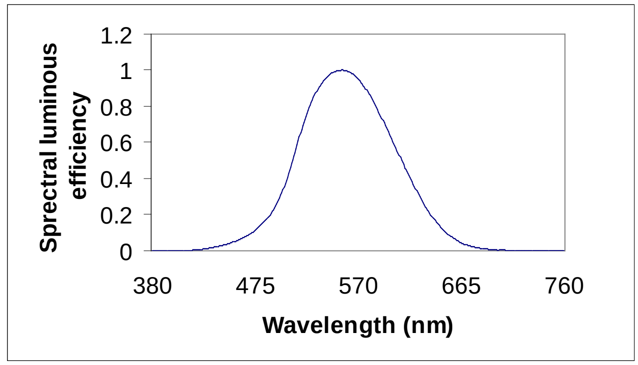

FIGURE 1. Plot of the spectral luminous efficiency values [10]. *V*(λ) against the wavelength λ.

C. Lunar luminous constant

The distance of the earth as well as moon from the Sun being almost the same the expression (7) will also be valid on the surface of the moon. The moon will reflect this flux according to its albedo [11] κ which when reaches the earth will be diluted by the factor

$$g = \frac{R_m^2}{\ell^2}. \qquad (8)$$

Here R_m is the radius of the moon and ℓ is the yearly mean distance between the moon and the earth. The value of lunar luminous constant will be given by

$$LLC = SLC \cdot \kappa \cdot g. \qquad (9)$$

Here it is assumed that the earth intercepts this light in the direction normal to the incidence. This condition is normally fulfilled during full moon nights.

D. Parameterization of $V(\lambda)$

This curve was parameterized by Agrawal, Leff and Menon [12] assuming a skewed Gaussian function

$$V_{approx}(\lambda) \approx \exp\left(-az^2 + bz^3\right), \qquad (10)$$

with

$$z \equiv \lambda/\lambda_m - 1, \lambda_m = 555nm, \qquad (11)$$

and

$$a = 87.868, b = 40.951, \chi^2 = 0.035. \qquad (12)$$

The above constants were obtained by Agrawal, Leff and Menon using un-weighted least squares fit of 381 values

[10] of $\ln V(\lambda)$. The values of *a* and *b* were reexamined both by using the 39 values of $V(\lambda)$ in between 380-760 nm at an interval of 10 nm given in Table I of reference 12 as well as 381 values [10] at an interval of 1 nm. There is practically no difference between these two cases and the better chi-square fit so obtained corresponds to the values

$$a = 88.90, b = 112.95, \chi^2 = 0.017. \qquad (13)$$

The curve corresponding to the above parameters overlaps with the experimental curve shown in figure 1 hence it has not been depicted.

III. NUMERICAL WORK

The solution of the integral of Eq. (6) is not possible analytically therefore this was evaluated numerically by Simpson's rule in the wavelength region $\lambda_i = 380nm$ to $\lambda_f = 760nm$ and substituting [7, 11, 13]

$$\begin{aligned} &R_S = 6.96x10^8\ \text{m},\ d = 1.5x10^{11}\ \text{m},\ T = 5776.0\ \text{K} \\ &R_m = 1.74x10^6\ \text{m},\ \ell = 3.82x10^8\ \text{m},\ \kappa = 7\% \\ &h = 6.63x10^{-34}\ \text{J.s},\ k = 1.38x10^{-23}\ \text{J/K}, \\ &c = 3.0x10^8\ \text{m/s} \\ &\sigma = 5.67051x10^{-8}\ \text{Wm}^{-2}.\text{K}^{-4}, \end{aligned} \qquad (14)$$

the values of dilution factors *f, g*, solar luminous constant *SLC* and lunar luminous constant *LLC* so obtained are as follows

$$\begin{aligned} &f = 2.153x10^{-5}, \\ &g = 2.075x10^{-5}, \\ &SLC = 122.686\ \text{k lux}, \\ &LLC = 0.18\ \text{lux}. \end{aligned} \qquad (15)$$

IV. CONCLUSIONS & DISCUSSIONS

The major conclusions of the present work are discussed below.

- The theoretical expressions for the solar luminous constant SLC [*cf.* Eq. (7)] and lunar luminous constant LLC [*cf.* Eq. (9)] are derived for the first time for the benefit of students.
- The values of dilution factors *f* and *g* [*cf.* Eqs. (4, 8)] are practically the same for the direct light from the Sun reaching to the earth and the indirect reflected light from the moon to the earth.
- The spectral luminous efficiency curve has been refitted by finding out a better choice of *a* and *b* [*cf.* Eq. (13)].
- The present theoretical estimate of the solar luminous constant 122.686 k lux [*cf.* Eq. (15)] is consistent with the reported [14] value of 133.334 k lux
- The reflected light from the moon provides the estimate of lunar luminous constant as 0.18 lux [*cf.* Eq. (15)] and this is also consistent with the accepted [2, 15] value of 0.25 lux at full moon. The value of the moonlight reaching the earth rapidly decreases to about 0.022 lux at first quarter or third quarter of the phase of the moon. This is because the earth does not intercept this light in a direction normal to it hence it should not be compared with the estimate presented here.
- The pedagogic theory presented here for the benefit of students and teachers of physics have also been successfully applied in determining the contributions of all the bands of electromagnetic spectrum in the value of the solar constant [16], finding out the value of photosynthetic solar constant [17] as well as estimating the value of biomass which could be produced on the earth through the photosynthesis [18] under ideal conditions.

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