Problem solving and writing I: The point of view of physics

Jorge Barojas [1, 2]
[1]*Department of Physics, School of Sciences. UNAM. Circuito Exterior, Ciudad Universitaria. A.P. 70542. C.P. 04510. México, D.F., México.*
[2]*Member of the Seminar on Hermeneutics, at the Institute of Research in Philology, UNAM.*

E-mail: jorgebarojas@yahoo.com

(Received 6 July 2007; accepted 12 September 2007)

Abstract

In this first paper, of a series of two, we relate the skills of problem solving and writing referring to two types of systems, those called physical and those concerning human learning. Then, the notion of semiotic representation registers is connected to the building of learning cycles. All these elements serve to propose a protocol for problem solving that contains four steps of cognitive nature and another one that is metacognitive. This protocol is applied by interpreting in detail the steps of the solution of a physics problem from the point of view of a physicist. Finally, some implications on science and technology education are discussed.

Key words: Problem solving, Problem-based learning, languages in problem solving.

Resumen

En este primer trabajo, de una serie de dos, relacionamos las habilidades de resolver problemas y de redactar por escrito su solución, refiriéndonos a dos tipos de sistemas, los denominados físicos y los de aprendizaje humano. Luego, conectamos la noción de registro de representación semiótica con la construcción de ciclos de aprendizaje. Esto nos permite proponer un protocolo de solución de problemas que comprende cuatro etapas de naturaleza cognitiva y una metacognitiva. La aplicación de este protocolo se ejemplifica interpretando las distintas etapas de la solución de un problema de física, resuelto desde el punto de vista de un físico. Concluimos con algunas implicaciones en educación en ciencia y tecnología.

Palabras clave: Resolución de problemas, aprendizaje basado en problemas, lenguaje en la solución de problemas.

PACS: 01.40.-d, 01.50-i, 01.40.gb

I. PROBLEM SOLVING AND WRITING AS CREATIVE DESIGNS FOR LEARNING PURPOSES

Educational programs try to build knowledge and also induce and monitor the development of skills, attitudes and values. In order to learn something we explore, provoke and transfer understanding concerning different types of systems. Wilson [1] considers four types of systems: natural or physical, artificially designed, for human activities, and socio-cultural. The first and second ones will be denoted as physical systems and the third and fourth ones as human learning systems [2]. In physical systems the components and interactions are rather well described in terms of organized structures, the parameters defining the system are known or can be determined with good precision because calculations and experiments can be performed, and we can estimate when the solutions have been obtained and with what kind of accuracy.

Problems in systems connected with education, training, production and management can not be treated as physical problems even if they concern physical situations. We will refer to them as human learning systems if the main aim is to learn how these systems work in order to improve their functioning by looking for the solution of specific problems. They imply planning, development and evaluation of different sorts of transformation activities in human organizations where learning communities are in operation. In these cases the practical solutions are the best possible ones under certain given conditions although it is understood that answers are rather rough and might change depending on the evolution of the context. Human learning systems can be understood in a cognitive space defined by the intersection of two intellectual domains: the building of knowledge and the organization of learning.

Problem solving and writing are different in academic and industrial contexts depending whether they concern physical systems or human learning systems. In any case, the most important outcomes are documents containing plans, procedures, results, recommendations… and these require an appropriate shaping of written texts that convey messages. Those messages involve chains of associations explicitly formulated and organized into data, information or knowledge that must be communicated and interpreted to produce some learning and understanding.

Problem solving in physical systems uses expressions written in natural languages as well as a great variety of symbolic representations; for instance, curves, graphs, diagrams, pictures, tables, equations, schemes, models, codes and even data coming out from experiments, computer simulations and calculations, as well as multimedia applications. Problem solving in human learning systems is much more qualitative in its descriptions and representations.

Depending on the type of system, the final written documents are presented in different forms and styles [3].

While solutions to problems in science are communicated in straightforward, precise and rigorous forms, narratives concerning interpretations of those solutions take advantage of the suggestive power of words and rather seldom use other forms of representations different from texts. Nevertheless, as any craft to be learned, problem solving and writing require practice as well as reflection on the experiences and works of the masters in the field. Also we do understand much better how children learn, what difficulties they have, and how they approach problem solving and writing [4,5].

Here we will refer neither to how to write nor to how to get training in problem solving. We are not concerned with the acquisition of the craftsmanship required to become a writer or a problem solver. We focus on the connections among writing, problem solving and creative design, and on some of their implications from the perspective of science and Hermeneutics: the study and interpretation of texts [6] (see Fig. 1).

- In the past writing was considered as a problem to be solved with three elements: an initial state to start with, a final state to which one has to arrive, and a procedure to go from one state to the other.
- **Assume that problem solving is like writing**, in the sense that it implies two creative processes: the design of a communication and its interpretation.
- Then, writing the solution of a problem means to communicate and interpret the narrative of the reasoning process leading to the solution.

Figure 1. Relationships among writing, problem solving and creative design.

II. LANGUAGES IN PROBLEM SOLVING

Here we consider that the solver of a science and technology problem behaves as a creative thinker with expertise in three types of languages: the natural language of everyday talking, the technical language of scientific disciplines with appropriate definitions of abstract terms and specific meanings according to rather well prescribed conditions of use, and, quite often the formal language of mathematics in which complex symbols usually represent difficult and complicated ideas that follow certain rules of operation under precise conceptual frameworks. The solver designs the scaffolding story of the solution and builds a discourse like a language producer managing linguistic resources (vocabulary, styles, structures...). In this sense, problem solvers behave as writers creating documents although very often they produce texts containing combinations of the three previously considered languages: natural, technical and mathematical.

Problem solvers usually follow certain procedures in order to generate ideas, create plans, draft texts, and review their works. In what follows we shall consider that problem solver prepare their written documents by working in two dimensions: (1) the cognitive dimension where the conceptual design used to solve the problem is formulated, applied and communicated, and (2) the metacognitive dimension where the author reflects on previous thoughts and actions as well as on their consequences.

A. Communication languages and semiotic representation registers.

Teaching and learning take place through written and verbal communications. Furthermore, the learning process very often starts and ends by using natural languages although in more advanced stages of science and engineering learning, technical and formal languages are used in all the documents written by authors who are quite familiar with such languages. For instance, written forms are usually employed to convey definitions, declare properties or present methods of solution assuming that the readers can interpret those texts. However, discussions, questions and answers involve written and verbal communications, but discursive practices concentrate on verbal expressions that leave no printed record of its existence. Sometimes we just observe their consequences leaving the door open to different interpretations.

Keeping track and understanding the mental representations that are conveyed through communication processes are ways of visualizing how discourses are organized and delivered under learning conditions. In what follows we comment more on the idea of representation registers and then connect the use of different languages to the cognitive activities employed to operate with those registers in the process of problem solving.

According to Duval [7], a semiotic representation register is a particular form of a mental register composed by any set of signs or symbols that describes the objects or events defining a given system. It serves to make explicit and to communicate to others the characteristic qualities of those objects. In mathematics we can identify semiotic representation registers of different nature: symbolic, algebraic, geometric…; therefore, conceptualization of mathematical ideas is the consequence of articulating representations belonging to different registers.

The generation of semiotic representations provides a means of tracking the corresponding cognitive activity that is responsible for its appearance because such representations are expressed in language forms, although different interpretations might be provoked. In many aspects, these characteristics of learning mathematics can be applied to devise practical applications of problem solving in natural sciences such as chemistry and physics as well as in engineering. Without any intention of proving it, we propose that to communicate the solution of a problem is equivalent to describing a discursive practice undertaken through cognitive activities involving semiotic representation registers.

Duval distinguishes three conceptual activities serving to handle semiotic representation registers [7]: formation, treatment and conversion. Formation activity is accomplished when a register exists where certain signs or symbols are used to describe an object and therefore that such description can be operated and modified. In the treatment activity there are explicit rules indicating how and when those symbols must be operated. Finally, the conversion activity implies the existence of certain signs that can be identified in two different representation registers and a transition occurs between them, although there might be no explicit rule controlling such an articulated transformation.

B. Learning cycles and a problem solving protocol

The semiotic representation registers that go through the conceptual activities of formation, treatment and conversion integrate learning cycles [8] which involve four stages (S) to be worked out in the context of problem solving in a physical system:

(S_1) everyday natural language is introduced by means of words in order to make interpretations or to establish consequences about the statement of the problem,

(S_2) the learners´ worldviews are presented in the technical language of a scientific or technological discipline by analyzing a problematic situation in abstract terms serving to describe possible scenarios that might lead to the solution of the problem,

(S_3) theoretical model structures are applied through the use of formal languages that lead to the presentation of a design of the solution, and

(S_4) changes among different representation registers expressed in any of the previous languages are produced when descriptions, calculations and predictions serve to implement the solution of the problem.

The transitions between these four stages define the conceptual activities that involve semiotic representation registers: formation corresponds to S_1 going into S_2, treatment means S_2 going into S_3 and conversion refers to S_3 going into S_4. Associated to these steps S_1 to S_4 we propose a problem solving protocol called TADIR, as a way of traveling through the learning cycle in which semiotic representation registers are formulated, treated and converted [9]. In this way the solver of a problem communicates thoughts, manipulates symbols, goes through representations by using different languages, and eventually makes some calculations or performs experiments to get and interpret the solution. The name of this protocol, TADIR, comes from the initials of the steps summarized in Table I.

Table I. Steps of the TADIR problem solving protocol applied to a physical system

STEPS	DESCRIPTION
T: Translation	If the statement describing the problem situation is formulated in everyday natural language, then such a statement needs to be reformulated in the technical language of a scientific or technological discipline by using abstract notions and conceptual relationships. In this reformulation the objects and events characterizing the physical system under consideration are identified.
A: Analysis	All the assumptions required to interpret the problematic situation concerning the physical system and to build the solution of the problem are explicitly described by taking into account models and theories of relevance. Although this description is mainly made in terms of a technical language it also might imply the use of formal languages. The general characteristics of possible answers are also considered.

D: Design	A scheme or conceptual diagram showing the line of reasoning expected to lead toward the solution is proposed mostly in terms of a formal language. The main components of this scheme can be related to the following types of knowledge: (1) the basic concepts serving to define and solve the problem, (2) the conceptual relationships required to describe the conditions defining the system, and (3) the ancillary calculations useful in answering the questions asked in the statement of the problem. A convenient procedure to arrive at such a scheme consists of preparing a narrative of the solution by organizing a short discourse in a verbal form describing its components then writing it in natural and technical languages and finally making a diagrammatic scheme that represents how it works.
I: Implementation	Appropriate criteria, definitions, information and procedures are applied in order to follow the Design, and if required, experiments and calculations are performed to obtain the solution. By devising the procedure to get the solution of the problem, quite often the formal path depicted in the Design step is expressed in natural language, but enriched by the transitions already accomplished among the natural, technical and formal languages.
R: Review	All the previous steps are considered again by applying metacognitive procedures in order to detect possible conceptual errors, false or unnecessary assumptions, improper reasoning, wrong calculations, results obtained under different conditions, and confrontation with predicted answers.

III. THE POINT OF VIEW OF A PHYSICIST SOLVING A PHYSICS PROBLEM

The problem to be solved is the following: *What fraction of the total volume of an iceberg remains over the surface of water?* We begin by presenting in Fig. 2 a typical answer obtained just by playing the game of plugging equations and numbers, as in high school physics lectures or textbooks.

(I) Basic equations

Weight of the iceberg:	$W = (\rho_I V)g$	(1)
Buoyant force:	$B = (\rho_W V_s)g$	(2)
Total Volume:	$V = V_f + V_s$	(3)
Buoyancy condition	$W = B$	(4)

(II) Algebraic and arithmetic operations

$$\rho_I(V_f + V_s) = \rho_W V_s \quad (5)$$

$$V_f / V_s = (\rho_W - \rho_I) / \rho_I \quad (6)$$

$$V_f / V_s = (1000 – 917) / 917 = 83 / 917 = 0.0905 \quad (7)$$

$$V_f / V_s \approx 9 \% \quad (8)$$

Figure 2. A formal solution to the iceberg buoyancy problem

For anyone foreign to physics this text is written in a strange language also it is full of nonsense if you do not understand and know how to apply Archimedes´ Principle. In what follows we shall see how the TADIR protocol makes these issues clearer.

A. The cognitive dimension of the solution of a physics problem

TRANSLATION: using the language of physics the text of the problem is interpreted and the objects and events that define its context are described.

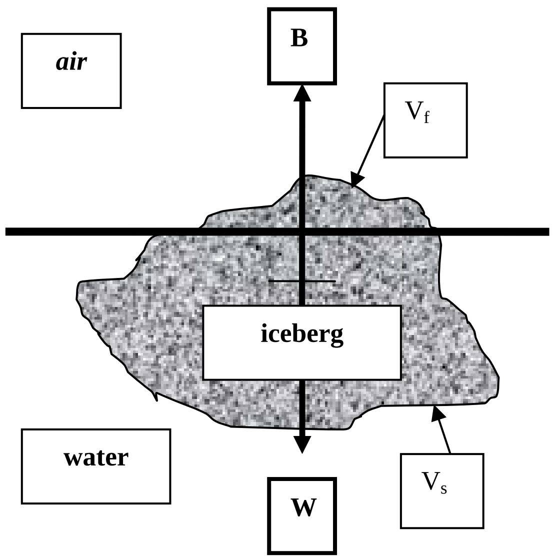
The physical system is shown in Fig. 3. It is composed by one *object* (the iceberg) surrounded by air and water, all of them under the action of the force of gravity. The *event* under consideration is the floating of the iceberg.

The physical situation is then described in the following terms:

The iceberg is a physical system characterized by a mass M and a volume V.

There is a part of the iceberg that is over the water surface and floats (V_f), and another part that is surrounded by water and sinks (V_s).

Two forces act on the iceberg: the downward weight (**W**) applied at the center of gravity, and the upward buoyant force (**B**) applied at the center of buoyancy.



V_f: volume that floats
V_s: volume that sinks
V: total volume
ρ_I: density of ice (917 kg/m^3)
ρ_W: density of water (1000 kg/m^3)
g: acceleration of gravity
W: weight of the iceberg
B: buoyant force (thrust)

Figure 3. Forces, volumes and notation for the iceberg problem

ANALYSIS: assumptions are made explicit by using models and theories of importance in the discipline and an expected answer is suggested.

A_1: The total volume of the iceberg is constant: $V = V_f + V_s$, because the volumes of the sunk part (V_s) and the floating part (V_f) are constant. We consider that the ice does not melt and there are no changes in the meteorological conditions.

A_2: The densities of ice (ρ_I) and water (ρ_W) are constant; both components of the physical system are homogeneous and do not suffer any transformation.

A_3: We do not take into account the composition of seawater and consider ρ_W = 1000 kg/m^3 instead of $\rho_W = \rho_{SW}$ = 1024 kg/m^3, where $_{SW}$ means salted water.

A_4: The iceberg floats on water in a region where the surface is flat compared to the curvature of Earth; therefore, the acceleration of gravity (g) is constant.

A_5: The buoyancy of the iceberg is due to the static equilibrium of two forces: the weight of the iceberg **W** and the buoyant force **B** produced by the water surrounding the iceberg. This condition implies that the magnitudes of these vectors are equal: W = B, where W = |**W**| and B = |**B**|.

A_6: The center of buoyancy and the center of gravity are along the same vertical line; there is no tilting and no restoration torque needs to be applied to maintain the iceberg in the same orientation with respect to that vertical line.

A_7: The conditions for the application of Archimedes´ Principle are satisfied: “a body wholly or partially immersed in a fluid will be buoyed up by a thrust force (B) equal to the weight of the fluid (V_s) that it displaces”, such that $B = (\rho_W V_s)g$.

Expected answer: the problem asks for the fraction of the total volume (V) that floats (V_f), but available data only allow us to express the ratio between the volumes V_f / V_s. This ratio is a number without dimensions and is usually given as a percentage.

DESIGN: a diagram describing the main components of the solution is considered.

The Design step, like flux diagrams in programming and conceptual maps in cognitive psychology only provide rough approximations. They are not unique diagrammatic representations of possible paths towards the solution of the problem. The solution to this problem is given in Figs. 4 and 5. The complete design of the solution integrates the three kinds of knowledge used to solve the problem as indicated in Fig. 4 (basic ideas, conceptual relationships and ancillary calculations).

The path describing one possible solution is depicted graphically by the Design in Fig. 5, where assumptions A1 to A7 are explicitly taken into account.

IMPLEMENTATION: by following the design the answer to the problem is found (see Fig. 6).

Although Figs. 2 and 6 contain the same number of equations (1) to (8), Fig. 6 comes after the application of the first three steps of TADIR: Translation (Fig. 3), Analysis (assumptions A_1 to A_7 and description of the expected answer) and Design (Figs. 4 and 5). This shows why and how the Implementation step requires the previous three steps (TAD) as a sort of a prerequisite to get the solution.

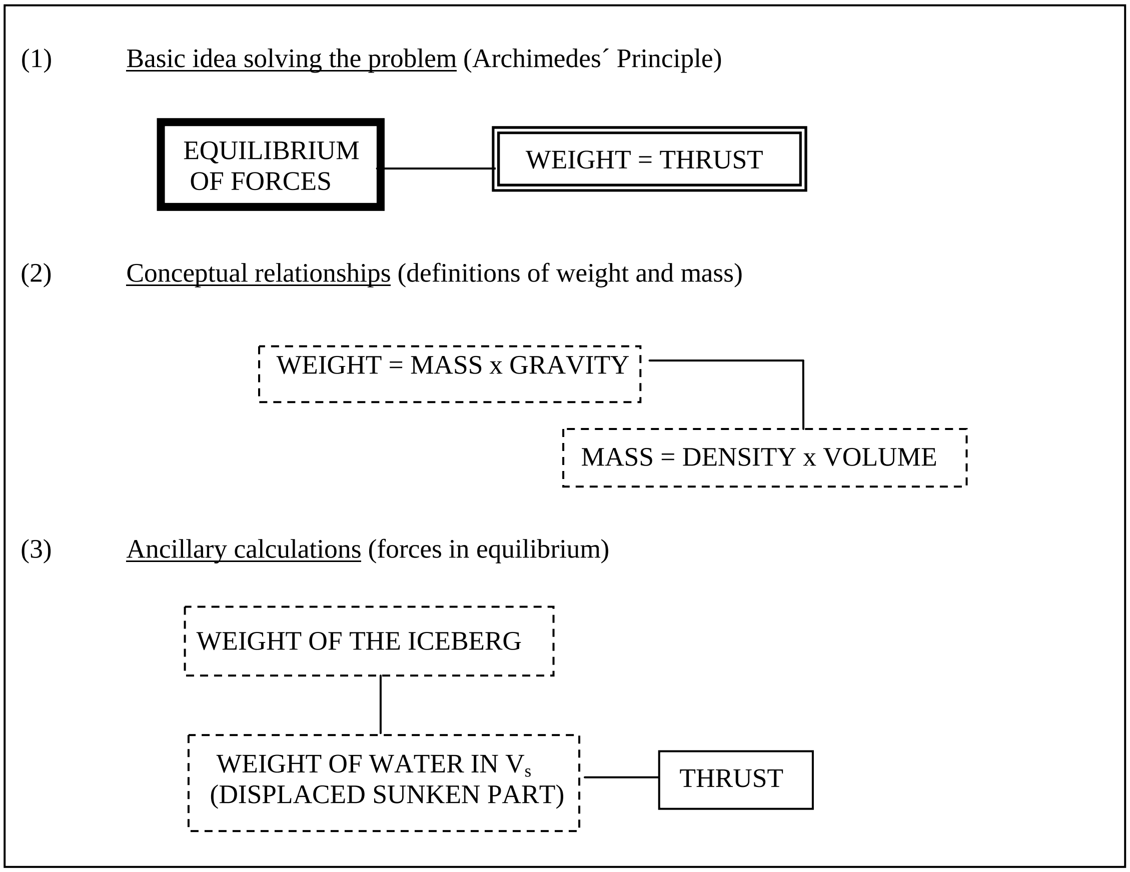


Figure 4. Elements of the design for the solution of the buoyancy problem.

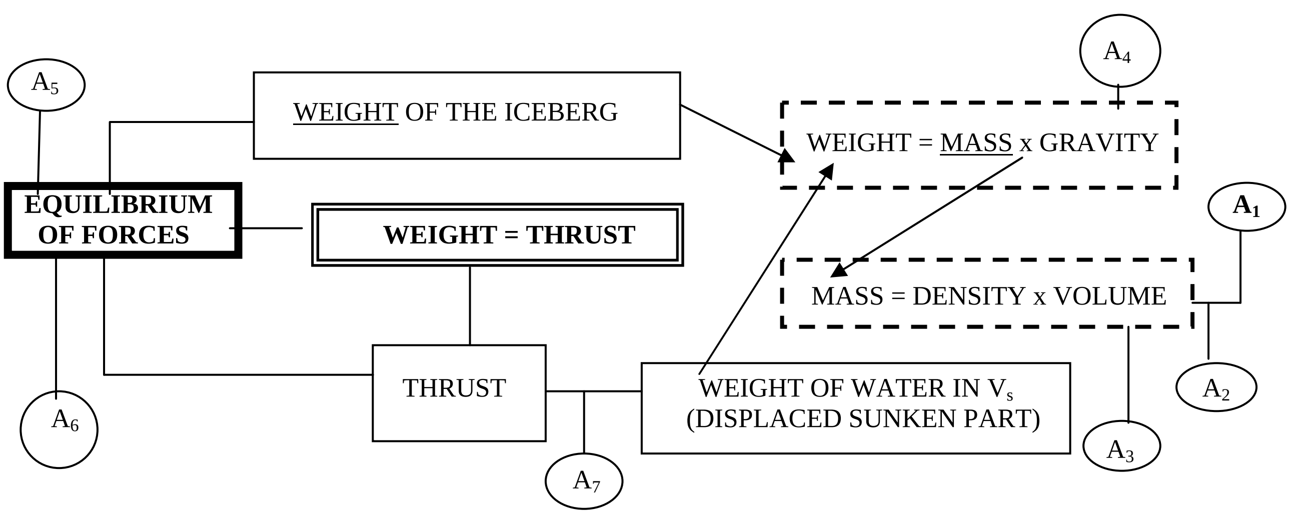


Figure 5. Design of the complete solution of the buoyancy problem.

Basic equations

Weight of the iceberg:	$W = (\rho_I V)g$	(1)
Buoyant force:	$B = (\rho_W V_s)g$	(2)
Total Volume:	$V = V_f + V_s$	(3)
Buoyancy condition	$W = B$	(4)

(II) Algebraic and arithmetic operations

(a) Replace (1) and (2) in (4): $W = (\rho_I V)g = B = (\rho_W V_s)g$

Use (3) in the previous equation: $\rho_I Vg = \rho_I(V_f + V_s)g = \rho_W V_s g$

Simplify the factor g: $\rho_I(V_f + V_s) = \rho_W V_s$ (5)

(b) Do the algebraic operations in (5): $\rho_I V_f + \rho_I V_s = \rho_W V_s$, and

$$\rho_I V_f = \rho_W V_s - \rho_I V_s = (\rho_W - \rho_I)V_s$$

Obtain the ratio of the two volumes: $\rho_I V_f / V_s = (\rho_W - \rho_I)$, and

$$V_f / V_s = (\rho_W - \rho_I) / \rho_I \quad (6)$$

(c) Substitute the numerical values of the densities in (6):

$$V_f / V_s = (1000 – 917) / 917 = 83 / 917 = 0.0905 \quad (7)$$

Obtain the percentage of the ratio (7):

$$V_f / V_s \approx 9 \% \quad (8)$$

Figure 6. Details of the complete solution of the buoyancy problem.

Equations (1) to (8) in Fig. 6 correspond to processes of different nature: while process I (Basic equations) deal with physical concepts and definitions belonging to the codified knowledge of the discipline, process II (Algebraic and arithmetic operations) implies routine work required for handling the equations and getting the solution. We could think that the problem solving process is already finished because the result obtained in equation (8) answers the question addressed by the problem. However, the TADIR methodology includes one last step (the Review: R) associated with metacognition [10].

B. The metacognitive dimension of the solution of a physics problem.

The metacognitive dimension of TADIR corresponds to the Review step (R) and can be used as a test in two ways: (1) to scrutinize the practical scaffoldings applied for chaining ideas while unfolding the solution procedure, and (2) to examine how the readers understand the discourse showing how the solution was obtained. Both ways are quite related to the issue of interpretation of the text of the solution provided by the physicist because we can present the results of our reflections on what has been done in problem solving in many different ways.

This metacognitive step has four components, which are identified by a sub index indicating the capital letter of the previous four cognitive steps (TADI) to which it is associated. In what follows we just present one of the many possibilities of interpreting this review step (R) for the buoyancy problem.

R_T: Inspect the original statement of the problem, examine if the elements given in the *Translation* serve to obtain correct answers to the problem, and check if the solution can be meaningfully interpreted in natural everyday language.

Going back to the statement of the problem we now judge if the answer given in equation (8) of Fig. 6 responds to what has been asked. This solution indicates that less than one tenth of the total volume of the iceberg is floating and nine tenths are under water. Unless we give the value of the total volume of the iceberg (V) we have no idea of the corresponding values of the volumes of the part of the iceberg that floats (V_f) and the one sunken (V_s); we are only able to calculate their ratio V_f / V_s.

We can regard other aspects of the result, such as order of magnitude, units, and values of certain quantities under limiting cases. For instance, we might wonder what happens in equation (6) when the densities of water and ice are equal ($\rho_W = \rho_I$), which implies that the ratio of volumes $V_f / V_s = 0$ and no part of the iceberg is floating ($V_f = 0$ and $V_s = V$). Using these values for the volumes in equations (1) and (2) we will find that again B = W, but in this case the buoyancy problem is meaningless.

R_A: Consider if all the assumptions made in the *Analysis* have been appropriately employed, look for the implications of modifying them or ponder the consequences of introducing different ones. We might also think about any differences between the expected answer and the solution obtained after applying the first four steps of the protocol (TADI).

The first four assumptions (A_1 - A_4) refer to the interacting bodies: the iceberg, the air surrounding the floating part, the water surrounding the sunken part, and Earth (responsible for the gravitational attraction field due to which physical bodies have weight). The other assumptions ($A_5 – A_7$) deal with buoyancy conditions.

If assumptions A_1 and A_2 are not maintained, we must know the variations of volumes and densities. However, if in A_3 we take into account salted water instead of pure water ($\rho_W \rightarrow \rho_{SW}$ = 1024 kg/m^3), then the ratio of volumes will be different and in equation (6) we will obtain V_f / V_s = (1024 – 917) / 917 = 107 / 917 = 0.1167, which corresponds to a percentage $V_f / V_s \approx 12$ % instead of the 9 % obtained previously, meaning that the iceberg sinks more in pure water than in salted water.

Although the acceleration of gravity g does not appear in equation (6), the assumption A_4 is still applicable; under these conditions the direction of the force of gravity is always perpendicular to the surface of the iceberg. The curvature of Earth does not matter due to the relatively small size of the iceberg.

A treatment of the problem at the level of complexity here considered implies that assumptions A_5 and A_6 are not modified: the iceberg is in stable equilibrium because weight and thrust are equal in magnitude, opposite in direction and applied on the same vertical line. Also assumption A_7 is basic, because what this assumption means is that the conditions for applying Archimedes´ Principle exist.

In this example we just applied Archimedes´ Principle, we are not concerned with instruction procedures or learning activities designed to promote its understanding. This Principle is based on the fact that the buoyant force is produced by differences in pressure of the water surrounding the iceberg. As the pressure under water increases with depth, the pressure on the top of the part of the iceberg that sinks is lower compared to the pressure at the bottom of that same part. This is why the buoyant force goes up, against the weight that goes down.

R_D: Be critical about other paths or procedures applied in the *Design* in order to obtain the solution under two extreme conditions: (1) the conceptual structure of the Design remains the same although certain simplifications or more direct paths towards the solution are introduced or (2) something completely different is taken into account and a new Design is implemented for instance when the solution to a given problem is taken as a first level approach to solve a more difficult problem in which the previous physical system is just a part of a more complex system.

If assumptions A_5 and A_6 do not apply (the forces B and W are not on the same vertical line), then the left hand side of the Design in Fig. 5 must be modified. Usually the center of gravity where the weight W of the iceberg is applied does not change. This cannot be the case with the center of buoyancy where thrust B is applied (the center of gravity of volume V_s when it is full of water).

If for any reason the center of buoyancy is on a line different from the vertical that goes through the center of gravity of the complete iceberg, then a torque is produced. Depending on the magnitude of this torque the iceberg might change its orientation, oscillate for a while and then recover its vertical equilibrium. It could also happen that the iceberg leans to one side, turns over and then sinks completely.

R_I: Check that all the mathematical operations required to obtain the solution are correct and that the *Implementation* has been done accurately and completely.

The review of this step relates to procedure II in Fig. 6 (Algebraic and arithmetic operations). Therefore, we expect possible modifications in the calculations only if one or more of the previous steps (Translation, Analysis and Design) are changed. However, under realistic conditions the problem is much more complex and will require the application of advanced knowledge in hydrostatics at the level of naval engineering. In such a case the interpretation will require the knowledge of a highly qualified expert.

IV. IMPLICATIONS IN SCIENCE AND TECHNOLOGY EDUCATION

In science and technology education, problem solvers and writers must attain productivity, functionality, ergonomics, and esthetics. Furthermore, we might request written products with the following qualities: scholar accuracy, technical soundness and pedagogical effectiveness [11]. Deliberate problem solving according to a given procedure, like TADIR or any other one, must neither obstruct creativity nor impede discoveries. What has been said of problem solving in Physics could be extended to other disciplines in natural science, mathematics and engineering.

For many centuries, communications were made from the mind of the author to the surface of printed pages in books and journals. Nowadays, mainly due to the progress in Information and Communication Technology (ICT), there is another surface: the computer screen [12,13]. Under these conditions, communications acquire the third dimension of electronic networks, the context where new and powerful scenarios for e-learning are available. The consequence is a kind of universal access to a broader and deeper universe that goes beyond the printed press [14,15,16].

It is in the previous contexts that the TADIR problem solving protocol is a useful instrument to interpret the solution of problems in physical systems. In the sequel of this paper (part II) we will see the solution of the buoyancy problem here considered but from the perspective of a writer. Also we will present the TADIR protocol from the perspective of Hermeneutics and apply it to a problem in a human learning system.

ACKNOWLEDGENMENTS

The author acknowledges information coming from the research projects on mathematics education by Nahina Dehesa.

REFERENCES

[1] Wilson, B. (*Systems: Concepts, Methodologies and Applications*, John Wiley, New York, 1990).
[2] Barojas, J., *Teacher training as collaborative problem solving*, Educational Technology and Society **6**, 1-10 (2003).
[3] Sharples, M. (*How we write*, Routledge, London, 1992).
[4] Karmiloff-Smith, A. (*Beyond Modularity: A Developmental Perspective on Cognitive Science*, MIT Press, Cambridge, 1992).
[5] Driver, R., Guesne, N. and Tiberghien, A. (*Children´s Ideas in Science*, Taylor and Francis, London, 1985*)*.
[6] Gadamer, H-G. (*Truth and Method*, Sheed and Ward, London, 1996).
[7] Duval, R., *Registres de répresentation sémiotique et functionement cognitif de la pensée (Registers of semiotic representations and cognitive functioning of thinking)*, Annales de Didactique et de Sciences Cognitives **5**, IREM Strasbourg (1993).
[8] Barojas, J. and Dehesa, N., *Mathematics for Social Scientists: Learning Cycles and Teaching Strategies*, Industry and Higher Education **15**, 269-277 (2001).
[9] Barojas, J, and Pérez y Pérez, R., *Physics and Creativity: Problem Solving and Learning Contexts.* Industry and Higher Education **15**, 431-439 (2001).
[10] Schoenfeld, A. (Learning to Think Mathematically: Problem Solving, Metacognition, and Sense Making in Mathematics, in Grouws, D. A. (Ed.), *Handbook of Research on Mathematics Teaching and Learning*, New York, 334-370, 1992).
[11] Stephenson, R. S. (The Open Course Model: Collaborative development of Online Content, in Santana, G. A. and Uskov, V. (Eds.) *Computers and Advanced Technology in Education*, IASTED ACTA Press, Zurich 150–153, 2002).
[12] Jonassen, D., Peck, H. K. L. and Wilson, B. G. (*Learning with technology: A Constructivist Perspective*, Englewood Cliffs, Merrill-Prentice Hall, New Jersey, 1999).
[13] Teasley, S. D. and Roschelle, J. (*Computers as cognitive tools*, Lawrence Erlbaum Associates, New Jersey, 1993).
[14] Forcier, R. C., (*The Computer as an Educational Tool: Productivity and Problem Solving*, Merrill - Prentice Hall, New Jersey, 1999).
[15] Novak, G. M., Patterson, E. T. (Gavrin, A. G. and Christian, W., *Just in Time Teaching. Blending Active Learning with Web Technology*, Prentice Hall, New Jersey, 1999).
[16] Linn, M.C. and Hsi, S. (*Computers, Teacher, Peers: Science learning partners*, Lawrence Erlbaum Associates, New Jersey, 2000).