Solar constant versus the electromagnetic spectrum

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Abstract

The electromagnetic spectrum emitted by the Sun and reaching the earth comprises of long-waves, radio waves, amateur, microwave, infrared, visible, ultraviolet, x-rays and gamma rays. The contributions of these waves in the evaluation of solar flux, solar constant and total solar irradiance are examined and the possible impacts as well as importance are discussed for the benefit of undergraduate students.

Keywords: Solar energy, solar constant, total solar irradiance, electromagnetic spectrum, wavelengths regions.

Resumen

El espectro electromagnético es emitido por el Sol y llega a la tierra compuesto de ondas largas, ondas de radio de aficionados, microondas, infrarrojo, visible, ultravioleta, rayos X y rayos gamma. Las contribuciones de estas ondas en la evaluación del flujo solar, la constante solar y la radiación solar total se examinan y su posible impacto, así también se discute la importancia en beneficio de los estudiantes universitarios.

Palabras claves: Energía solar, constante solar, radiación solar total, espectro electromagnético, regiones de longitudes de onda.

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I. INTRODUCCTION

The study and measurement of solar energy [1] is important in astrophysics, climatology, meteorology, atmospheric physics, geophysics, thermodynamics, automobile industry, agriculture, medicine as well as for the existence of life on the earth. This energy is electromagnetic in nature which is characterized by wavelength λ frequency v and velocity C satisfying the relation

$$C = \lambda v, 0 \leq \lambda \leq \infty, \infty \leq v \leq 0. \qquad (1)$$

The electromagnetic spectrum [2, 3] extends from below the radio frequencies at the long-wavelength end through gamma radiation at the short-wavelength end covering wavelengths from thousands of kilometers down to a fraction of the size of an atom. The amount of sunshine intercepted by the earth per second per unit area is called the solar constant [1, 4, 5] and its value is distributed among the various wavelength regions very much like the radiation emitted by a blackbody at a temperature 5776 K from the Sun [6]. It is well known [7] that the yearly mean solar irradiance on a surface oriented towards the Sun above the atmosphere is 1340 W/m^2.

The aim of the present paper is to study the contributions in the value of solar flux, solar constant and total solar irradiance from long-waves, radio waves, amateur, microwave, infrared, visible, ultraviolet, x-rays and gamma rays present in the electromagnetic spectrum of solar radiation for the benefit of undergraduate students. The study is important because the above mentioned waves of spectrum have varying impacts [1] on devices, plants, bacteria, human beings, insects, animals, diseases, climate, etc. Section 2 will take up the formulation part of the present work followed by the numerical work in Section 3. The conclusions and importance will be summarized in the last Section 4.

II. FORMULATION

A. Preliminaries

It is assumed that the Sun has as a uniform temperature T over its surface obeying the Planck's radiation law [8]

$$I(\lambda,T)d\lambda = \frac{\varepsilon(\lambda,T)A(2\pi hc^2)d\lambda}{\lambda^5\left[\exp(hc/\lambda kT)-1\right]} W, \qquad (2)$$

$I(\lambda,T)d\lambda$ is the power radiated between the wavelengths λ and $\lambda + d\lambda$, A is the surface area, ε is the emissivity and the constants h and k, respectively are Planck's constant and Boltzmann's constant. For simplicity, considering the Sun to be an ideal blackbody (ε=1) the solar flux Q emitted over all the wavelengths from the unit area $(A = 1m^2)$ of the Sun is

$$Q = \int_0^{\infty} I(\lambda,T)d\lambda = \sigma T^4 W / m^2 , \quad (3)$$

where σ is the Stefan-Boltzmann constant. When this flux reaches the earth [9] this is diluted by a factor

$$f = \frac{R_S^2}{d^2}, \quad (4)$$

giving rise to the value of solar constant as

$$S = \sigma T^4 f\, W/m^2 , \quad (5)$$

and the total solar irradiance [10] (*TSI*) on the earth would be

$$TSI = \sigma T^4 f \pi R_E^2 W . \quad (6)$$

Here R_S and R_E are the radii of the Sun and the earth, respectively and d is the yearly mean distance between them. Now the calculation of the contributions in the value of solar flux, solar constant and total solar irradiance from the above mentioned nine waves bands present in the solar radiation will be taken up.

B. Expression of solar flux, solar constant and TSI in the region λ_i and λ_f

The expression for the solar flux $Q(\lambda_i \rightarrow \lambda_f)$ emitted from an unit area of the Sun in between wavelengths λ_i and λ_f according to (3) would be

$$Q\left(\lambda_i \rightarrow \lambda_f\right) = \int_{\lambda_i}^{\lambda_f} I(\lambda,T)d\lambda\, W/m^2 . \quad (7)$$

This will be diluted by the factor f [*cf.* Eq. (4)] when it reaches the earth's atmosphere. Therefore its contribution in the value of solar constant and total solar irradiance in the said wavelengths region would be

$$S\left(\lambda_i \rightarrow \lambda_f\right) = Q\left(\lambda_i \rightarrow \lambda_f\right) f\, W/m^2 , \quad (8)$$

$$TSI = \left(\lambda_i \rightarrow \lambda_f\right) = S\left(\lambda_i \rightarrow \lambda_f\right) f .\pi R_E^2 W . \quad (9)$$

The above partial solar constant and partial total solar irradiance when summed over all the wavelength regions would give

$$S = \sum_{wavelengthregions} S\left(\lambda_i \rightarrow \lambda_f\right) W/m^2 , \quad (10)$$

$$TSI = \sum_{wavelengthregions} TSI\left(\lambda_i \rightarrow \lambda_f\right) W . \quad (11)$$

Now the solution of the integral (7) will be taken up.

C. Solution of the integral (7)

The analytical solution of the integral (3) is possible whereas that of (7) was achieved approximately by Agrawal, Leff and Menon [8]. Following their procedure and notations one can write

$$Q\left(\lambda_i \rightarrow \lambda_f\right) = F\left(0, \lambda_f T\right) - F\left(0, \lambda_i T\right), \quad (12)$$

where

$$F\left(0,\lambda_0 T\right) = \frac{15\left(hc\right)^4}{\left(\pi k\right)^4} \int_0^{\lambda_0 T} \frac{dx}{x^5 \left[\exp\left(hc/kx\right) - 1\right]}, \quad (13)$$

$\lambda_0 \equiv \lambda_i, \lambda_f$ and $x \equiv \lambda T$. Making the variable change $y = hc / kx$ the above expression takes the form

$$F\left(0,\lambda_0 T\right) = \frac{15}{\pi^4} \int_{hc/k\lambda_0 T}^{\infty} \frac{y^3 dy}{[\exp\left(y\right) - 1]} . \quad (14)$$

In the low and high wavelength limits a vanishingly small fraction of the power is emitted. This follows mathematically from Eqs. (12) and (13) because for $\lambda_0 \rightarrow 0, F(0,\lambda_0 T) \rightarrow 0$ and for $\lambda_0 \rightarrow \infty, F\left(0,\lambda_0 T\right) \rightarrow 1$ Here the integrand in Eq. (14) can be written as $y^3 \exp\left(-y\right)[1 - \exp\left(-y\right)]^{-1}$ and $[1 - \exp\left(-y\right)]^{-1}$ can be expanded as the infinite series $1 + \exp\left(-y\right) + \exp\left(-2y\right) + ...$ Term by term integration of this series leads to the result

$$F\left(0,\lambda_0 T\right) = \frac{15}{\pi^4} \sum_{n=1}^{\infty} \exp\left(-ny_0\right)[\frac{y_0^3}{n} + \frac{3y_0^2}{n^2} + \frac{6y_0}{n^3} + \frac{6}{n^4}], \quad (15)$$

where $y_0 \equiv hc / kT\lambda_0$. This series converges rapidly and lends itself to straightforward evaluation by computer for a finite number of terms; truncation at n = 5 gives excellent results.

III. NUMERICAL WORK

Taking the numerical values [7, 8] as

$$R_S = 6.96x10^8\, m,\ R_E = 6.37x10^6\, m,\ d = 1.5x10^{11}\, m$$

$$h = 6.63x10^{-34} Js,\ k = 1.38x10^{-23} J/K,$$
$$c = 3.0x10^{8} m/s,$$
$$\sigma = 5.67051x10^{-8} Wm^{-2}K^{-4}, \qquad (16)$$

the partial solar fluxes [*cf.* Eq. (7)], partial solar constants [*cf.* Eq. (8)] and partial solar irradiances [*cf.* Eq. (9)] between all the wavelengths regions are computed by making use of the relations (12) and (15). It is well known [2, 3] that there is no sharp demarcation between the various regions of the electromagnetic spectrum a finite value in between the wavelengths has been assumed for the purpose of estimating $Q(\lambda_i \rightarrow \lambda_f)$, $S(\lambda_i \rightarrow \lambda_f)$ and $TSI(\lambda_i \rightarrow \lambda_f)$.

Table I. Tabulation of the calculated values of solar flux, solar constant, and total solar irradiance using the formulae (7), (8) and (9), respectively along with the relations (12) and (15) over the classified regions of electromagnetic spectrum radiated from the Sun.

Name	Wavelengths	Solar flux $Q(\lambda_i \rightarrow \lambda_f)$ W/m^2 [*cf.* Eq. (7)]	$S(\lambda_i \rightarrow \lambda_f)$ W/m^2 [*cf.* Eq. (8)]	$TSI(\lambda_i \rightarrow \lambda_f)$ W [*cf.* Eq. (9)]	Corresponding Percentage	Role
Gamma-rays	$0 \rightarrow 10^{-14}$ m	~ 0	~ 0	~ 0	~0	Practically absent
X-rays	$10^{-14} \rightarrow 10^{-10}$ m	~ 0	~ 0	~ 0	~0	Practically absent
Ultra-violet	$10^{-10} \rightarrow 400x10^{-9}$ m	7.6x10^6	164.0	2.1x10^{16}	12.1 %	Formation of Ozone layer
Visible	$400x10^{-9} \rightarrow 700x10^{-9}$ m	2.3x10^7	495.0	6.3x10^{16}	36.7 %	Illumination and Photosynthesis
Infrared	$700x10^{-9} \rightarrow 10^{-3}$ m	3.2x10^7	690.6	8.8x10^{16}	51.2 %	Heating the earth
Microwave	$10^{-3} \rightarrow 0.1$ m	0.38	8.1x10^{-6}	1.0x10^9	Negligible	Negligible
Amateur	$0.1 \rightarrow 10^{2}$ m	2.6	5.6x10^{-5}	7.2x10^9	Negligible	Negligible
Radio waves	$10^{2} \rightarrow 10^{4}$ m	1.4	3.0x10^{-5}	3.8x10^9	Negligible	Reflected back
Long waves	$10^{4} \rightarrow \infty$ m	0.02	3.7x10^{-7}	4.7x10^7	Negligible	Reflected back
	TOTAL	6.3x10^7	1349.6	1.7x10^{17}		

The results are reported in columns 3-5 of Table I along with the corresponding percentage contribution is mentioned in the 6th column. The major role played by these wavelengths regions are also mentioned in 7th column. The last row of the table shows the sum of the each column 3-5 which corresponds to the solar flux *Q*, solar constant *S* and total solar irradiance *TSI*, respectively.

IV. CONCLUSIONS

The conclusions and importance of the present work may be summarized as follows.

- The gamma and X-rays are practically absent in the solar flux and hence they do not show up their presence in the earth's atmosphere (rows 2 and 3 in Table I).
- The ultraviolet ray's shows 12.1 % presence in the Sun radiation and its major role [11] is in the formation and maintenance of Ozone layer in our earth's atmosphere. This is also responsible for the production of vitamin D in our body. The Ozone layer saves us from the ultraviolet radiation as well otherwise the life on earth would suffer [11] from Sun burn, skin cancer, skin aging effect, etc. (row 4)
- The contribution of visible part of radiation is 36.7 % (row 5) which is responsible for the illumination of our globe [12]. This range is also responsible for the photosynthesis but its value 6.3x10^{16} Watts (column 5, row 5) over the globe is quite large compared to the reported [10] available value 40x10^{12} Watts. This is because the whole visible range does not participate uniformly in the photosynthesis [13].
- The major contribution of 51.2 % (row 6) comes from the infrared region and this is basically responsible for the heating of the globe.
- The contributions of microwaves, amateur waves, radio waves and long waves are negligible (rows 6-9). The last two waves are reflected [14] back from over the atmosphere.
- The last 10th row lists the total of all the said radiations and it shows that the values of solar constant *S* and total solar irradiance *TSI* are consistent with the reported values [7, 10] in the literature.
- The present formulation can be utilized for finding out the available wattage for a desired wavelengths region from the Sun to study its absorption characteristics as it pass through the atmosphere.
- The present approach can also be applied for any other star/body at temperature *T*.
- The present work is pedagogic in nature because the results can easily be reproduced by the undergraduate students.

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