Hydrostatic equilibrium in modeling the neutral atmosphere

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Abstract

This report focuses on demonstrating the approach of hydrostatic equilibrium in modeling the neutral atmosphere and ionization intensity. We showed how the altitude variations of the neutral density can be described; altitude and density at maximum ion production and optical depth of the atmosphere can be modeled using hydrostatic equilibrium. Moreover MSIS-E-90 atmospheric model results are employed for comparison purposes to see the trends of outputs of the hydrostatic approximations. As a result, we came to conclude that hydrostatic approximations could be useful at least to explain the theoretical science behind the Earth's atmosphere.

Keywords: Hydrostatic equilibrium, momentum equation, neutral atmosphere.

Resumen

Este informe se centra en la demostración del enfoque de equilibrio hidrostático en la modelización de la atmósfera neutra y la intensidad de ionización. Mostramos cómo se pueden describir las variaciones en la altitud de la densidad neutra, la altitud y la densidad máxima en la producción de iones y profundidad óptica de la atmósfera pueden ser modeladas utilizando el equilibrio hidrostático. Por otra parte, los resultados del modelo atmosférico MSIS-E-90 se emplean para propósitos de comparación para ver las tendencias de los resultados de las aproximaciones hidrostáticas. Como resultado, hemos llegado a la conclusión de que las aproximaciones hidrostáticas puede ser útil al menos para explicar la ciencia teórica detrás de la atmósfera de la Tierra.

Palabras clave: Equilibrio hidrostático, ecuación de momento, atmósfera neutral.

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I. INTRODUCTION

It is clearly understood that ionization in Earth's atmosphere can be taking place depending on the strength of the solar radiation as well as the neutral density of the atmosphere. This is to say that ionization can vary accordingly with the variation of the intensity of the solar radiation and the density of the neutral particles (gases) of the atmosphere. Nowadays, characterizing the neutral atmosphere, which has a direct impact on the ionospheric phenomena of our planet, becomes the concern of many scholars.

To model the density and other parameters in the atmosphere, number of investigations have been conducted and came out with smart atmospheric models such as MSIS-E-90. However, theoretically, the science of neutral atmosphere is being conducted based on hydrostatic approximations.

In this report, we tried to demonstrate the gap between MSIS-E-90 model and the hydrostatic approximation by taking density of atmosphere of Bhair Dar (11^0N lat, and 37^0E long), Ethiopia, at a reference height of 150km, on Oct. 15, 2009 at 14UT. In doing so, we computed the hydrostatic approximations of the neutral atmospheric density (n), altitude of maximum ionization (z_m), density at maximum ionization (n_m) and the optical depth (τ). Particularly, the comparison between MSIS-E-90 model and hydrostatic approximation results of atmospheric density and optical depth are carried out. The general picture of our approach to carry this atmospheric model is demonstrated by figure 1. Moreover the cross-section of the neutral particles used in this hydrostatic modeling is displayed by table I.

Including the introduction, this paper has four major sections. The second section gives brief descriptions on MSIS-90 atmospheric model. The third presents atmospheric theories related to hydrostatic approximations. And then, possible results and conclusions are dealt under the fourth section of our paper.

II. MSISE-90 MODEL DESCRIPTION

The MSISE model describes the neutral temperature and densities in Earth's atmosphere from ground to thermospheric heights. Below 72.5 km, the model is based on the MAP Handbook tabulation of zonal average temperature and pressure [1]. Below 20 km these data were supplemented with averages from the National Meteorological Center (NMC). Besides this, measurements of pitot tube, falling sphere, and grenade sounder rocket during 1947 to 1972 were treated.

Above 72.5 km, MSISE-90 is essentially a revised MSIS-86 model taking into account data derived from space shuttle flights and newer incoherent scatter results [2]. The authors of MSISE-90 model recommend the MSIS-86 model for those who are interested only in the thermosphere (above 120 km).

III. HYDROSTATIC APPROXIMATIONS

The momentum equation for a gas of density ρ and velocity $\boldsymbol{v}$ can be defined by

$$\rho\left[(\boldsymbol{v}.\nabla)+\frac{\partial \boldsymbol{v}}{\partial t}\right]=f, \tag{1}$$

where f stands for total force density [5]. Basically, we consider gravitational and pressure gradient force densities that can be given by

$$f=\rho\boldsymbol{g}-\nabla p. \tag{2}$$

For static atmosphere, the left hand side of the momentum equation, Eq. (1), becomes zero and we left only with a balance between gravitational force density and pressure gradient. This approximation is known as hydrostatic equilibrium [4] and it leads to have a hydrostatic equation given by

$$\rho\boldsymbol{g}=\nabla p. \tag{3}$$

The above equation can be written in number density by introducing $\rho = mn$ as

$$mn\boldsymbol{g}=\nabla p, \tag{4}$$

where n is the number density and m is the molecular mass of the neutral species of the atmosphere.

A. Number density of the neutral atmosphere

Let us now try to formulate the number density of the atmosphere based on the hydrostatic approximation (equation). For spherical symmetry, the pressure gradient can be given by

$$\nabla p=\frac{dp}{dr}\boldsymbol{e}_r. \tag{5}$$

So Eq. (4) can be rewritten as

$$mn\boldsymbol{g}=\frac{dp}{dr}\boldsymbol{e}_r. \tag{6}$$

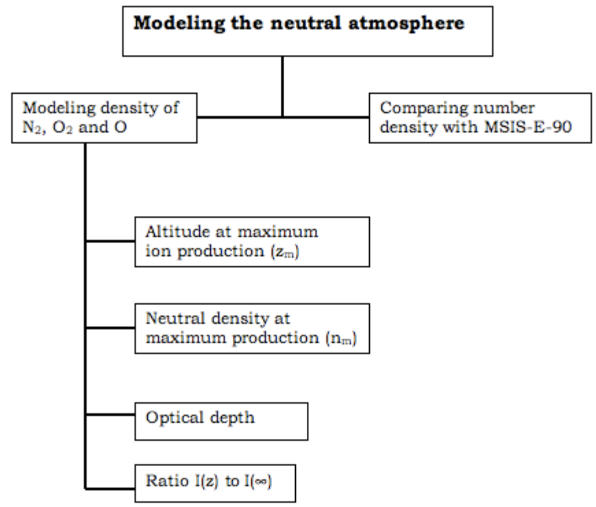

FIGURE 1. Flowchart used for modeling the neutral atmosphere.

It is known that the Earth (any planet) attracts its atmosphere by a gravitational acceleration of

$$\boldsymbol{g}=-\frac{GM_e}{r^2}\boldsymbol{e}_r=-\frac{GM_e}{R^2}\frac{R^2}{r^2}\boldsymbol{e}_r=-g_o\frac{R^2}{r^2}\boldsymbol{e}_r, \tag{7}$$

where g_o is the gravitational acceleration at the surface of the Earth and R is the radius of the Earth. Substituting Eq. (7) into Eq. (6) gives

$$-mng_o\frac{R^2}{r^2}\boldsymbol{e}_r=\frac{dp}{dr}\boldsymbol{e}_r. \tag{8}$$

This can be written as

$$\frac{dp}{dr}=-mng_o\frac{R^2}{r^2}. \tag{9}$$

From ideal gas law, we have $n=\dfrac{p}{k_BT}$.

Using this relation and integrating with respect to r gives the altitude dependence of the atmospheric pressure and temperature as well. That is

$$p=p_o\left[-\int_R^r\frac{mg_o}{k_BT(r)}\frac{R^2}{r^2}dr\right]. \tag{10}$$

If we consider the isothermal atmosphere (T=constant) and use $p=nk_BT\rightarrow p_o=n_ok_BT$, then we can have a hydrostatic approximation based model for the number density of the atmosphere. Thus we can have

$$n = n_o \exp\left(-\frac{z - z_o}{H}\right), \qquad (11)$$

where $H = \frac{k_B T}{mg}$ is the scale height of the neutral atmosphere $z = r - R$ and z_o is the reference altitude (height).

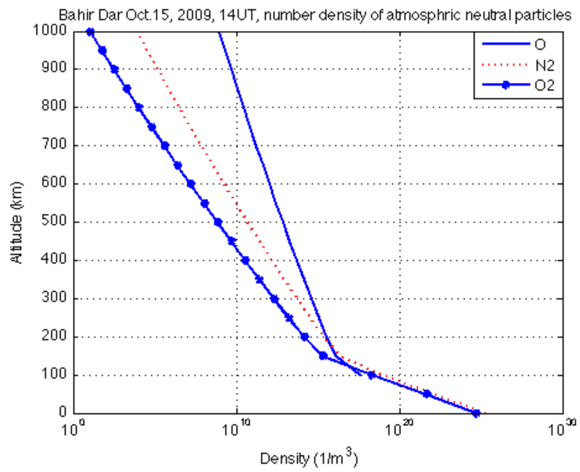

FIGURE 2. MSIS-E-90 model approximations of number densities of N_2, O_2 and O species.

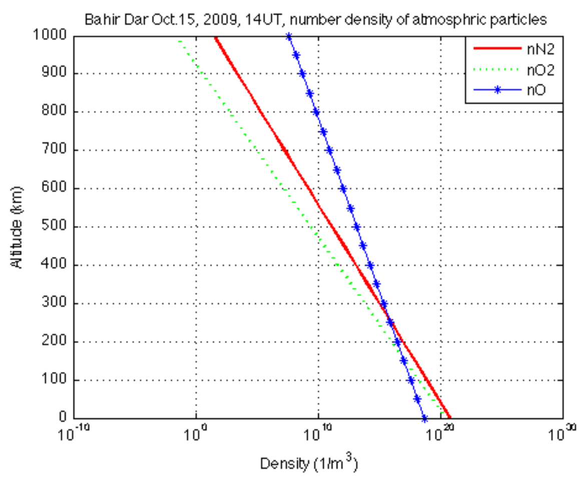

FIGURE 3. Hydrostatic approximations of number densities of N_2, O_2 and O species

B. Altitude and density at maximum ionization

To model the altitude and density of the neutral atmosphere at maximum ion production, let us start from the definition of production rate. That is

$$q = q_{m,0} \cos\theta \exp\left(1 - h' - e^{-h'}\right), \qquad (12)$$

where $h' = h - \ln\sec\theta$, θ is the zenith angle and, $q_{m,0}$ is the production rate at θ=0. Maximum ion production can be attained at an altitude of h_m where $\frac{dq}{dh'} = 0$. Hence we have

$$q = q_{m,0} \cos\theta \exp\left(1 - h' - e^{-h'}\right)\left(-1 + e^{-h'}\right) = 0. \qquad (13)$$

This leads to

$$\begin{aligned} -1 + e^{-h'} &= 0, \\ e^{-h'} &= 0, \\ -h' &= \ln 1, \\ h' &= 0. \end{aligned} \qquad (14)$$

Since $h' = h - \ln\sec\theta$, then we have

$$h_m - \ln\sec\theta = 0\,, \qquad (15)$$

$$\Rightarrow h_m = \ln\sec\theta. \qquad (16)$$

Furthermore, since $h = \frac{z_m - z_o}{H}$, then

$$\frac{z_m - z_o}{H} = \ln\sec\theta\,, \qquad (17)$$

$$\Rightarrow z_m = z_o + H\ln\sec\theta. \qquad (18)$$

Eq. (18) refers to altitude at which maximum ion production can be taking place.

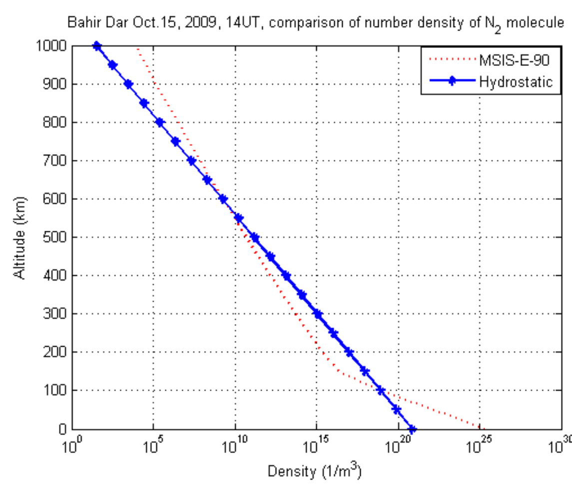

FIGURE 4. Number density of N_2 molecule.

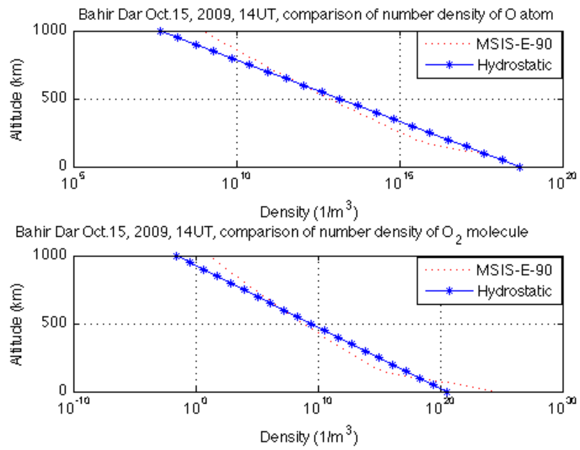

FIGURE 5. Number density of O_2 molecule and O atom.

At this altitude, the neutral atmosphere has a particular density (n_m) that can be computed from the fact that the optical depth at maximum production becomes one [3]. That is

$$\tau = \sigma \sec\theta Hn(z)\,, \tag{19}$$

$$1 = \sigma \sec\theta Hn_m. \tag{20}$$

From this relation one can easily solve for n_m as

$$n_m = \frac{1}{\sigma \sec\theta H}. \tag{21}$$

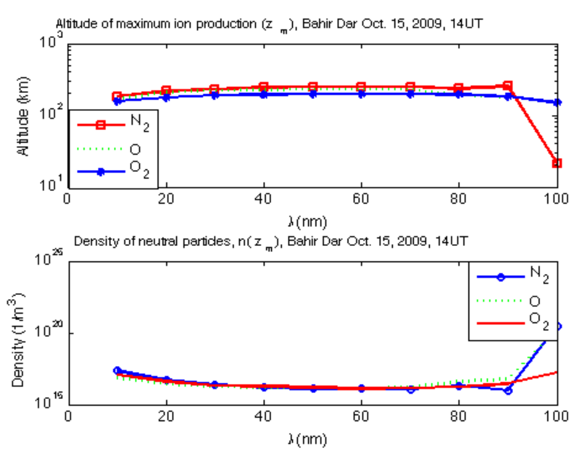

FIGURE 6. Hydrostatic approximations of altitude and density at maximum ion production.

We can also write the altitude of maximum production, z_m, in terms of the corresponding density of the neutral atmosphere, n_m, as

$$n = n_o \exp\left(-\frac{z - z_o}{H}\right)$$
$$\Rightarrow n_m = n_o \exp\left(-\frac{z_m - z_o}{H}\right). \tag{22}$$

This results

$$\ln\frac{n_m}{n_o} = -\frac{z_m - z_o}{H} \Rightarrow$$
$$z_m = z_o + H\ln(H\sigma n_o \sec\theta). \tag{23}$$

Optical depth in the Earth's atmosphere relates the intensity of solar radiation at a certain altitude z to that of the intensity at infinity [6]. It depends on both altitude and wavelength (absorption cross-section of the atmosphere).

If we have j neutral species in the atmosphere then the optical depth at an altitude z and wavelength λ can be defined as

$$\tau(\lambda,z) = \sec\theta \sum_j \sigma_j(\lambda) n_j(z) H_j. \tag{24}$$

C. Optical depth

For instance if the atmosphere is basically consists of N_2, O_2 and O species then the optical depth at 100km, $\theta = 30^0$ and λ= 50nm can be computed as

$$\tau(50,100) = \sec 30^0 \begin{bmatrix} \sigma_{N_2}(50) n_{N_2}(100) H_{N_2} \\ +\sigma_{O_2}(50) n_{O_2}(100) H_{O_2} \\ +\sigma_O(50) n_o(100) H_o \end{bmatrix}. \tag{25}$$

The ratio of the intensity at an altitude z to intensity at ∞ can be computed from optical depth, τ, as

$$\frac{I(z)}{I(\infty)} = \exp(-\tau). \tag{26}$$

This fraction can tell us the amount of radiation absorbed in the ionization process at a specific altitude compared with intensity at the source, $I(\infty)$.

IV. RESULTS AND DISCUSSIONS

In this report, we demonstrated how neutral atmosphere parameters such as number density could be modeled using hydrostatic approximations. We considered a specific atmosphere and simulated temperature and number density at one particular reference altitude for N_2, O_2 and O neutral species using MSIS-E-90 atmospheric model. After that, we modeled the altitude variation of the number density of the atmosphere using hydrostatic conditions. Consequently, we computed the altitude and the density of the neutral atmosphere at which maximum ion production can be attained. Moreover, we showed the amount of radiation intensity that can be received at a certain altitude by computing optical depth of the atmosphere. In all above cases, we implemented the hydrostatic approximations by considering the density of the atmosphere at reference height 150km, Bahir Dar (11^0N lat, and 37^0E long), Ethiopia, Oct. 15, 2009 at 14UT.

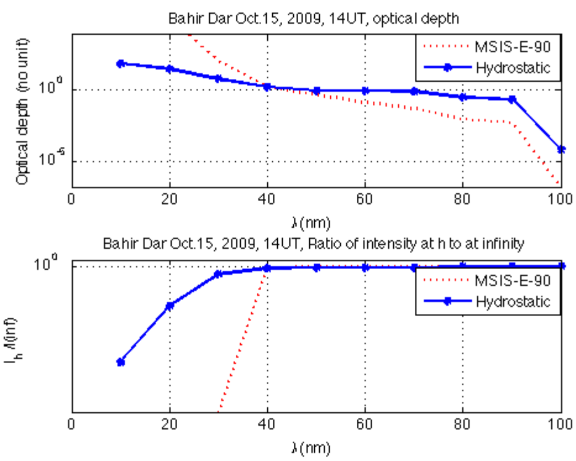

FIGURE 7. Hydrostatic approximations of optical depth of the atmosphere.

TABLE I. Cross-sections of neutral particles used in modeling.

λ(nm)	σ_{N2} x $10^{-22}(m^2)$	σ_{O2} x $10^{-22}(m^2)$	σ_O x $10^{-22}(m^2)$
10	1.3	2.9	2.3
20	6.4	8.7	6.7
30	11.6	16.0	9.7
40	19.3	18.7	10.9
50	23.3	22.6	11.7
60	23.0	26.0	12.0
70	26.3	25.0	11.4
80	15.0	20.9	4.4
90	31.0	12.0	3.0
100	0.001	2.0	0.001

The results of our hydrostatic modeling are illustrated by figures 3, 4, 5, 6 and 7. Accordingly, figure 3 shows the altitude variation of the number density of the atmosphere modeled by hydrostatic approximations. The comparison of this result with MSIS-E-90 model density outputs is illustrated by figures 4 for N_2 and 5 for O and O_2 species of the atmosphere. As these results show, there is some agreement between hydrostatic and MSIS-E-90 model results. This confirms the fact that hydrostatic approximations can still be considered as a means to model Earth's atmosphere.

After modeling the number density of the atmosphere using hydrostatic approach, we computed parameters such as altitude and density at maximum ion production. As we can see from figure 6, the maximum ion production is occurred at lower altitudes for wavelength ranges of 10nm to 20nm and 90nm to 100nm. This is because in these wavelength ranges, the cross-sections of the neutral atmosphere are so small. Hence, the direct proportionality between altitude (z_m) and the cross-section (σ) yields maximum ion production at lower altitudes (see Eq. (29)). As it is described by Eq. (24), the number density at maximum ion production is inversely proportional to the cross-section (wavelength) and then the density at wavelength ranges of 10 nm to 20 nm and 90 nm to 100 nm became greater than that of the densities at the intermediate wavelength ranges and this is clearly demonstrated by the bottom panel of figure 6.

The optical depth and ratio of intensity at a particular altitude z to intensity at ∞ are demonstrated by figure 7. As it can be seen from this figure, the hydrostatic and MSIS-E-90 model results of optical depth are compared. Still the comparison seems to be good because the curves made by hydrostatic approximation showed a similar trend with that of MSIS-E-90 model curves.

In conclusion to this report, one can use hydrostatic approximation to model parameters of the atmosphere at least for theoretical purposes such as explaining the physics behind the neutral atmosphere that has a direct influence on ionospheric explorations.

ACKNOWLEDGEMENTS

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